ICIC Express Letters
Volume **10**, Number **1**, January **2016** ICIC International ©2016 ISSN 1881-803X
pp. 73–81

REPRESENTATION OF UNCERTAIN INFORMATION AND APPLICATIONS BASED ON GAUSSIAN TYPE FUZZY NUMBERS

Yumei Zhou and Guixiang Wang

Institute of Operations Research and Cybernetics
Hangzhou Dianzi University
Xiasha Higher Education Zone, Hangzhou 310018, P. R. China
g.x.wang@hdu.edu.cn

Received June 2015; accepted August 2015

Abstract. *In this paper, a special kind of fuzzy numbers which are called Gaussian type fuzzy numbers are studied, their properties are investigated, and a method of constructing such fuzzy numbers to represent uncertain or imprecise digital signal information is given. Then the calculation formulas (which can be easy realized by computer program in applications) of the means, discrete degrees and difference values are obtained as they are respectively restricted to the Gaussian type fuzzy number space. And then, a practical example is given to demonstrate the application of Gaussian type fuzzy numbers.*
Keywords: Fuzzy number, Gaussian type fuzzy number, Mean, Discrete degree, Difference value

1. **Introduction.** The concept of fuzzy set was first put forward by Zadeh [1] in 1965. In 1972, Chang and Zadeh [2] proposed the concept of fuzzy numbers to study the properties of probability functions. With the development of theories and applications of fuzzy numbers, it becomes more and more important. For example, in 2009, Wang et al. [6] protested the representation of uncertain multichannel digital signal spaces and study of pattern recognition based on metrics and difference values on fuzzy n-cell number space. In 2013, Ramesh and Jena [10] discussed the fuzzy clustering with Gaussian type member ship function.

For the convenience of application, the concepts of the triangle fuzzy numbers and trapezoid fuzzy numbers were defined, and the theories and applications of them have developed very well. For example, in 2005, Abbasbandy and Asady [3] gave the nearest trapezoid fuzzy number to a fuzzy quantity. In 2008, Adrian [4] introduced approximation of fuzzy numbers by trapezoid fuzzy numbers preserving the expected interval. In 2009, Abbasbandy and Hajjari [5] studied a new approach for ranking of trapezoidal fuzzy numbers. In 2010, Xu et al. [7] introduced a method for fuzzy risk analysis based on the new similarity of trapezoidal fuzzy numbers. In 2013, Zhang et al. [8] discussed a grey relational projection for multi-attribute decision making based on intuitionistic trapezoidal fuzzy number. In 2013, Wu and Cao [9] studied same families of geometric operators with intuitionistic trapezoidal fuzzy number. And then, in 2015, Wang and Wang [11] studied the trapezoid fuzzy integers and triangle fuzzy integers and their application.

However, it is not always suitable to use linear function to represent the membership function of a fuzzy quantity. Therefore, although the trapezoid fuzzy numbers and triangle fuzzy numbers can be conveniently used in applications, sometimes, there are some defects in accuracy as they are used to represent some uncertain or imprecise digital signals. For example, if we use triangle fuzzy number u with normal point 72 (unit: cm) and support set $(66, 78)$ (unit: cm) to represent fuzzy quantity "the height of man of medium height (in a certain area)", then $u(71) = 0.83$, and it means that the degree of membership of a man with height 71 cm belonging to "the height of man of medium height" is only 0.83, which seems a bit small compared with the actual situation. In this paper, we discuss Gaussian type fuzzy numbers, which not only inherit the convenience of trapezoid fuzzy numbers and triangle fuzzy numbers (see Remark 3.1), but also can be used more precisely to represent some uncertain or imprecise digital signals. The specific arrangements of this paper are as follows. In Section 2, we briefly review some basic notions, definitions and results about fuzzy numbers. In Section 3, we give the definition of Gaussian type fuzzy numbers, investigate their properties, and give a method of constructing such fuzzy numbers to represent uncertain or imprecise digital signal information. Then, in Section 4, for the sake of convenience in application, we work out the calculation formulas (which can be easily realized by computer program in applications) of the means, discrete degrees and difference values as they are respectively restricted to the Gaussian type fuzzy number space. And then, in Section 5, we give a practical example to demonstrate the application of Gaussian type fuzzy numbers. At last, we make a conclusion in Section 6.

