

SIMPLIFIED NEUTROSOPHIC WEIGHTED AVERAGE OPERATORS AND THEIR APPLICATION TO E-COMMERCE

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ABSTRACT. *With respect to the simplified neutrosophic multicriteria decision making problems, some basic concepts and operational laws of the simplified neutrosophic values are introduced. Then, we develop some simplified neutrosophic weighted average operators called the simplified neutrosophic weighted arithmetic average (SNWAA) operator, the simplified neutrosophic ordered weighted average (SNOWA) operator and the simplified neutrosophic hybrid weighted average (SNHWA) operator. We study some of their characteristic, and prove that the SNWAA operator and the SNOWA operator are two special cases of the SNHWA operator. Based on the proposed aggregation operators, a multicriteria decision making method is established in which the evaluation values of alternatives with respect to criteria are represented by the form of the simplified neutrosophic sets (SNSs). Finally, a practical application of the developed method is given.*

Keywords: Neutrosophic set, Simplified neutrosophic set, Aggregation operator, E-commerce

1. Introduction. To deal with the incomplete information, the indeterminate information and the inconsistent information simultaneously, Smarandache [1] proposed the neutrosophic set (NS), which generalizes many famous sets, such as fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set. The most important characteristic of the neutrosophic set is that its indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership, and falsity-membership are independent. This characteristic is very important in many applications such as in information fusion in which the data are combined from different sensors. Since its appearance, the NS has been studied by many scholars and applied in various fields, such as image thresholding [2], image denoise applications [3], and multicriteria decision making [4-7].

From scientific or engineering point of view, the neutrosophic set needs to be specified. Otherwise, it will be difficult to apply the NS to the real applications. Therefore, many reduced neutrosophic sets are developed, such as the interval neutrosophic set (INS) [4], the single valued neutrosophic set (SVNS) [5,6], and the simplified neutrosophic set (SNS) [7]. In this paper, we focus on the application of the SNS in multicriteria decision making (MCDM) problem. In any MCDM, the final solution must be obtained from the synthesis of performance degrees of criteria [6], which can be accomplished by the aggregation of information. In this process, the aggregation measures and operators are fundamental. Ye [4] defined the Hamming and Euclidean distances between INSs and developed the similarity measures between INSs, and applied the similarity measures to MCDM in which criterion values with respect to alternatives are evaluated by the form of INVs. In [5], Ye

presented the information energy of SVNS, correlation of SVNSs, correlation coefficient of SVNSs, and weighted correlation coefficient of SVNSs based on the extension of the correlation of intuitionistic fuzzy sets, and then applied them to SVNS decision making problems. Ye [6] proposed a single valued neutrosophic cross-entropy of SVNSs, which is an extension of the cross entropy of fuzzy sets. Then, an MCDM method is developed in which criteria values for alternatives are SVNSs. In [7], Ye defined some aggregation operators, based on which, a multicriteria decision-making method is established.

In this paper, we shall go on studying the application of the SNSs in MCDM. To do so, the remainder of the paper is set out as follows. Section 2 gives some basic knowledge of SNS and the aggregation operators. Section 3 develops three simplified neutrosophic weighted aggregation operators named the SNWAA operator, the SNOWA operator and the SNHWA operator. In Section 4, we apply our proposed operators to MCDM under simplified neutrosophic environments. In Section 5, a practical example is provided to illustrate the use of our proposed method. The paper ends with some conclusions in Section 6.

2. Some Concepts of Neutrosophic Sets. Neutrosophic set is a part of neutrosophy, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra [1], and is a powerful general formal framework, which generalizes many classical sets from philosophical point of view. Smarandache [1] developed the following definition of a neutrosophic set.

Definition 2.1. [1] *Let X be a space of points(objects), with a generic element in X denoted by x . A neutrosophic set (NS) A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is $T_A(x) : X \rightarrow]0^-, 1^+[$, $I_A(x) : X \rightarrow]0^-, 1^+[$, and $F_A(x) : X \rightarrow]0^-, 1^+[$.*

It is noted that there is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.2. [1] *A neutrosophic set A is contained in the other neutrosophic set B , $A \subseteq B$ if and only if $\inf T_A(x) \leq \inf T_B(x)$, $\sup T_A(x) \leq \sup T_B(x)$, $\inf I_A(x) \geq \inf I_B(x)$, $\sup I_A(x) \geq \sup I_B(x)$, $\inf F_A(x) \geq \inf F_B(x)$, and $\sup F_A(x) \geq \sup F_B(x)$ for every x in X .*

In [7], Ye reduced NSs of nonstandard intervals into a kind of SNSs of standard intervals, which is defined as follows.

