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A NEW CHAOTIC ATTRACTOR GENERATED BY FEEDBACK CONTROLLING METHOD AND ITS CHAOS CONTROL

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Abstract. *A new chaotic system is found by feedback controlling method in this paper. The Lyapunov exponents of the system, chaotic behavior and the stability at equilibrium points are studied. Based on feedback controlling method, a nonlinear feedback controller is designed for the novel system. In addition, according to Lyapunov stability theory, the sufficient condition for stabilizing chaos to unstable equilibria is obtained. The theoretical analysis and simulation results are given to show the effectiveness of the method.*
Keywords: New chaotic system, Feedback controlling method, Lyapunov exponents, Lyapunov stability theory

1. **Introduction.** Since Lorenz found the first chaotic attractor [1] in a smooth three-dimensional autonomous system, considerable research interests have been made in searching for new chaotic attractors. In 1976, Rössler found a special three-dimensional chaotic system with only one nonlinear term [2]. Later, more and more chaotic attractors were found, such as Chen system [3], Lv system [4], and Liu system [5]. Due to great potential in chemical reactions, electrical engineering, information processing and so on, it is important to generate new chaotic systems and analyze their dynamical behaviors and dynamical properties.

Since chaotic systems are very sensitive to initial conditions and show irregular, unpredictable behaviors, it is sometimes desirable to control and eliminate it from the system [6,7]. Inspired by the pioneering work of Ott, Grebogi and Yorke (OGY) in 1990 [8], chaos control has attracted great attention in the past decades. Subsequently, a series of chaos control methods [9-11] have been generated. Based on adaptive backstepping method, chaos control of oscillator circuit is realized in [12]. The work in [13] presents chaos control of a permanent magnet synchronous motor for eliminating the chaotic phenomena using an improved sliding mode controller. In [14], adaptive fuzzy control is applied to chaos control for hyperchaotic Lorenz systems. However, in practical application, it is required that the control method is simple and effective and easy to implement, and the inherent characteristics of the system must be remained.

Recently, attention has shifted towards detecting and constructing chaotic attractors with complex dynamical behaviors. A chaotic system is proposed in [15], which uses exponential function instead of the nonlinear term of the Lv system. An autonomous chaotic system is presented by substituting absolute term with square term in Sprott system [16]. In this paper, a new chaotic system is gained by feedback controlling method. The proposed system has more complicated topological structure and dynamical behaviors. Some basic dynamical properties are studied. Based on these properties, a simple but effective feedback controller is designed, and the system ultimately in a relatively short time stabilizes to the unstable equilibrium.

This paper is organized as follows. In Section 2, a new chaotic system is proposed. The basic dynamical properties of the new chaotic system are studied in Section 3. Section 4 studies the stabilization of the unstable equilibrium under the effective controller. Section 5 gives the conclusions of the paper and further study directions.

2. Finding the New Chaotic System. The chaotic system with pulse-excitation [17] is described by

$$\begin{cases} \dot{x} = y - x \\ \dot{y} = xz + a\mathrm{sgn}\left(\sin\left(bx\right)\right) \\ \dot{z} = k - xy \end{cases} \tag{1}$$

where x, y and z are state variables, and a, b and k are positive real parameters.

The pulse-excitation $\mathrm{sgn}\left(\sin\left(bx\right)\right)$ is defined as

$$\mathrm{sgn}\left(\sin\left(bx\right)\right) = \begin{cases} 1 & \sin\left(bx\right) \geq 0 \\ -1 & \sin\left(bx\right) < 0 \end{cases} \tag{2}$$

When the system parameters are given as $k = 1$, $a = 1$ and $b = 10$, two separate chaotic attractors from different initial values $(1, 1, 0.9)$ (red) and $(-1, -1, 0.9)$ (blue) as their phase planes and time series shown in Figure 1 coexists in system (1).

From Figure 1, we can find that system (1) shows such a weird phenomenon, which has the different area of attractors for different initial values $(1, 1, 0.9)$ (red) and $(-1, -1, 0.9)$ (blue).