2. **Basic Definitions and Notations.** Let R be the real number set, and I be the integer set. And let $K(R)$ denote the collection of non-empty compact subsets of R. The addition, scalar multiplication and multiplication on the space $K(R)$ are respectively defined as $A+B=\{a+b|a\in A, b\in B\}$, $\lambda A=\{\lambda a|a\in A\}$ and $AB=\{ab|a\in A, b\in B\}$ for any $A, B\in K(R)$, $\lambda\in R$.

A fuzzy subset (for short, a fuzzy set) of R is a function $u\colon R\to[0,1]$, and for all fuzzy sets, we denote by $F(R)$. For each such fuzzy set u, we denote by $[u]^r=\{x\in R^n : u(x)\geq r\}$ for any $r\in(0,1]$, its r-level set. By supp$u$ we denote the support of u, i.e., the $\{x\in R^n : u(x)>0\}$. By $[u]^0$ we denote the closure of the supp$u$, i.e., $[u]^0=\overline{\{x\in R : u(x)>0\}}$.

If u is a normal and fuzzy convex fuzzy set of R, $u(x)$ is upper semi-continuous, and $[u]^0$ is compact, then we call u a (1-dimensional) fuzzy number, and denote the collection of all fuzzy numbers by E.

It is obvious that if $u\in E$ and $r\in[0,1]$, then $[u]^r$ is a closed interval, and we denote it as $[u]^r=[\underline{u}(r),\overline{u}(r)]$.

For any $u\in E$, we define $\underline{M}(u)=2\int_0^1 r\underline{u}(r)dr$, $\overline{M}(u)=2\int_0^1 r\overline{u}(r)dr$, and $M(u)=\frac{\underline{M}(u)+\overline{M}(u)}{2}=\int_0^1 r\left[\underline{u}(r)+\overline{u}(r)\right]dr$, and call $M(u)$ the mean of u.

For any $u\in E$, we define $\underline{D}(u)=\int_0^1 r\left[\frac{\underline{u}(1)+\overline{u}(1)}{2}-\underline{u}(r)\right]dr$, $\overline{D}(u)=\int_0^1 r\left[\overline{u}(r)-\frac{\underline{u}(1)+\overline{u}(1)}{2}\right]dr$, and $D(u)=\underline{D}(u)+\overline{D}(u)=\int_0^1 r\left[\overline{u}(r)-\underline{u}(r)\right]dr$, and call $D(u)$ the discrete degree of u.

3. **Gaussian Type Fuzzy Numbers and Their Construction.** In order to introduce the Gaussian type fuzzy numbers, we give the following theorem.

Theorem 3.1. *Let a, c, $b\in R$ with $a\leq c\leq b$, and $\alpha,\beta\in(0,+\infty)$. If fuzzy set u of R is defined as follows:*

$$u(x)=\begin{cases}\exp\left(-\dfrac{(x-c)^2}{\alpha}\right), & x\in[a,c]\\ \exp\left(-\dfrac{(x-c)^2}{\beta}\right), & x\in(c,b]\\ 0, & x\notin(a,b)\end{cases}\tag{1}$$

then u is a fuzzy number.

Proof: (1) When $x=c$, $u(x)=\exp\left(-\frac{(c-c)^2}{\alpha}\right)=1$, so we have u being a regular fuzzy set of R.

(2) Let $x,y\in R$, $t\in[0,1]$. In the following, we only show $u(tx+(1-t)y)\geq\min\{u(x),u(y)\}$ as $x\leq y$ (it can be similarly seen as $x>y$), i.e., u is a convex fuzzy set:

(i) If $x, y \in [a, c]$, then $tx + (1-t)y \in [x, y] \subset [a, c]$ and $tx + (1-t)y \geq x$, so by the increasing of function $u(x)$ on $[a, c]$, we have

$$u(tx + (1-t)y) \geq u(x) = \min\{u(x), u(y)\}$$

(ii) If $x, y \in (c, b]$, then $tx + (1-t)y \in [x, y] \subset (c, b]$ and $tx + (1-t)y \leq y$, so by the decreasing of function $u(x)$ on $(c, b]$, we have

$$u(tx + (1-t)y) \geq u(y) = \min\{u(x), u(y)\}$$

(iii) If $x \in [a, c]$ and $y \in (c, b]$, then $tx+(1-t)y \in [x, y] \subset [x, c] \cup (c, y]$. As $tx+(1-t)y \in [x, c]$, by the increasing of function $u(x)$ on $[a, c]$, we have

$$u(tx + (1-t)y) \geq u(x) = \min\{u(x), u(y)\}$$

As $tx + (1-t)y \in (c, y]$, by the decreasing of function $u(x)$ on $(c, b]$, we have

$$u(tx + (1-t)y) \geq u(y) = \min\{u(x), u(y)\}$$

By the above (i)-(iii), we know that $u(tx + (1-t)y) \geq \min\{u(x), u(y)\}$ holds, so u is a convex fuzzy set.