Definition 2.3. [7] *Let X be a space of points(objects), with a generic element in X denoted by x . A simplified neutrosophic set (SNS) A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. If the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are singleton subintervals/subsets in the real standard $[0, 1]$, that is $T_A(x) : X \rightarrow [0, 1]$, $I_A(x) : X \rightarrow [0, 1]$, and $F_A(x) : X \rightarrow [0, 1]$. Then, an SNS A is denoted by*

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle | x \in X \}.$$

In the following, we shall use SNS whose $T_A(x)$, $I_A(x)$ and $F_A(x)$ values are single points in the real standard $[0, 1]$ instead of subintervals/subsets in the real standard $[0, 1]$.

Definition 2.4. [1] *An SNS A is contained in the other SNS B , $A \subseteq B$ if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, and $F_A(x) \geq F_B(x)$ for every x in X .*

Based on the operational laws of the intuitionistic fuzzy sets, in the following, we shall define some operational laws of SNSs.

Definition 2.5. Let $A = \langle T_A(x), I_A(x), F_A(x) \rangle$, $B = \langle T_B(x), I_B(x), F_B(x) \rangle$ be two SNSs, and $\lambda > 0$, and then

$$A + B = \langle T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x) \rangle. \quad (1)$$

$$\lambda A = \langle 1 - (1 - T_A(x))^\lambda, I_A(x)^\lambda, F_A(x)^\lambda \rangle. \quad (2)$$

When $X = \{x\}$, we call $A = \langle T_A(x), I_A(x), F_A(x) \rangle$ a simplified neutrosophic value (SNV), denoted as $A = \langle T_A, I_A, F_A \rangle$.

Definition 2.6. Let $A = \langle T_A, I_A, F_A \rangle$ be an SNV, and then its score function $S(A)$ and accuracy function $H(A)$ are defined as follows:

$$S(A) = \frac{1}{3}(T_A - pI_A - F_A), \quad (3)$$

$$H(A) = \frac{1}{3}(T_A + pI_A + F_A), \quad (4)$$

where $p \in [0, 1]$, which is decided by the decision maker. Specially let $p = 1$, $p = 0$, $p = 0.5$, respectively, then we have three choosing schemes: pessimistic, optimistic, and neutral.

Without loss of generality, in the following, we shall set $p = 1$. Obviously, the larger the value of $H(A)$ is, the more the degree of accuracy of the SNV A is. Based on the score function S and the accuracy function H , we shall give an order relation between two SNVs, which is defined as follows:

Definition 2.7. Let A and B be two SNVs, and then

- If $S(A) < S(B)$, then A is smaller than B , denoted by $A < B$.
- If $S(A) = S(B)$, then
 - (1) If $H(A) = H(B)$, then A and B represent the same information, denoted by $A = B$.
 - (2) If $H(A) < H(B)$, then A is smaller than B , denoted by $A < B$.

3. Some Simplified Neutrosophic Weighted Average Operators. In this section, we shall introduce some simplified neutrosophic weighted average operators to aggregate the simplified neutrosophic information. First, based on the classical weighted average operator, we can give a simplified neutrosophic weighted arithmetic average (SNWAA) operator.

Definition 3.1. Let A_j ($j = 1, 2, \dots, n$) be a collection of SNSs. The SNWAA operator is defined by

$$\text{SNWAA}(A_1, A_2, \dots, A_n) = \sum_{j=1}^n \lambda_j A_j, \quad (5)$$

where $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)^\top$ is the weight vector of A_j ($j = 1, 2, \dots, n$), $\lambda_j \in [0, 1]$ and $\sum_{j=1}^n \lambda_j = 1$.

Based on the operational laws of SNVs described in Definition 2.5, we can obtain the following Theorem 3.1.

