ICIC Express Letters
Volume **10**, Number **1**, January **2016**
 ISSN 1881-803X
pp. **43–49**

APPLICATION OF FUZZY ANT COLONY ALGORITHM TO ROBOTICS ARM INVERSE KINEMATICS PROBLEM

Ming Zhao and Yong Dai

School of Electronic and Information Engineering
University of Science and Technology Liaoning
No. 185, Qianshan Middle Road, Anshan 114051, P. R. China
as_zmyli@163.com; 578916135@qq.com

Received June 2015; accepted September 2015

Abstract. *For solving inverse kinematics problem of robotics serial manipulators, a solution of fuzzy control ant colony algorithm is put forward. That is to say, the inverse kinematics problem is transformed into the multidimensional function maximum problem, and the independent variables of multidimensional function are gridded. Fuzzy control ant colony algorithm makes a 'path-planning' on the gridded multidimensional domain of the function, which is transformed into a new multidimensional function optimization algorithm. The improvement of the conventional ant colony algorithm with fuzzy control intelligent pheromone updating method successfully increases convergence, accuracy and heuristic of the algorithm. A practical inverse kinematics experiment of 6 freedom structure robotics manipulator is made, of which goal forward matrix to real forward matrix* $\left\|A^6 - T\right\|_1$ *can be below 0.1 precision in 100 iterations, and the deviation between them will be less with increased iterations. Therefore, practice has proved that the feasible method of solving robotics multi-freedom manipulators inverse kinematics problem is presented by this paper.*
Keywords: Inverse kinematics, Fuzzy control ant colony algorithm, Multidimensional function optimization, Robotics manipulator, Independent variables gridded

1. **Introduction.** Research of robot arm control is to study the two problem – forward kinematics and inverse kinematics. There are a variety of forward kinematics problem solving methods, and one of them that is the most simple and effective is Denavit and Hartenberg Notation [1,2], which is a general solving method proposed in 1955. However, there are ever-changing methods of solving inverse kinematics problem and different structure of the robot arm even has its own unique solution, still not having one general approach. Inverse kinematics problem is different from forward kinematics problem, mainly because of the nonlinear kinematic equations, it is difficult to obtain closed-form solutions, as well as the existence of solutions and the problem of multiple solutions. Common methods of solving inverse kinematics problem are geometric method, the matrix inversion method, the Lie algebra method and intelligent optimization algorithms, etc. The first three methods for most of complex mechanical structure of manipulators, the solution cannot be found because of its algorithm limitation [1,3]. Intelligent optimization algorithm has been in high hopes, and some intelligent algorithms become examples of the successful resolution of the robot arm inverse kinematics problem, such as genetic algorithm [4,5], and neural network algorithm [6,7]. However, genetic algorithm is essentially a non-heuristic optimization algorithm, and non-heuristic offspring cannot solve complex multi-DOF robot arm inverse kinematics problem in ideal. Neural network algorithm has limitations for the complex structure of robot arm in data acquisition and training, and it is difficult to train neural network function accurately. Therefore, this paper presents a new intelligent optimization algorithm, which uses fuzzy control [8,9] to improve ant colony algorithm [10,11] to solve the problem of inverse kinematics.

2. Fuzzy Ant Colony Algorithm for the Inverse Kinematics.

2.1. Establishment of fuzzy ant colony algorithm objective function. Conversion Formula (1) from D-H Notation method to the Cartesian coordinate system based on rotation and translation theory [1] is:

$$T_i^{i-1} = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i} = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where s_{θ_i} refers to $\sin(\theta_i)$, c_{θ_i} refers to $\cos(\theta_i)$, s_{α_i} refers to $\sin(\alpha_i)$, and c_{α_i} refers to $\cos(\alpha_i)$. The target robot arm joint structure is known, so D-H Notation parameters are known. According to Formula (1), n degrees of freedom forward kinematics equation can be built as Equation (2) shows

$$A^n = T_1^0(\theta_1) \cdot T_2^1(\theta_2) \cdot T_3^2(\theta_3) \cdots T_n^{n-1}(\theta_n) \tag{2}$$