(3) By the definition of the membership function, we can directly prove that $u(x)$ is upper semi-continuous functions.

(4) The closure of the supp$u$, i.e., $[u]^0 = \overline{\{x \in R : u(x) > 0\}}$ is $[a, b]$, which is bounded closed set, i.e., $[u]^0$ is compact.

By the above (1)-(4), we know that u is a fuzzy number. The proof of the theorem is completed.

Definition 3.1. *We call the fuzzy number u defined by Formula (1) Gaussian type fuzzy number, and denote it as $u = (a, c, b, \alpha, \beta)$, denoting the collection of all Gaussian type fuzzy numbers by $G(E)$.*

Remark 3.1. *In the first section, we gave an example of using triangle fuzzy number u with normal point 72 (unit: cm) and support set $(66, 78)$ (unit: cm) to represent fuzzy quantity "the height of man of medium height (in a certain area)" to illustrate that sometimes, there are some defects in accuracy as trapezoid fuzzy numbers or triangle fuzzy numbers are used to represent some uncertain or imprecise digital signals. If we use Gaussian type fuzzy number $v = (66, 72, 78, 6^2, 6^2)$ with normal point 72 and support set $(66, 78)$ to represent fuzzy quantity "the height of man of medium height (in a certain area)", then $v(71) = 0.9726$, and this means that the degree of membership of a man with height 75 cm belonging to "the height of man of medium height" is 0.9726, which is much more reasonable than 0.83. Therefore, sometimes, using Gaussian type fuzzy numbers to represent some uncertain or imprecise quantity is indeed more reasonable than using trapezoid fuzzy numbers or triangle fuzzy numbers.*

In the following, we introduce a method of constructing Gaussian type fuzzy numbers to represent uncertain or imprecise digital information.

Suppose that the following data come from an object (denoted by O) which is characterized by an uncertain or imprecise digital information:

$$O: \begin{array}{cccc} \textit{Sample } 1 & \textit{Sample } 2 & \cdots & \textit{Sample } n \\ o_1 & o_2 & \cdots & o_n \end{array}$$

The problem to be solved is constructing a Gaussian type fuzzy number to represent the object.

Constructing method:

The first step: We calculate

$$c = \frac{1}{n} \sum_{i=1}^{n} o_i \tag{2}$$

and

$$\alpha = \frac{1}{N_\alpha}\sum_{o_i<c}(c-o_i), \quad \beta = \frac{1}{N_\beta}\sum_{o_i>c}(o_i-c) \tag{3}$$

where N_α and N_β are the number of the character values o_i, $i=1,2,\cdots,n$ which satisfy $o_i<c$ and the number of the character values o_i, $i=1,2,\cdots,n$ which satisfy $o_i>c$, respectively.

The second step: Making a domain $\left[\underline{O},\overline{O}\right]$ with $\underline{O}\le c\le\overline{O}$ (such that all the possible character values of O are in it) of the character values of O according to the practical case, taking out a proper parameter ρ (usually $\rho\in[2,4]$), and denoting $a=\max\{c-\rho\alpha,\underline{O}\}$ and $b=\min\left\{c+\rho\beta,\overline{O}\right\}$, we construct a Gaussian type fuzzy number u as follows:

$$u(x)=\begin{cases}\exp\left(-\dfrac{(x-c)^2}{\alpha}\right), & x\in[a,c]\\ \exp\left(-\dfrac{(x-c)^2}{\beta}\right), & x\in(c,b]\\ 0, & x\notin(a,b)\end{cases} \tag{4}$$

Then the constructed Gaussian type fuzzy number $u=(a,c,b,\alpha,\beta)$ can be used to express the object O.