Theorem 3.1. For a collection of SNVs $A_j = \langle T_{A_j}, I_{A_j}, F_{A_j} \rangle$ ($j = 1, 2, \dots, n$), we have

$$\text{SNWAA}(A_1, A_2, \dots, A_n) = \left\langle 1 - \prod_{j=1}^n (1 - T_{A_j})^{\lambda_j}, \prod_{j=1}^n I_{A_j}^{\lambda_j}, \prod_{j=1}^n F_{A_j}^{\lambda_j} \right\rangle \quad (6)$$

Proof: The proof is omitted for succinct.

Example 3.1. Let $A_1 = \langle 0.3, 0.6, 0.2 \rangle$, $A_2 = \langle 0.2, 0.7, 0.1 \rangle$, $A_3 = \langle 0.9, 0.1, 0 \rangle$ and $A_4 = \langle 0.3, 0.1, 0.6 \rangle$ be a collection of SNVs. Suppose that $\lambda = (0.3, 0.2, 0.3, 0.2)^\top$, and then by (6), we can get the aggregated result of A_j ($j = 1, 2, 3, 4$): $\text{SNWAA}(A_1, A_2, A_3, A_4) = \langle 0.5990, 0.2526, 0 \rangle$.

The SNWAA operator has the following properties, and their proofs are standard, which are omitted here.

Theorem 3.2. (Idempotency) If $A_j = A = \langle T_A, I_A, F_A \rangle$ ($j = 1, 2, \dots, n$), then $\text{SNWAA}(A_1, A_2, \dots, A_n) = A$.

Theorem 3.3. (Monotonicity) If $A_j \subseteq A_j^*$ ($j = 1, 2, \dots, n$), then $\text{SNWAA}(A_1, A_2, \dots, A_n) \leq \text{SNWAA}(A_1^*, A_2^*, \dots, A_n^*)$.

Theorem 3.4. (Boundedness) Let A_j ($j = 1, 2, \dots, n$) be a collection of SNVs, and let $A^- = \langle \min_j T_{A_j}, \max_j I_{A_j}, \max_j F_{A_j} \rangle$, and $A^+ = \langle \max_j T_{A_j}, \min_j I_{A_j}, \min_j F_{A_j} \rangle$, then $A^- \leq \text{SNWAA}(A_1, A_2, \dots, A_n) \leq A^+$.

Notice that the SNWAA operator only considers the importance of the given arguments, while it cannot emphasize the importance of their ordered positions. By the famous OWA [8] operator, we will develop following a simplified neutrosophic ordered weighted average (SNOWA) operator, which can overcome the aforementioned issue.

Definition 3.2. Let A_j ($j = 1, 2, \dots, n$) be a collection of SNVs. A simplified neutrosophic ordered weighted average (SNOWA) operator of dimension n is defined by

$$\text{SNOWA}(A_1, A_2, \dots, A_n) = \sum_{j=1}^n w_j A_{\sigma(j)}, \quad (7)$$

where $(\sigma(1), \sigma(2), \dots, \sigma(n))$ is a permutation of $(1, 2, \dots, n)$, such that $A_{\sigma(j-1)} \geq A_{\sigma(j)}$ for all $j = 2, 3, \dots, n$. Furthermore, $w = (w_1, w_2, \dots, w_n)^\top$ is the weight vector of the ordered positions of $A_{\sigma(j)}$ ($j = 1, 2, \dots, n$) such that $w_j \geq 0$, $\sum_{j=1}^n w_j = 1$.

Based on the operational laws of SNVs, we have the following theorem.

Theorem 3.5. For a collection of SNVs $A_j = \langle T_{A_j}, I_{A_j}, F_{A_j} \rangle$ ($j = 1, 2, \dots, n$), we have

$$\text{SNOWA}(A_1, A_2, \dots, A_n) = \left\langle 1 - \prod_{j=1}^n (1 - T_{A_{\sigma(j)}})^{w_j}, \prod_{j=1}^n I_{A_{\sigma(j)}}^{w_j}, \prod_{j=1}^n F_{A_{\sigma(j)}}^{w_j} \right\rangle \quad (8)$$