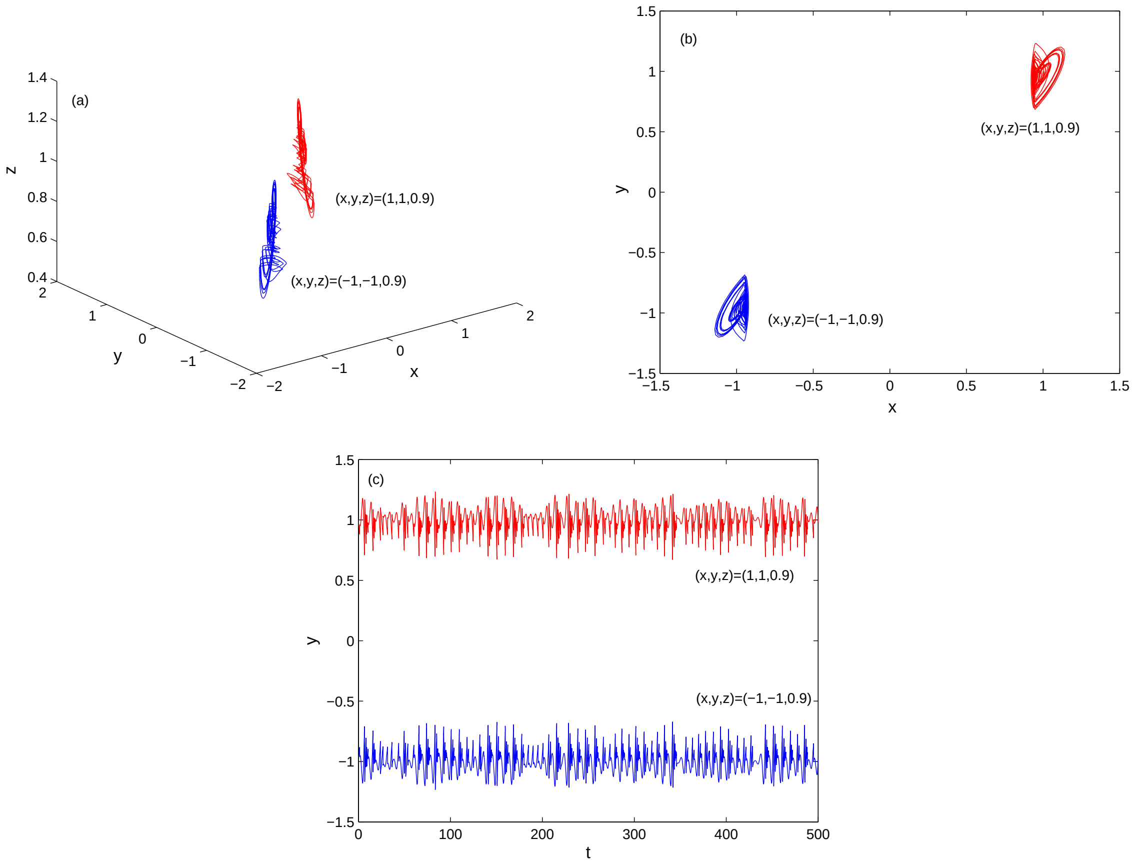


Figure 1. The coexisting two chaotic attractors of system (1) from initial values $(1, 1, 0.9)$ (red) and $(-1, -1, 0.9)$ (blue): (a) $x - y - z$; (b) $x - y$; (c) time series of y

Since chaos can be exploited for potential technological applications, it is important to develop techniques for designing chaotic attractors with complicated topological structures and complex shapes [18]. To this purpose, we provide a bridge between two separate chaotic attractors to construct more complex chaotic attractor.

Now, by introducing cy which is a linear state-feedback term to the first equation of system (1), we get:

$$\begin{cases} \dot{x} = y - x + cy \\ \dot{y} = xz + a\operatorname{sgn}(\sin(bx)) \\ \dot{z} = k - xy \end{cases} \tag{3}$$

where x, y and z are state variables, and a, b, c and k are positive real constants.

When the parameters $k = 1$, $a = 1$, $b = 10$ and $c = 5$, a chaotic attractor exists in the new system (3). The shape of the chaotic attractor is displayed in Figure 2.