4. **Calculation Formulas of Some Aggregation Operators.** For the sake of convenience in application, we work out the calculation formulas (which can be easy realized by computer program in applications) of the means, discrete degrees and difference values as they are respectively restricted to the Gaussian type fuzzy number space. For this reason, we first give the following Lemma 4.1.

Lemma 4.1. $\int_0^1 r\sqrt{-\ln r}dr=\frac{\sqrt{2\pi}}{8}$

Proof:

$$\begin{aligned}\int_0^1 r\sqrt{-\ln r}dr &= \int_{+\infty}^0 e^{-x}\sqrt{x}de^{-x}=\int_0^{+\infty}e^{-2x}\sqrt{x}dx \text{ (where let } r=e^{-x})\\ &= \int_0^{+\infty}e^{-2y^2}ydy^2=-\frac{1}{2}\int_0^{+\infty}yde^{-2y^2} \text{ (where let } x=y^2)\\ &= -\frac{1}{2}\left(ye^{-2y^2}\Big|_0^{+\infty}-\int_0^{+\infty}e^{-2y^2}dy\right)=\frac{1}{2}\int_0^{+\infty}e^{-2y^2}dy\end{aligned}$$

Let

$$s=\frac{1}{2}\int_0^{+\infty}e^{-2y^2}dy$$

we else have

$$s=\frac{1}{2}\int_0^{+\infty}e^{-2x^2}dx$$

then

$$s^2=\frac{1}{4}\int_0^{+\infty}\int_0^{+\infty}e^{-2\left(x^2+y^2\right)}dxdy$$

Then let

$$x=R\cos\theta, \quad y=R\sin\theta$$

so

$$\begin{aligned} s^2 &= \frac{1}{4}\int_0^{\frac{\pi}{2}}\int_0^{+\infty} Re^{-2R^2}dRd\theta = \frac{1}{4}\int_0^{\frac{\pi}{2}}\int_0^{+\infty}\left(-\frac{1}{4}\right)(-4R)e^{-2R^2}dRd\theta \\ &= -\frac{1}{16}\int_0^{\frac{\pi}{2}}\int_0^{+\infty} de^{-2R^2}d\theta = -\frac{1}{16}\int_0^{\frac{\pi}{2}}\left(e^{-2R^2}\Big|_0^{+\infty}\right)d\theta \\ &= \frac{1}{16}\int_0^{\frac{\pi}{2}} d\theta = \frac{1}{16}\times\frac{\pi}{2} = \frac{\pi}{32} \end{aligned}$$

Therefore, we get

$$s = \frac{\sqrt{2\pi}}{8}, \text{ i.e., } \textstyle\int_0^1 r\sqrt{-\ln r}dr = \frac{\sqrt{2\pi}}{8}$$

The proof of the lemma is completed.

The following result is the calculation formula of the means as they are restricted to the Gaussian type fuzzy number space.

Theorem 4.1. *Let $u = (a, c, b, \alpha, \beta) \in G(E)$. Then the mean $M(u)$ of u is*

$$M(u) = c + \frac{\sqrt{2\pi}}{8}\left(\sqrt{\beta} - \sqrt{\alpha}\right)$$

Proof: By the definition of Gaussian type fuzzy number $u = (a, c, b, \alpha, \beta)$, we know that

$$\underline{u}(r) = c - \sqrt{\alpha}\sqrt{-\ln r}, \quad \overline{u}(r) = c + \sqrt{\beta}\sqrt{-\ln r}$$

Then by the definitions of $\underline{M}(u)$ and $\overline{M}(u)$ and Lemma 4.1, we have

$$\begin{aligned} \underline{M}(u) &= 2\int_0^1 r\underline{u}(r)dr = 2\int_0^1 r\left(c - \sqrt{\alpha}\sqrt{-\ln r}\right)dr \\ &= 2\int_0^1 crdr - 2\sqrt{\alpha}\int_0^1 r\sqrt{-\ln r}dr \\ &= 2c\times\frac{1}{2}r^2\Big|_0^1 - 2\sqrt{\alpha}\times\frac{\sqrt{2\pi}}{8} = c - \frac{\sqrt{2\pi\alpha}}{4} \end{aligned}$$

$$\begin{aligned} \overline{M}(u) &= 2\int_0^1 r\overline{u}(r)dr = 2\int_0^1 r\left(c + \sqrt{\beta}\sqrt{-\ln r}\right)dr \\ &= 2\int_0^1 crdr + 2\sqrt{\beta}\int_0^1 r\sqrt{-\ln r}dr \\ &= 2c\times\frac{1}{2}r^2\Big|_0^1 + 2\sqrt{\beta}\times\frac{\sqrt{2\pi}}{8} = c + \frac{\sqrt{2\pi\beta}}{4} \end{aligned}$$