Example 3.2. Let $A_1 = \langle 0.5, 0.2, 0.1 \rangle$, $A_2 = \langle 0.4, 0.3, 0.2 \rangle$, $A_3 = \langle 0.5, 0.3, 0.2 \rangle$ and $A_4 = \langle 0.7, 0.2, 0.2 \rangle$ be a collection of SNVs, and then $S(A_1) = 0.0667$, $S(A_2) = -0.3333$, $S(A_3) = 0$, $S(A_4) = 0.1$. Therefore, $\sigma(1) = 4$, $\sigma(2) = 1$, $\sigma(3) = 3$, $\sigma(4) = 2$. Suppose that $w = (0.2, 0.3, 0.3, 0.2)^\top$, and then by (8), we can get the aggregated result of A_j ($j = 1, 2, 3, 4$):

$$\text{SNOWA}(A_1, A_2, A_3, A_4) = \langle 0.5318, 0.2449, 0.1625 \rangle.$$

Similar to the classical OWA operator, the SNOWA operator has the following desired property besides the idempotency, monotonicity, and boundedness properties.

Theorem 3.6. (Commutativity) $\text{SNOWA}(A_1, A_2, \dots, A_n) = \text{SNOWA}(A'_1, A'_2, \dots, A'_n)$, where $(A'_1, A'_2, \dots, A'_n)$ is any permutation of (A_1, A_2, \dots, A_n) .

In the following, motivated by the hybrid weighted aggregation operators in [9], we shall propose a simplified neutrosophic hybrid weighted average (SNHWA) operator which weights both the given arguments and their ordered position.

Definition 3.3. Let A_j ($j = 1, 2, \dots, n$) be a collection of SNVs. A simplified neutrosophic hybrid weighted average (SNHWA) operator is a mapping SNHWA, defined by an associated weight vector $w = (w_1, w_2, \dots, w_n)^T$ with $w_j \geq 0$, $\sum_{j=1}^n w_j = 1$, such that

$$\text{SNHWA}(A_1, A_2, \dots, A_n) = \frac{\sum_{j=1}^n \lambda_{\sigma(j)} w_j A_{\sigma(j)}}{\sum_{j=1}^n \lambda_{\sigma(j)} w_j}, \quad (9)$$

where $(\sigma(1), \sigma(2), \dots, \sigma(n))$ is a permutation of $(1, 2, \dots, n)$, such that $A_{\sigma(j-1)} \geq A_{\sigma(j)}$ for all $j = 2, 3, \dots, n$. Furthermore, $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)^T$ is the weighting vector of the SNVs A_j ($j = 1, 2, \dots, n$), with $\lambda_j \in [0, 1]$ and $\sum_{j=1}^n \lambda_j = 1$.

Theorem 3.7. For a collection of SNVs $A_j = \langle T_{A_j}, I_{A_j}, F_{A_j} \rangle$ ($j = 1, 2, \dots, n$), we have

$$\text{SNHWA}(A_1, A_2, \dots, A_n) = \left\langle 1 - \prod_{j=1}^n (1 - T_{A_{\sigma(j)}})^{\frac{\lambda_{\sigma(j)} w_j}{\sum_{j=1}^n \lambda_{\sigma(j)} w_j}}, \right. \\ \left. \prod_{j=1}^n I_{A_{\sigma(j)}}^{\frac{\lambda_{\sigma(j)} w_j}{\sum_{j=1}^n \lambda_{\sigma(j)} w_j}}, \prod_{j=1}^n F_{A_{\sigma(j)}}^{\frac{\lambda_{\sigma(j)} w_j}{\sum_{j=1}^n \lambda_{\sigma(j)} w_j}} \right\rangle \quad (10)$$

Example 3.3. Let $A_1 = \langle 0.6, 0.3, 0.2 \rangle$, $A_2 = \langle 0.3, 0.2, 0.3 \rangle$, $A_3 = \langle 0.6, 0.3, 0.1 \rangle$ and $A_4 = \langle 0.4, 0.3, 0.1 \rangle$ be a collection of SNVs, then $S(A_1) = 0.0333$, $S(A_2) = -0.0667$, $S(A_3) = 0.0667$, $S(A_4) = 0$. Therefore, $\sigma(1) = 3$, $\sigma(2) = 1$, $\sigma(3) = 4$, $\sigma(4) = 2$. Suppose that $w = (0.2, 0.3, 0.3, 0.2)^T$ and $\lambda = (0.1, 0.2, 0.3, 0.4)^T$, and then by (10), we can get the aggregated result of A_j ($j = 1, 2, 3, 4$): $\text{SNHWA}(A_1, A_2, A_3, A_4) = \langle 0.4464, 0.2694, 0.1927 \rangle$.