Set target matrix T as follows:

$$T = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Solving inverse kinematics problem needs to meet two conditions, namely: (1) θ joints meet the actual range of motion allowed; (2) to find $\theta = [\theta_1, \theta_2, \theta_3, \cdots, \theta_n]^T$ makes A^n so close to target matrix T. According to the above conditions, the establishment of a bounded continuous objective function F is:

$$F = \frac{1}{1 + \|A^n - T\|_1} + C_1 + C_2 + \cdots + C_n; \text{ if } \theta_n \in \theta_n^{range} \text{ then } C_n = 1, \text{ or } C_n = 0 \tag{4}$$

where θ_n^{range} is joint n actual permit range of motion, and T is a known matrix of each element in the clear target point and grabbing condition. In this way, Equation (4) establishes a matrix function of $\theta = [\theta_1, \theta_2, \theta_3, \cdots, \theta_n]^T$, and the range of F is $(0, n+1]$. If and only if $F = n+1$, i.e., $\|A^n - T\|_1 = 0$ and $C_1; C_2; \cdots; C_n = 1$ (i.e., θ in the allowed condition of the actual joint range of motion), it happens to be the optimal θ for inverse kinematics problem solution. Therefore, objectives can be transformed into finding θ corresponding to the maximum value of the F-function.

2.2. The path-planning of fuzzy ant colony algorithm. Firstly, maximum problem of the F-function of variable $\theta = [\theta_1, \theta_2, \theta_3, \cdots, \theta_n]^T$ is transformed into ant colony algorithm "path-planning" problem. As shown in Figure 1, each variable in θ is to establish a grid net $10 \times d$ (d is the ranging accuracy of the variable). For example, in Figure 1, variable θ_1 establishes a 10×5 grid net, and the provisions of each column grid points have numbers of 0-9, each variable sign column is regarded as the sign bit of the variable, and then the θ_1 values from -9.999 to 9.999. Connect the variables θ_1 to θ_n together, and add Start, Destination points in both ends of the grid net, and a diagram of $10 \times (d \times n)$ lattice points of θ can be obtained as shown in Figure 1. And the numbered grid points are feasible.

If you let the ant K from the Start point position always follow through each grid point to reach the Destination point position, shown in Figure 1, you can get a set of digital information about each grid points, namely: 4739303528…15933. Thus, decoding can be established by formula below:

$$\theta_q = (-1)^{\theta(i,1+d\cdot(q-1))} \cdot \sum_{k=0}^{d-1} \theta(i, 2 + d\cdot(q-1) + k) \cdot 10^{-k} \tag{5}$$

where $\theta(i, j)$ is the grid net with numbers from Start to Destination, such as, $\theta(3, 4) = 2$. $q = 1, 2 \ldots, n$, $i = 1, 2, \ldots, 10$, $j = 1, 2, \ldots, 10 \times d$, d is the variable accuracy, as shown in Figure 1, $d = 5$. According to the decoding Formula (5), the path corresponding to each variable in Figure 1 can be decoded respectively as follows:

$$\theta_1 = (-1)^4 \cdot (7 \times 10^0 + 3 \times 10^{-1} + 9 \times 10^{-2} + 3 \times 10^{-3}) = 7.393$$
$$\theta_2 = (-1)^0 \cdot (3 \times 10^0 + 5 \times 10^{-1} + 2 \times 10^{-2} + 8 \times 10^{-3}) = 3.528$$
$$\ldots$$
$$\theta_n = (-1)^1 \cdot (5 \times 10^0 + 9 \times 10^{-1} + 3 \times 10^{-2} + 3 \times 10^{-3}) = -5.933$$