Therefore, by the definition of $M(u)$, we obtain

$$\begin{aligned} M(u) &= \frac{\underline{M}(u) + \overline{M}(u)}{2} = \frac{c - \frac{\sqrt{2\pi\alpha}}{4} + c + \frac{\sqrt{2\pi\beta}}{4}}{2} \\ &= c + \frac{\sqrt{2\pi\beta} - \sqrt{2\pi\alpha}}{8} = c + \frac{\sqrt{2\pi}}{8}\left(\sqrt{\beta} - \sqrt{\alpha}\right) \end{aligned}$$

The proof of the theorem is completed.

The following result is the the calculation formula of the discrete degrees as they are restricted to the Gaussian type fuzzy number space.

Theorem 4.2. *Let $u = (a, c, b, \alpha, \beta) \in G(E)$. Then the discrete degree $D(u)$ of u is*

$$D(u) = \frac{\sqrt{2\pi}}{8}\left(\sqrt{\alpha} + \sqrt{\beta}\right)$$

Proof: By the definition of Gaussian type fuzzy number $u = (a, c, b, \alpha, \beta)$, we know that

$$\underline{u}(1) = c,\ \overline{u}(1) = c,\ \underline{u}(r) = c - \sqrt{\alpha}\sqrt{-\ln r}, \quad \overline{u}(r) = c + \sqrt{\beta}\sqrt{-\ln r}$$

Then by the definitions of $\underline{D}(u)$ and $\overline{D}(u)$ and Lemma 4.1, we have

$$\begin{aligned}\underline{D}(u) &= \int_0^1 r\left[\frac{\underline{u}(1)+\overline{u}(1)}{2} - \underline{u}(r)\right]dr = \int_0^1 r\left[\frac{c+c}{2} - \left(c - \sqrt{\alpha}\sqrt{-\ln r}\right)\right]dr \\ &= \int_0^1 r\sqrt{\alpha}\sqrt{-\ln r}dr = \sqrt{\alpha}\int_0^1 r\sqrt{-\ln r}dr = \frac{\sqrt{2\pi\alpha}}{8}\end{aligned}$$

$$\begin{aligned}\overline{D}(u) &= \int_0^1 r\left[\overline{u}(r) - \frac{\underline{u}(1)+\overline{u}(1)}{2}\right]dr = \int_0^1 r\left[\left(c + \sqrt{\beta}\sqrt{-\ln r}\right) - \frac{c+c}{2}\right]dr \\ &= \int_0^1 r\sqrt{\beta}\sqrt{-\ln r}dr = \sqrt{\beta}\int_0^1 r\sqrt{-\ln r}dr = \frac{\sqrt{2\pi\beta}}{8}\end{aligned}$$

Therefore, by the definition of $D(u)$, we obtain

$$D(u) = \underline{D}(u) + \overline{D}(u) = \frac{\sqrt{2\pi\alpha}}{8} + \frac{\sqrt{2\pi\beta}}{8} = \frac{\sqrt{2\pi}}{8}\left(\sqrt{\alpha} + \sqrt{\beta}\right)$$

The proof of the theorem is completed.

In [6], authors introduced the concepts about difference value in fuzzy n-cell number space. If $n = 1$, i.e., $u, v \in E$ with $M(u), M(v) \geq 0$ and $M(u)+M(v) \neq 0$, and $\lambda \in (0, \infty)$, then the left difference value $L_\lambda(u, v)$ and the right difference value $R_\lambda(u, v)$ of u and v (with respect to parameter λ) are respectively

$$L_\lambda(u, v) = \frac{\int_0^1 2r|\underline{u}(r) - \underline{v}(r)|dr}{[M(u) + M(v)]^\lambda}, \quad R_\lambda(u, v) = \frac{\int_0^1 2r|\overline{u}(r) - \overline{v}(r)|dr}{[M(u) + M(v)]^\lambda}$$

and the difference value $\Delta_\lambda(u, v)$ of u and v (with respect to parameter λ) is

$$\Delta_\lambda(u, v) = \frac{1}{2}\left[L_\lambda(u, v) + R_\lambda(u, v)\right]$$

$$\left(\text{i.e., } \Delta_\lambda(u, v) = \frac{\int_0^1 r[|\underline{u}(r) - \underline{v}(r)| + |\overline{u}(r) - \overline{v}(r)|]dr}{\left(\int_0^1 r[\underline{u}(r) + \underline{v}(r) + \overline{u}(r) + \overline{v}(r)]dr\right)^\lambda}\right)$$