Similar to the SNOWA operator, the SNHWA operator is commutative, idempotent, monotonic and bounded. Furthermore, the SNWAA operator and the SNOWA operator are two special cases of the SNHWA operator.

4. An Approach to Multicriteria Decision Making Based on the SNHWA Operator. For a multicriteria decision making (MCDM) problem with simplified neutrosophic information, let $A = \{A_1, A_2, \dots, A_m\}$ be a set of m alternatives, and $G = \{G_1, G_2, \dots, G_n\}$ be the set of n criteria. The decision maker determines the importance degrees λ_j ($j = 1, 2, \dots, n$) for the relevant criteria according to his/her preferences. Meanwhile, since different criteria may have different focuses and advantages, to reflect this issue, the decision maker also gives the ordering weights w_j ($j = 1, 2, \dots, n$) for different criteria. Suppose that $R = (\alpha_{ij})_{m \times n}$ is the decision matrix, where $\alpha_{ij} = \langle T_{\alpha_{ij}}, I_{\alpha_{ij}}, F_{\alpha_{ij}} \rangle$ is a preference value, which takes the form of the SNV, given by the decision maker for the alternative $A_i \in A$ with respect to the criterion $G_j \in G$. Then, based on the SNHWA operator, we will propose an approach to solve this multicriteria decision making problem, which involves the following steps.

Step 1. Utilize the SNHWA operator:

$$\alpha_i = \text{SNHWA}(\alpha_{i1}, \alpha_{i2}, \dots, \alpha_{in}) = \frac{\sum_{j=1}^n \lambda_{\sigma(ij)} w_j \alpha_{\sigma(ij)}}{\sum_{j=1}^n \lambda_{\sigma(ij)} w_j}, \quad i = 1, 2, \dots, m, \quad (11)$$

to aggregate all the preference values, α_{ij} ($j = 1, 2, \dots, n$), of the i th row, and get the overall preference value α_i , which is correspondent to the alternative A_i .

Step 2. Calculate the score value $S(\alpha_i)$ and the accuracy value $H(\alpha_i)$ of α_i ($i = 1, 2, \dots, m$) by Definition 2.7.

Step 3. Get the priority of the alternatives A_i ($i = 1, 2, \dots, m$) by ranking $S(\alpha_i)$ and $H(\alpha_i)$ ($i = 1, 2, \dots, m$), and select the best one(s).

Step 4. End.

5. Illustrative Example. In this section, we present an empirical case study of evaluating the customer satisfaction of e-commerce websites (adapted from [10]). The project's aim is to evaluate the best e-commerce website from the different companies of e-commerce website, which provides alternatives of e-commerce websites to e-commerce enterprise. The customer satisfaction of five possible alternatives of e-commerce websites A_i ($i = 1, 2, \dots, 5$) is evaluated. Assume that an e-commerce enterprise newly identified an investment with e-commerce websites, and in order to maximize the expected profit, we need to determine the customer satisfaction of the five e-commerce websites so as to choose the optimal one. The investment company must take a decision according to the following five attributes: G_1 is the platform characteristics of e-commerce website; G_2 is the store characteristics of e-commerce website; G_3 is the pre-sale and after-sale service of e-commerce website; G_4 is the trading pay and logistics distribution of e-commerce website. The five alternatives A_i ($i = 1, 2, \dots, 5$) are to be evaluated using the SNVs under the above four attributes. The decision matrix is listed in the following matrices $D = (\alpha_{ij})_{5 \times 4}$ as follows:

$$D = \begin{bmatrix} \langle 0.4, 0.2, 0.3 \rangle & \langle 0.5, 0.1, 0.3 \rangle & \langle 0.2, 0.2, 0.5 \rangle & \langle 0.6, 0.1, 0.2 \rangle \\ \langle 0.6, 0.1, 0.2 \rangle & \langle 0.3, 0.2, 0.2 \rangle & \langle 0.7, 0.1, 0.2 \rangle & \langle 0.5, 0.3, 0.2 \rangle \\ \langle 0.3, 0.4, 0.1 \rangle & \langle 0.5, 0.2, 0.3 \rangle & \langle 0.5, 0.1, 0.2 \rangle & \langle 0.6, 0.3, 0.1 \rangle \\ \langle 0.7, 0.0, 0.1 \rangle & \langle 0.6, 0.1, 0.2 \rangle & \langle 0.4, 0.3, 0.2 \rangle & \langle 0.5, 0.2, 0.1 \rangle \\ \langle 0.4, 0.3, 0.3 \rangle & \langle 0.7, 0.2, 0.1 \rangle & \langle 0.3, 0.2, 0.3 \rangle & \langle 0.6, 0.3, 0.2 \rangle \end{bmatrix}$$