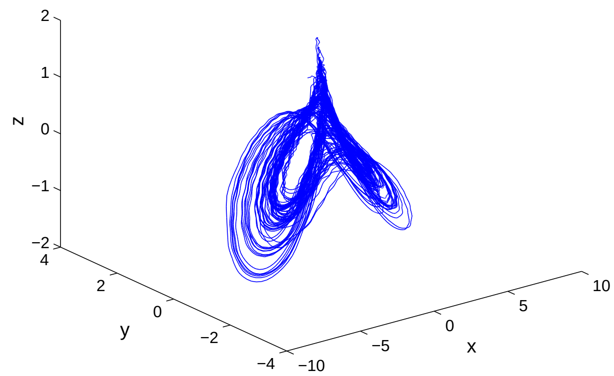


FIGURE 2. The new chaotic attractor

3. **Some Basic Properties of the New System.** In this section, we will investigate some basic properties of the new system (3) by careful theoretical analysis. These dynamical behaviors include the divergence, chaotic behavior, sensitivity to initial conditions, equilibrium points and their stability.

(a) Divergence

The state space of system (3) is three-dimensional. The vector field on the right-hand sides of system (3) is defined by

$$f(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{pmatrix} = \begin{pmatrix} y - x + cy \\ xz + a\operatorname{sgn}(\sin(bx)) \\ k - xy \end{pmatrix}.$$

The divergence of the vector field f is easily calculated as

$$div f(x) = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} = -1.$$

Therefore, dynamical system (3) is one dissipative system, and an exponential contraction of system (3) is $\frac{df}{dt} = e^{-1}$.

That is, in the dynamical system (3), a volume element V_0 is apparently contracted by the flow into a volume element $V_0 e^{-t}$ in time t. This means that each volume containing the trajectories of this dynamical system shrinks to zero as $t \to +\infty$ at an exponential rate. Therefore, all these dynamical system orbits will be eventually confined to a special subset that has zero volume, and the asymptotic motion of system (3) will settle onto an attractor of the system.

(b) Chaotic behavior

There exists a chaotic attractor for $k = 1$, $a = 1$, $b = 10$ and $c = 5$, and the shape of the chaotic attractor is shown in Figure 2. The dynamics of the corresponding Lyapaunov exponents of the novel system (3) is shown in Figure 3.

(c) Sensitivity to initial conditions

A basic and prominent property of a chaotic system is its seemingly erratic behavior, where a key element is the sensitivity of a trajectory to initial conditions. Different chaotic systems have different degrees of sensitivity to initial conditions, which differentiate their degrees of disorder.

Select initial values $(1, 1, 0.9)$ (red) and $(1.00002, 1, 0.9)$ (blue), utilizing Matlab mathematical software, and the simulation results are shown in Figure 4.

From Figure 4(b), we can see that the state responses of the proposed chaotic system (3) are completely different after $t = 12$s. By comparison, we can find that the new system has a higher degree of sensitivity to initial conditions.

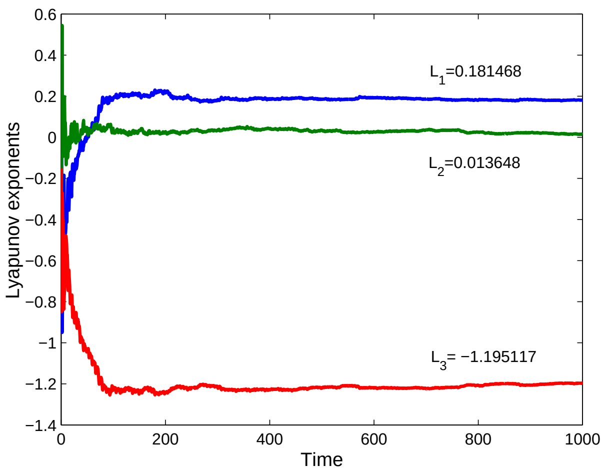
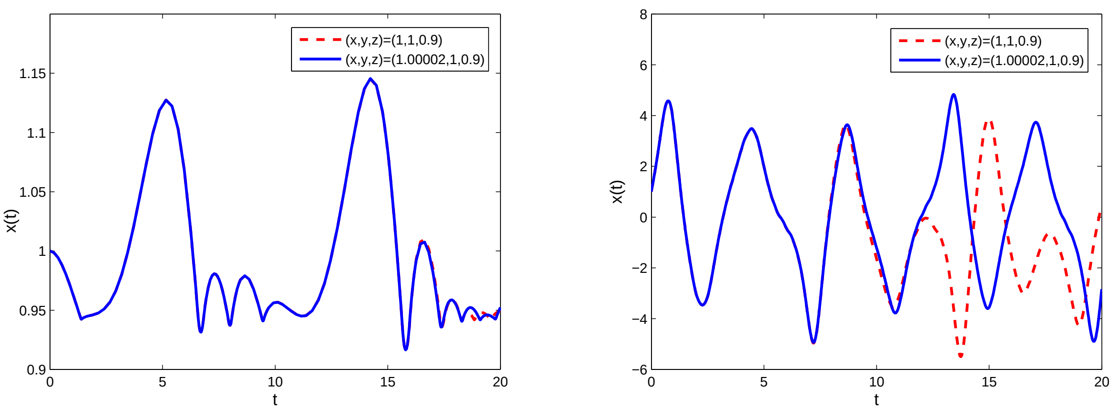