The following result is the the calculation formula of the difference value of two Gaussian type fuzzy numbers.

Theorem 4.3. *Let $u = (a_1, c_1, b_1, \alpha_1, \beta_1)$, $v = (a_2, c_2, b_2, \alpha_2, \beta_2) \in G(E)$ with $M(u)$, $M(v) \geq 0$ and $M(u)+M(v) \neq 0$, and $\lambda \in (0, \infty)$. Then the difference value of the Gaussian type fuzzy numbers u and v is*

$$\Delta_\lambda(u, v) = \frac{\left|\frac{1}{2}(c_1 - c_2) + \frac{\sqrt{2\pi}}{8}\left(\sqrt{\alpha_2} - \sqrt{\alpha_1}\right)\right| + \left|\frac{1}{2}(c_1 - c_2) + \frac{\sqrt{2\pi}}{8}\left(\sqrt{\beta_1} - \sqrt{\beta_2}\right)\right|}{\left[c_1 + c_2 + \frac{\sqrt{2\pi}}{8}\left(\sqrt{\beta_1} + \sqrt{\beta_2} - \sqrt{\alpha_1} - \sqrt{\alpha_2}\right)\right]^\lambda}$$

Proof: The result can be directly obtained by the definition of Δ_λ, Theorem 4.1 and Lemma 4.1, and the specific derivation process is omitted.

5. **Applications in Pattern Recognition.** In the section, we give a practical example of application of Gaussian type fuzzy numbers.

Example 5.1. *Let there be two kinds of ores that are from different regions A and B, we take 10 samples arbitrarily from two kinds of ores respectively, and measure their iron content (unit: kg/100kg):*

$$A : 13.8, 11.76, 8.31, 9.02, 9.63, 8.65, 11.36, 12.3, 12.03, 7.98$$
$$B : 52.33, 79.34, 34.51, 62.34, 82.26, 28.36, 17.37, 25.32, 10.11, 8.34$$

Now there is a batch of ore (denoted by O) which comes from region A or region B. The following set of data (of the iron content) comes from the sample measuring to the batch of ore O.

$$O : 18.7, 20.81, 76.02, 9.36, 29.06, 23.50, 16.86, 21.01, 13.77, 20.82$$

The problem to be solved is to identify that the batch of ore O is from A or from B.

By Formulas (2) and (3), we work out the following c_A, c_B, c_O, α_A, α_B, α_O, β_A, β_B and β_O which are the values of c, α and β in Formula (2) or (3) corresponding to the A, B and O, respectively.

$$\begin{aligned} &c_A = 10.48, \quad \alpha_A = 1.76, \quad \beta_A = 1.77, \\ &c_B = 40.03, \quad \alpha_B = 19.36, \quad \beta_B = 29.04, \\ &c_O = 25.00, \quad \alpha_O = 6.90, \quad \beta_O = 27.54 \end{aligned}$$

Taking $\underline{A} = \underline{B} = \underline{O} = 0$, $\overline{A} = \overline{B} = \overline{O} = 100$ and $\rho = 2$, by Formulae (4), *we can construct the following Gaussian fuzzy numbers u_A, u_B and u_O to represent A, B and O respectively*

$$u_A(x) = \begin{cases} \exp\left(-\dfrac{(x-10.48)^2}{1.76}\right), & x \in [6.96, 10.48] \\ \exp\left(-\dfrac{(x-10.48)^2}{1.77}\right), & x \in (10.48, 14.02] \\ 0, & x \notin (6.96, 14.02) \end{cases}$$

$$u_B(x) = \begin{cases} \exp\left(-\dfrac{(x-40.03)^2}{19.36}\right), & x \in [1.31, 40.03] \\ \exp\left(-\dfrac{(x-40.03)^2}{29.04}\right), & x \in (40.03, 98.11] \\ 0, & x \notin (1.31, 98.11) \end{cases}$$

$$u_O(x) = \begin{cases} \exp\left(-\dfrac{(x-25.00)^2}{6.90}\right), & x \in [11.20, 25.00] \\ \exp\left(-\dfrac{(x-25.00)^2}{27.54}\right), & x \in (25, 80.08] \\ 0, & x \notin (11.20, 80.08) \end{cases}$$