The combination of the decoded variables can get a set of $\theta = [\theta_1, \theta_2, \theta_3, \cdots, \theta_n]^T$. Visiblily, as long as any ant from Start position on the grid net reaches the Destination position can be decoded by Equation (5) to obtain a set of variables solution. Therefore, seeking F Maximization on $\theta = [\theta_1, \theta_2, \theta_3, \cdots, \theta_n]^T$ is equivalent to the path-planning problem in Figure 1. Secondly, the establishment of the transition function of fuzzy ant colony algorithm. Each ant K transfers from the grid point of the j-th column to the grid point of the c-th with transition function probability formula as follows:

$$P^k_{(i,j;r,c)} = \frac{\tau_{(i,j;r,c)}}{\sum\limits_{r=1}^{10} \tau_{(i,j;r,c)}} \tag{6}$$

where $i, r = 0, 1, \ldots, 9$, $c = j + 1$, $j = 0, 1, \ldots, (d \times n + 2)$, $\tau_{(i,j;r,c)}$ is the pheromone value of ant k from the grid point $\theta(i, j)$ to the grid point $\theta(r, c)$. Finally, updating methods of the pheromone $\tau_{(i,j;r,c)}$ can be classified into local and global two sorts. Local updating method is that each ant K takes every step grid point to update once, and formula is:

$$\tau^{new}_{(i,j;r,c)} = (1 - \rho) \cdot \tau^{old}_{(i,j;r,c)} + \rho \cdot Tau \tag{7}$$

where ρ is local pheromone evaporation factor, and Tau is local pheromone carrying capacity of each ant K, and Tau size value cooperates with global updating Q size value. Global updating method is that the optimal path among each group of ants path-planning is selected (F function takes the optimal solution). The pheromone updating method is every step of the path from Start to Destination. Global updating formula is:

$$\tau^{new}_{(i,j;r,c)} = (1 - \alpha) \cdot \tau^{old}_{(i,j;r,c)} + Q \tag{8}$$

where α is global pheromone evaporation factor, and Q is the global updating carrying amount of fuzzy pheromone. Because this parameter is very important, the paper uses