FIGURE 3. Dynamics of the Lyapaunov exponents of system (3)



(a) (b)

FIGURE 4. Sensitivity of the system state response versus time: (a) time series of state variable x of system (1); (b) time series of state variable x of system (3)

(d) The existence and stability of equilibrium

$$\begin{cases} y - x + cy = 0 \\ xz + a\mathrm{sgn}\left(\sin\left(bx\right)\right) = 0 \\ k - xy = 0 \end{cases} \tag{4}$$

Let $k = 1$, $a = 1$, $b = 10$ and $c = 5$. Equation (4) has two solutions as follows:

$$E_1(-2.44949, -0.40825, 0.40825), \quad E_2(2.44949, 0.40825, 0.40825).$$

The Jacobian matrix of system (3) is defined as

$$J = \begin{pmatrix} -1 & 1+c & 0 \\ z & 0 & x \\ -y & -x & 0 \end{pmatrix} = \begin{pmatrix} -1 & 6 & 0 \\ z & 0 & x \\ -y & -x & 0 \end{pmatrix}.$$

For equilibrium point E_1, let $|\lambda E - J_{E_1}| = 0$, we gain its eigenvalues as

$$\lambda_1 = -2.0753, \quad \lambda_2 = 0.5377 + 2.3437i, \quad \lambda_3 = 0.5377 - 2.3437i.$$

Similarly, the corresponding eigenvalues at E_2 are

$$\lambda_1 = -2.0753, \quad \lambda_2 = 0.5377 + 2.3437i, \quad \lambda_3 = 0.5377 - 2.3437i.$$

By Lyapunov stability theory, we know that E_1 and E_2 are unstable equilibrium points.

4. **Controlling Chaos via Feedback Controller.** In this section, simple but effective feedback controller is designed to drive the chaotic trajectories to the unstable equilibrium.

In order to stabilize the equilibrium E_1 and E_2, we select the controller $U = (u_1, u_2, u_3)^T$ as follows:

$$\begin{cases} u_1 = \bar{x} - \bar{y} - c\bar{y} - y\bar{z} + \bar{y}z \\ u_2 = -\bar{x}\bar{z} - a\mathrm{sgn}\left(\sin\left(bx\right)\right) + k_1(y - \bar{y}) \\ u_3 = \bar{x}\bar{y} - k + k_1(z - \bar{z}) \end{cases} \tag{5}$$

where k_1 is control parameter.

Under the action of U, at the unstable equilibrium E_1 and E_2, the controlled system is supposed to be

$$\begin{cases} \dot{x} = y - x + cy + u_1 \\ \dot{y} = xz + a\mathrm{sgn}\left(\sin\left(bx\right)\right) + u_2 \\ \dot{z} = k - xy + u_3 \end{cases} \tag{6}$$

Let

$$\begin{cases} e_1 = x - \bar{x} \\ e_2 = y - \bar{y} \\ e_3 = z - \bar{z} \end{cases} \tag{7}$$

Then the system error is

$$\begin{cases} \dot{e}_1 = y - x + cy + u_1 \\ \dot{e}_2 = xz + a\mathrm{sgn}\left(\sin\left(bx\right)\right) + u_2 \\ \dot{e}_3 = k - xy + u_3 \end{cases} \tag{8}$$