We have

$$\begin{aligned} &\underline{u_A}(r) = 10.48 - \sqrt{1.76}\sqrt{-\ln r} = 10.48 - 1.33\sqrt{-\ln r} \\ &\overline{u_A}(r) = 10.48 + \sqrt{1.77}\sqrt{-\ln r} = 10.48 + 1.33\sqrt{-\ln r} \\ &\underline{u_B}(r) = 40.03 - \sqrt{19.36}\sqrt{-\ln r} = 40.03 - 4.40\sqrt{-\ln r} \\ &\overline{u_B}(r) = 40.03 + \sqrt{29.04}\sqrt{-\ln r} = 40.03 + 5.39\sqrt{-\ln r} \\ &\underline{u_O}(r) = 25 - \sqrt{6.90}\sqrt{-\ln r} = 25 - 2.63\sqrt{-\ln r} \\ &\overline{u_O}(r) = 25 + \sqrt{27.54}\sqrt{-\ln r} = 25 + 5.25\sqrt{-\ln r} \end{aligned}$$

Then we have

$$
\begin{aligned}
&\Delta_a(A,O)\\
&= \frac{\int_0^1 r\left[\left|\underline{u_A}(r)-\underline{u_O}(r)\right|+\left|\overline{u_A}(r)-\overline{u_O}(r)\right|\right]dr}{\left(\int_0^1 r\left[\underline{u_A}(r)+\underline{u_O}(r)+\overline{u_A}(r)+\overline{u_O}(r)\right]dr\right)^a}\\
&= \frac{\left|\frac{1}{2}(c_A-c_O)+\frac{\sqrt{2\pi}}{8}\left(\sqrt{\alpha_O}-\sqrt{\alpha_A}\right)\right|+\left|\frac{1}{2}(c_A-c_O)+\frac{\sqrt{2\pi}}{8}\left(\sqrt{\beta_A}-\sqrt{\beta_O}\right)\right|}{\left[c_A+c_O+\frac{\sqrt{2\pi}}{8}\left(\sqrt{\beta_A}+\sqrt{\beta_O}-\sqrt{\alpha_A}-\sqrt{\alpha_O}\right)\right]^a}\\
&= \frac{\left|\frac{1}{2}(10.48-25)+\frac{\sqrt{2\pi}}{8}\left(\sqrt{6.9}-\sqrt{1.76}\right)\right|+\left|\frac{1}{2}(10.48-25)+\frac{\sqrt{2\pi}}{8}\left(\sqrt{1.77}-\sqrt{27.54}\right)\right|}{\left[10.48+25+\frac{\sqrt{2\pi}}{8}\left(\sqrt{1.77}+\sqrt{27.54}-\sqrt{1.76}-\sqrt{6.90}\right)\right]^a}\\
&= \frac{15.9872}{36.9472^a}\\
&\Delta_a(B,O)\\
&= \frac{\int_0^1 r\left[\left|\underline{u_B}(r)-\underline{u_O}(r)\right|+\left|\overline{u_B}(r)-\overline{u_O}(r)\right|\right]dr}{\left(\int_0^1 r\left[\underline{u_B}(r)+\underline{u_O}(r)+\overline{u_B}(r)+\overline{u_O}(r)\right]dr\right)^a}\\
&= \frac{\left|\frac{1}{2}(c_B-c_O)+\frac{\sqrt{2\pi}}{8}\left(\sqrt{\alpha_O}-\sqrt{\alpha_B}\right)\right|+\left|\frac{1}{2}(c_B-c_O)+\frac{\sqrt{2\pi}}{8}\left(\sqrt{\beta_B}-\sqrt{\beta_O}\right)\right|}{\left[c_B+c_O+\frac{\sqrt{2\pi}}{8}\left(\sqrt{\beta_B}+\sqrt{\beta_O}-\sqrt{\alpha_B}-\sqrt{\alpha_O}\right)\right]^a}\\
&= \frac{\left|\frac{1}{2}(40.03-25)+\frac{\sqrt{2\pi}}{8}\left(\sqrt{6.9}-\sqrt{19.36}\right)\right|+\left|\frac{1}{2}(40.03-25)+\frac{\sqrt{2\pi}}{8}\left(\sqrt{29.04}-\sqrt{27.54}\right)\right|}{\left[40.03+25+\frac{\sqrt{2\pi}}{8}\left(\sqrt{29.04}+\sqrt{27.54}-\sqrt{19.36}-\sqrt{6.9}\right)\right]^a}\\
&= \frac{14.1172}{67.0516^a}
\end{aligned}
$$