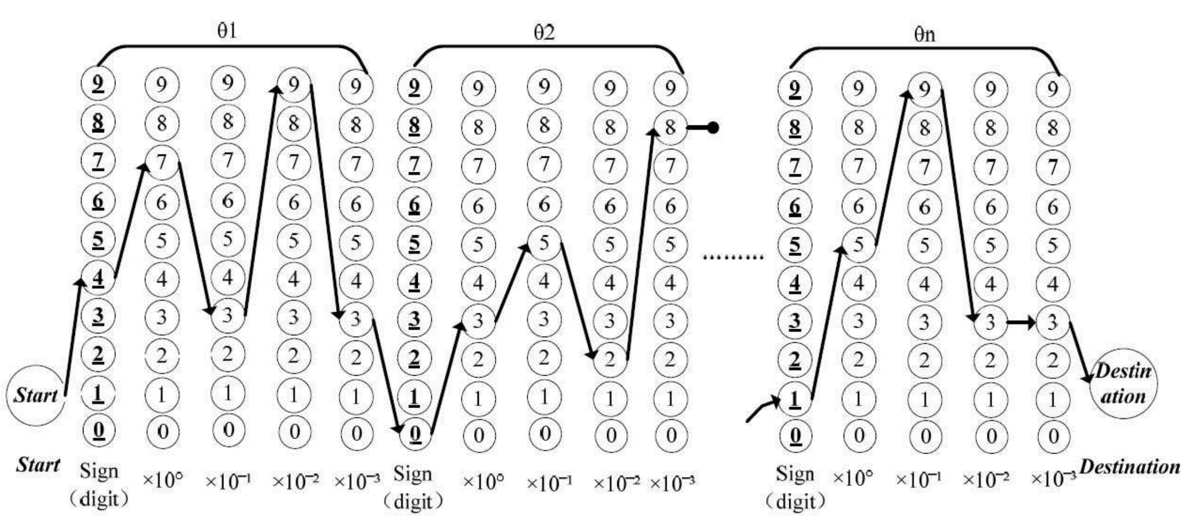

Figure 1. n element meshing function arguments

fuzzy reasoning methods to control Q value of each time in order to get reasonable values. Q has two parameters determining its amount of value. The first one is NC the ant colony algorithm iteration, and the second one is the result of each set of ants in optimal path-planning decoded into the objective function F value. At the beginning of the pheromone updating, if the Q value is too large, ant colony algorithm will be quickly into local optimum; if the Q value is set too low, the ant colony algorithm convergence speed is too slow. So the best way is to set the Q value smaller when NC is less, and the Q value larger when NC is larger. If you have an ant's path very close to the maximum value of the objective function decoded, the Q value of the ant would be the bigger, which makes next ant choose this path in all probability. Making optimization again on this path by next ant can improve the convergence speed. The provision of the number of iterations NC and the value of the objective function F decoded by the optimal path are regarded as fuzzy inputs, Q as the fuzzy output, and then a fuzzy inference rule shows in Table 1. The global nature of Gaussian function [8] is smooth, so Gaussian function is used for the input and output membership functions.

Mamdani reasoning is applied to fuzzy reasoning, shown in Figure 2.

TABLE 1. Fuzzy inference table

Q	FS	FM	FB
NCS	QS	QS	QM
NCM	QS	QS	QM
NCB	QS	QS	QB

References: S is small, M is middle, and B is big

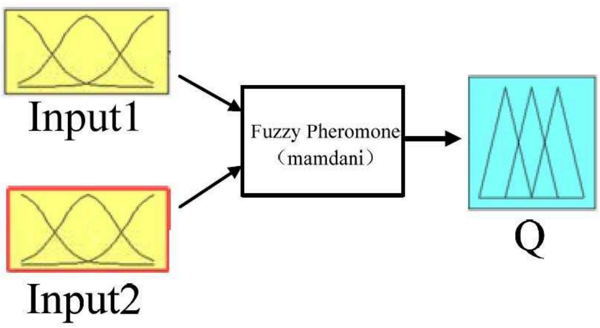

FIGURE 2. Fuzzy pheromone reasoning

2.3. Inverse kinematics solution steps of fuzzy ant colony algorithm.

Step 1 Objective function F constructed, the n variables of the objective function gridded, parameters initialized for α, ρ, d, Max_Iter (maximum iterations), Tau, fuzzy inference variable Q each group with ant amount m, $k = 1$, Iter = 1.

Step 2 Beginning from the Start point, according to the probability of ant k transition Equation (6) selecting the next point of the grid.

Step 3 Local updating according to Formula (7).

Step 4 If the ant k reaches the Destination point, according to Equation (5) calculate the decoded F value, $k = k+1$, otherwise, continue to follow the probability of transition Equation (6) to select the next column grid point and proceed with **Step 3**.

Step 5 If $k = m$, then the optimal path corresponding to F value is selected in the group of m ants, according to Equation (8), Globally update this path, Iter = Iter + 1, or the process moves to **Step 2**.

Step 6 If Iter = Max_Iter, then output the result of the F value corresponding to the optimal set of ants, otherwise $k = 1$, transfer to **Step 2**.

3. Robotic Arm Inverse Kinematics Experiment.

3.1. Forward kinematics for robotic arm. A 6 DOF physical mechanical arm is as Figure 3 shows. Sketch frames and D-H parameter table are established in Figure 4 and Table 2 according to the D-H Notation.