Consider the following Lyapunov function:

$$V(e_1, e_2, e_3) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right)$$

Then the differentiation of V is

$$\begin{aligned}\dot{V}(e_1,e_2,e_3) &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= e_1e_2 - e_1^2 + ce_1e_2 - y\bar{z}\,(x-\bar{x}) + \bar{y}z\,(x-\bar{x}) \\ &\quad + (y-\bar{y})\,(xz-\bar{x}\bar{z}) + k_1e_2^2 + (z-\bar{z})\,(\bar{x}\bar{y}-xy) + k_1e_3^2 \\ &= e_1e_2 - e_1^2 + ce_1e_2 + k_1e_2^2 + k_1e_3^2 \\ &= -\,e^TP_1e\end{aligned}$$

where $e=(e_1,e_2,e_3)^T$, $P_1 = \begin{pmatrix} 1 & -\frac{c+1}{2} & 0 \\ -\frac{c+1}{2} & -k_1 & 0 \\ 0 & 0 & -k_1 \end{pmatrix}$.

Clearly if

$$\begin{cases} -k_1 - \frac{(c+1)^2}{4} > 0 \\ -k_1 > 0 \end{cases}$$

we gain $k_1 < -9$, so the matrix P_1 is positive definite, which leads to $\lim_{t\to\infty}\|e(t)\| = 0$. Therefore, the equilibrium solution $O=(0,0,0)$ of sysyem (8) is asymptotically stable. According to (7), the equilibrium $E_{1,2}\,(\bar{x},\bar{y},\bar{z})$ of the controlled system (6) is globally asymptotically stable for the choice $k_1 < -9$ and Equation (5).

To verify the effectiveness of the control, select $a=1$, $b=10$, $c=5$, $k=1$, $k_1=-10$, the initial value $x=3$, $y=2$, $z=-1$, utilizing Matlab mathematical software, the simulation results of the controlled system (6) at the equilibrium $E_{1,2}\,(\bar{x},\bar{y},\bar{z})$ are shown in Figure 5.

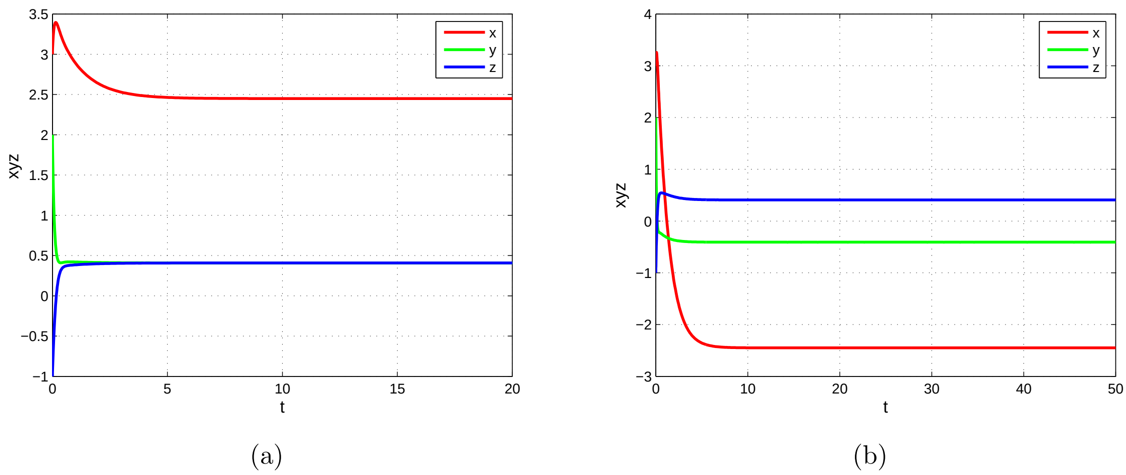


FIGURE 5. State curves of the controlled system (6) under the control law (5): (a) stabilizing chaos to equilibrium E_1 under the control law (5), the control is achieved at $t=8$s; (b) stabilizing chaos to equilibrium E_2 under the control law (5), the control is achieved at $t=10$s

5. **Conclusions.** In this paper, the new chaotic system is obtained by feedback controlling method. Some basic dynamical properties of the new system are studied. Based on these properties, simple but effective controller is designed for stabilizing chaos to unstable equilibria. According to Lyapunov stability theory, specific control parameter is given. Simulation results show that the chaos can be stabilized to the unstable equilibrium easily. In addition, for system (3), we will consider the bifurcation, the chaos synchronization and its application in secure communication using simple and effective controllers.