The information about the attribute weight is determined by the decision maker as $\lambda = (0.2, 0.1, 0.3, 0.4)^\top$. In addition, the ordered position weight vector is $w = (0.2, 0.3, 0.3, 0.2)^\top$. In the following, we utilize the approach developed to get the most desirable e-commerce website(s).

Step 1. We can obtain the weighted hybrid average value α_i for A_i ($i = 1, 2, \dots, 5$) by (11):

$$\alpha_1 = \langle 0.3967, 0.1634, 0.3324 \rangle, \alpha_2 = \langle 0.5004, 0.1902, 0.2000 \rangle, \alpha_3 = \langle 0.4611, 0.2472, 0.1646 \rangle, \\ \alpha_4 = \langle 0.5255, 0, 0.1374 \rangle, \alpha_5 = \langle 0.4651, 0.2449, 0.2375 \rangle.$$

Step 2. Compute the score values $S(\alpha_i)$ ($i = 1, 2, \dots, 5$):

$$S(\alpha_1) = -0.0330, S(\alpha_2) = 0.0367, S(\alpha_3) = 0.0164, \\ S(\alpha_4) = 0.1294, S(\alpha_5) = -0.0058.$$

Step 3. Rank all the alternatives A_i ($i = 1, 2, \dots, 5$) in accordance with the scores α_i ($i = 1, 2, \dots, 5$): $A_4 \succ A_2 \succ A_3 \succ A_5 \succ A_1$. Thus the most desirable alternative is A_4 .

6. Conclusion. In this paper, we defined some operational laws, score function and accuracy function of simplified neutrosophic values (SNVs). Then, we developed three aggregation operators for SNVs. We also discussed their desirable properties and relationships. The proposed aggregation operators were applied to multicriteria decision making problems under the SNV environment. Finally, a numerical example is provided to illustrate the application of the developed approach. In the future, we expect to develop other types of aggregation operators, such as geometric average operator, and induced aggregation operator. In addition, we shall apply the simplified neutrosophic aggregation operators to solve practical applications in other areas such as pattern recognition, and medical diagnoses.

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REFERENCES

- [1] F. Smarandache, *A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic*, American Research Press, Rehoboth, 1999.
- [2] H. D. Cheng and Y. Guo, A new neutrosophic approach to image thresholding, *New Mathematics and Natural Computation*, vol.4, no.3, pp.291-308, 2008.
- [3] Y. Guo, H. D. Cheng, Y. Zhang and W. Zhao, A new neutrosophic approach to image denoising, *New Mathematics and Natural Computation*, vol.5, no.3, pp.653-662, 2009.
- [4] J. Ye, Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making, *Journal of Intelligent and Fuzzy Systems*, vol.26, no.1, pp.165-172, 2014.
- [5] J. Ye, Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment, *International Journal of General Systems*, vol.42, no.4, pp.386-394, 2013.
- [6] J. Ye, Single valued neutrosophic cross-entropy for multicriteria decision making problems, *Applied Mathematical Modelling*, vol.38, no.3, pp.1170-1175, 2013.
- [7] J. Ye, A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets, *Journal of Intelligent and Fuzzy Systems*, vol.26, no.5, pp.2459-2466, 2013.
- [8] R. R. Yager, On ordered weighted averaging aggregation operators in multicriteria decision making, *IEEE Transactions on Systems, Man and Cybernetics*, vol.18, no.1, pp.183-190, 1988.
- [9] J. Lin and Y. Jiang, Some hybrid weighted averaging operators and their application to decision making, *Information Fusion*, vol.16, pp.18-28, 2014.
- [10] L. Y. Zhou, X. F. Zhao and G. W. Wei, Hesitant fuzzy Hamacher aggregation operators and their application to multiple attribute decision making, *Journal of Intelligent and Fuzzy Systems*, vol.26, no.6, pp.2689-2699, 2014.