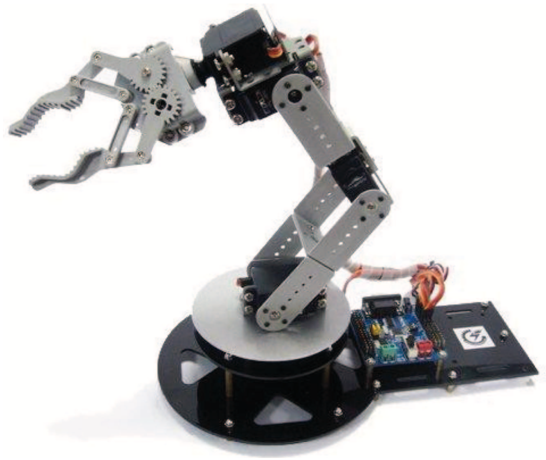

FIGURE 3. 6 DOF arms control physical picture

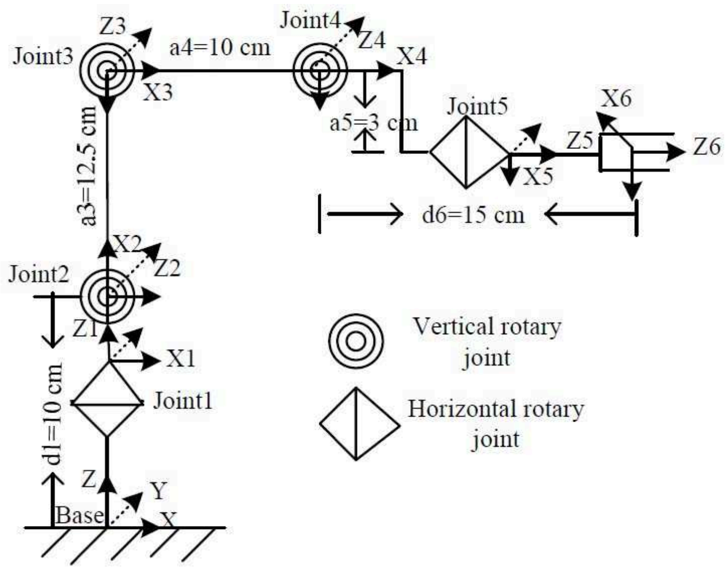

FIGURE 4. 6 degrees of freedom arm frames

TABLE 2. D-H parameter table

$Joint_{i-1}$	a_i (cm)	α_i	d_i (cm)	θ_i	Joints limitation (rad)
1	0	0	10	θ_1	$0 \sim \pi$
2	0	$-\pi/2$	0	θ_2	$0 \sim \pi$
3	12.5	0	0	θ_3	$-\pi/2 \sim 0$
4	10	0	0	θ_4	$-\pi/2 \sim \pi/2$
5	3	$\pi/2$	0	θ_5	$0 \sim \pi/2$
6	0	0	15	θ_6	$-\pi/2 \sim \pi/2$

3.2. **Fuzzy ant colony algorithm inverse kinematics for robotic arm.** Under the base frame, the target grabbing point is set $P = (3, 0, 47.5)$, and set base coordinate system rotation $\theta_1 = 0$; θ_6 wrist rotation is set $\theta_6 = \pi/2$; target matrix T can be obtained, according to the Cartesian coordinate system. Therefore, inverse kinematics problem in the real environment can be transformed into an inverse solution of 4 joints θ_2, θ_3, θ_4, θ_5. According to Equation (4), establishing an objective function F, set $\alpha = 0.8$; $\rho = 0.8$; $d = 6$. Take the maximum iterations Max_Iter = 1000, and there are $m = 20$ ants in each group, $Tau = 0.01$; Gaussian membership function is taken in fuzzy control, and NC membership ranges $[0, 1000]$, and F ranges $(0, 4 + 1]$. The fuzzy membership pheromone increment Q ranges $[0, 1]$, and the fuzzy inference rules are as Table 1 shows. According to realistic joints limitation in Table 2, the optimal solution of iterations 1000 is obtained in Table 3.

TABLE 3. Fuzzy ant colony optimization results

$\|A^6 - T\|_1$	θ_2	θ_3	θ_4	θ_5
0.0890	0.0034	−1.5700	0.0021	1.5701

The robot arm forward kinematics matrix equation A^6 can be obtained by Equation (1) and Equation (2), according to the D-H Notation in Table 2. With the number of iterations increasing, $\|A^6 - T\|_1$ optimization trend is shown in Figure 5, and the specific optimization computing time and convergence are in Table 4.