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REFERENCES

[1] E. N. Lorenz, Deterministic non-periodic flows, *Journal of the Atmospheric Sciences*, vol.20, pp.130-141, 1963.
[2] O. E. Rössler, An equation for continuous chaos, *Physics Letter A*, vol.57, no.4, pp.397-398, 1976.
[3] G. R. Chen and T. Ueta, Yet another chaotic attractor, *International Journal of Bifurcation and Chaos*, vol.9, no.6, pp.1465-1466, 1999.
[4] J. H. Lv and G. R. Chen, A new chaotic attractor coined, *International Journal of Bifurcation and Chaos*, vol.12, no.3, pp.659-661, 2002.
[5] C. X. Liu, T. Liu, L. Liu and K. Liu, A new chaotic attractor, *Chaos, Solitons and Fractals*, vol.22, no.5, pp.1031-1038, 2004.
[6] D. Tomic and B. Pernaric, Control and optimization of complex biological systems, *The 37th International Convention on Information and Communication Technology, Electronics and Microelectronics*, pp.222-227, 2014.
[7] M. L. Ma, F. H. Min, W. D. Huang and Y. Cao, Dynamic analysis and controlling chaos of the electrical power system with the disturbance of electromagnetic power, *The 33rd Chinese Control Conference*, pp.1994-1998, 2014.
[8] E. Ott, C. Grebogi and J. A. Yorke, Controlling chaos, *Phys. Rev. Lett.*, vol.64, pp.1196-1199, 1990.
[9] A. Jimenez-Triana, G. Chen and A. Gauthier, A parameter-perturbation method for chaos control to stabilizing UPOs, *IEEE Trans. Circuits and Systems II: Express Briefs*, 2014.
[10] Q. Huang, X. G. Liu, T. W. Li, K. Wang and S. X. Wang, On impulsive parametric perturbation control techniques for chaotic ship steering, *IEEE Chinese Guidance, Navigation and Control Conference*, pp.428-433, 2014.
[11] W. Xiang, X. Liu, H. Liu and Y. Huangfu, An adaptive fuzzy terminal sliding mode control scheme for uncertain chaotic systems, *ICIC Express Letters, Part B: Applications*, vol.4, no.5, pp.1391-1397, 2013.
[12] Z. H. Zhang, Chaos control of oscillator circuit based on adaptive backstepping method, *The 26th Chinese Control and Decision Conference*, pp.3136-3139, 2014.
[13] L. L. Li, S. D. Huang, Y. Wang, G. Z. Cao and Y. Liu, Chaos control of the permanent magnet synchronous motor using an improved sliding mode controller, *The 17th International Conference on Electrical Machines and Systems*, pp.2936-2939, 2014.
[14] H. Wang, Y. Hou, B. Chen and W. Qian, Adaptive fuzzy control for hyperchaotic Lorenz systems, *ICIC Express Letters, Part B: Applications*, vol.5, no.5, pp.1285-1290, 2014.
[15] C. L. Zhang and Z. L. Wang, Design and realization of a new chaotic system, *International Conference on Sensor Network Security Technology and Privacy Communication System*, pp.101-104, 2013.
[16] Q. Q. Wu and T. Li, Analysis, circuit implementation and synchronization of a new chaotic system, *Proc. of the 33rd Chinese Control Conference*, pp.6070-6073, 2014.
[17] Q. Lai, M. Chi, R. Q. Liao, Z. H. Guan and Y. H. Wu, A new chaotic system with pulse-excitation, *The 33rd Chinese Control Conference*, pp.2559-2563, 2014.
[18] Q. F. Chen, Y. G. Hong and G. R. Chen, Chaotic behaviors and toroidal/spherical attractors generated by discontinuous dynamics, *Physica A: Statistical Mechanics and Its Applications*, vol.371, no.2, pp.293-302, 2006.