Taking $a=1$, we can obtain $\frac{15.9872}{36.9472} > \frac{14.1172}{67.0516}$, i.e., $\Delta_a(A,O) > \Delta_a(B,O)$, so we know the ore O is from region B.

6. **Conclusions.** In this paper, we defined a special kind of fuzzy numbers which are called Gaussian type fuzzy numbers, and gave the expression of their membership function (Theorem 3.1). Then, we gave the method of constructing of Gaussian type fuzzy numbers to represent uncertain or imprecise digital information (Formulas (2)-(4)). And then, for the sake of convenience in application, we worked out the calculation formulas (which can be easily realized by computer program in applications) of the means, discrete degrees and difference values as they are respectively restricted to the Gaussian type fuzzy number space (Theorems 4.1, 4.2 and 4.3). At last, we gave a practical example (Example 5.1) to show the application of Gaussian type fuzzy numbers. From the view of structure, generally, using Gaussian type fuzzy numbers to represent uncertain or imprecise digital information is more rational than using triangle fuzzy numbers. In the future, we are going to the problems related with studying the multi-dimensional Gaussian type fuzzy numbers.

Acknowledgment. This work is partially supported by the Natural Science Foundation of China (Nos. 61273077 and 61433001), the Natural Science Foundation of Zhejiang Province, China (No. LY12A01001). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] L. A. Zadeh, Fuzzy sets, *Information and Control*, vol.8, no.3, pp.338-353, 1965.
[2] S. S. L. Chang and L. A. Zadeh, On fuzzy mappings and control, *IEEE Trans. Syst. Man Cybernet*, vol.2, pp.30-34, 1972.
[3] S. Abbasbandy and B. Asady, The nearest trapezoid fuzzy number to a fuzzy quantity, *Applied Mathematics and Computation*, vol.156, pp.381-386, 2005.
[4] B. Adrian, Approximation of fuzzy numbers by trapezoid fuzzy numbers preserving the expected interval, *Fuzzy Sets and Systems*, vol.159, pp.1327-1344, 2008.
[5] S. Abbasbandy and T. Hajjari, A new approach for ranking of trapezoidal fuzzy numbers, *Computers and Mathematics with Applications*, vol.57, pp.413-419, 2009.
[6] G. Wang, P. Shi and P. Messenger, Representation of uncertain multichannel digital signal spaces and study of pattern recognition based on metrics and difference values on fuzzy n-cell number spaces, *Trans. Fuzzy Systems*, vol.17, pp.421-439, 2009.
[7] Z. Xu, S. Shang, W. Qian and W. Shu, A method for fuzzy risk analysis based on the new similarity of trapezoidal fuzzy numbers, *Expert Systems with Applications*, vol.37, pp.1920-1927, 2010.
[8] X. Zhang, F. Jin and P. Liu, A grey relational projection for multi-attribute decision making based on intuitionistic trapezoidal fuzzy number, *Applied Mathematical Modelling*, vol.37, pp.3467-3477, 2013.
[9] J. Wu and Q. Cao, Same families of geometric operators with intuitionistic trapezoidal fuzzy number, *Fuzzy Sets and Systems*, vol.37, pp.318-327, 2013.
[10] C. R. Ramesh and G. Jena, Fuzzy clustering with Gaussian-type member ship function, *International Conference on Intelligent Systems and Signal Processing*, Gujarat, India, pp.393-396, 2013.
[11] G. Wang and J. Wang, Trapezoid and triangle fuzzy integers and their application, *ICIC Express Letters*, vol.9, no.2, pp.365-370, 2015.