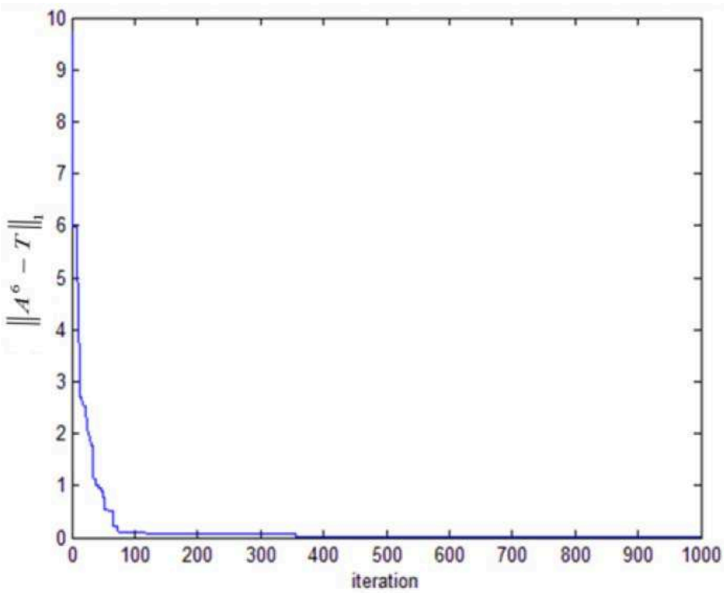

FIGURE 5. Fuzzy ant colony algorithm optimization in trends

TABLE 4. The convergence computing time table

Iterations	100	400	700	1000
Run time (s)	0.38	0.82	1.77	2.42
$\|A^6 - T\|_1$	0.0977	0.0921	0.0890	0.0890

4. **Conclusions.** The algorithm using heuristic characteristics of ant colony algorithm, plus putting inspired pheromone factor with intelligent fuzzy control, effectively improves the convergence, heuristic and stability of the algorithm and successfully solves the problem of complex multi-degree robot arm inverse kinematics equation. Thus, the proposed

algorithm provides a general method of solving the inverse kinematics problem for any complex multi-DOF robot arm. Future study direction is using this method to make robot arm avoid obstacles under circumstances to reach a target, and it will be even more promising.

REFERENCES

[1] J. J. Craig, *Introduction to Robotics: Mechanics and Control*, 3rd Edition, 2006.
[2] J. Denavit and R. S. Hartenberg, A kinematic notation for lower-pair mechanisms based on matrices, *Journal of Applied Mechanics*, pp.215-221, 1955.
[3] Q. Chen, S. Zhu, X. Wang and X. Zhang, Inverse kinematics sub-problem solution algorithm for serial robot based on screw theory, *Journal of Zhejiang University (Engineering Science)*, vol.48, no.1, pp.8-14, 2014.
[4] F. Chapelle and P. Bidaud, Closed form solutions for inverse kinematics approximation of general 6R manipulators, *Mechanism and Machine Theory*, vol.39, no.3, pp.323-338, 2004.
[5] Z. S. Hammour, N. M. Mirza, S. M. Mirza et al., Cartesian path generation of robot manipulators using continuous genetic algorithms, *Robotics and Autonomous Systems*, vol.41, no.4, pp.179-223, 2002.
[6] S. S. Chiddarwar and N. R. Babu, Comparison of RBF and MLP neural networks to solve inverse kinematic problem for 6R serial robot by a fusion approach, *Engineering Applications of Artificial Intelligence*, vol.23, no.7, pp.1083-1092, 2010.
[7] R. Köker, C. Öz, T. Çakar et al., A study of neural network based inverse kinematics solution for a three-joint robot, *Robotics and Autonomous Systems*, vol.49, no.3, pp.227-234, 2004.
[8] X. Shi and Z. Hao, *Fuzzy Control and Matlab Simulation*, The Tsinghua University Press, 2008.
[9] E. Amiri, H. Keshavarz, M. Alizadeh, M. Zamani, T. Khodadadi and S. Khan, Energy efficient routing in wireless sensor networks based on fuzzy ant colony optimization, *International Journal of Distributed Sensor Networks*, 2014.
[10] M. Dorigo, V. Maniezzo and A. Colorni, Ant system: Optimization by a colony of cooperating agents, *IEEE Trans. Systems, Man, and Cybernetics – Part B: Cybernetics*, 1996.
[11] M. Dorigo and T. Stutzle, *Ant Colony Optimization*, The MIT Press, 2004.