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A MULTI-PERSON DECISION-MAKING METHOD BASED ON WEIGHTED FUZZY SOFT SET

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Abstract. *According to the researches about multi-person decision-making methods based on fuzzy soft set theory, in order to solve the problems that decision-making results are too many and unreasonable, a method of determining parameter weight is given. Based on it, a multi-person decision-making method based on weighted fuzzy soft set is proposed. Finally, compared with other multi-person decision-making methods, this new multi-person decision-making method not only is feasible, but also can make the decision-making result more reasonable and effective.*

Keywords: Fuzzy soft set, Multi-person decision-making, Parameter weight, Soft set

1. **Introduction.** Much information in our lives is imprecise and vague. Probability theory, rough set theory [1,2] and fuzzy set theory [3,4] can solve these problems. However, they all have some limitations (lack of parametric tool), so that these limitations limit their applications in practical problems widely. In order to solve these problems, Molodtsov proposed soft set [5] in 1999. Subsequently, Maji et al. gave the important concept of fuzzy soft set [6]. At present, fuzzy soft set theory has been applied to decision-making problems widely.

There have been fruitful researches about fuzzy soft set in terms of decision-making problems. Fuzzy soft set theory was applied to decision-making problems firstly [7]. Miu proposed 5 kinds of different multi-person decision-making methods based on fuzzy soft set [8], who extended soft set to fuzzy soft set. Multi-expert group decision-making methods based on fuzzy soft matrix were proposed [9,10]. So fuzzy soft matrix has important significance for future multi-person decision-making problems. Yue et al. considered all persons' preferences and proposed a new multi-person decision-making method based on fuzzy soft set [11]. It can show subjective judgment of different experts and solve imprecise and vague problems.

Based on above researches, in order to solve the problems that multi-person decision-making methods exist, a new multi-person decision-making method based on weighted fuzzy soft set is proposed. An example can verify that this new multi-person decision-making method is feasible, reasonable and effective.

2. **Improved Multi-Person Decision-Making Method.** The parameter weight is so important that it affects the decision-making result greatly. Presently, other methods of determining parameter weight are often used, such as AHP. However, they cannot reflect real intentions of decision makers. In order to solve this problem, this paper gives a new method of determining parameter weight based on fuzzy soft set. Based on it, a new multi-person decision-making method is proposed.

Definition 2.1. [12] *Given U is a finite non-empty universe of objects and E is a finite non-empty set of parameters. (F, E) is called a fuzzy soft set over U, where $F : E \rightarrow F(U)$*

is a mapping from E *to* $F(U)$. *Namely,* $\forall e \in E$, $F(e) = \{< x, \mu_{F(e)}(x) >: x \in U\} \in F(U)$, $\mu_{F(e)}(x) \in [0,1]$.

Because the standards that each person requests each object are different, λ-soft set is introduced.

Definition 2.2. [8] *Given* $S = (F, A)$ *is a fuzzy soft set over* U, λ *is a fuzzy set, to any* $a \in A$, $x \in U$, λ*-soft set is defined as:*

$$\mu_{F(a)}(\lambda) = \{x \in U | \mu_{F(a)}(x) \geq \lambda(a)\} \tag{1}$$

Definition 2.3. [13] *Given* $S = (F, A)$ *is a soft set over* U*, to any* $x_i, x_j \in U$*, discernibility parameter set of* x_i *and* x_j *is defined as:*

$$d(x_i, x_j) = \{a \in A : g(x_i, a) \neq g(x_j, a)\} \tag{2}$$

$D(S) = \{d(x_i, x_j) : x_i, x_j \in U\}$ *is called a discernibility matrix of* S.

Definition 2.4. *Given* $S = (F, A)$ *is a soft set over* U*, let* $|U| = n$, $d(x_i, x_j)$ *is discernibility parameter set of the objects* x_i *and* x_j*, to any* $a \in A$*, the importance degree of parameter* a *is defined as:*

$$D_a = \frac{\sum\limits_{i,j=1,2,\cdots,n} \frac{|a \cap d(x_i,x_j)|}{|d(x_i,x_j)|}}{K} \tag{3}$$

where $K = \frac{n \times (n-1)}{2}$ *is the total number of discernibility parameter sets in the upper triangle or the lower triangle of the discernibility matrix (we only need calculate the half* $d(x_i, x_j)$*, because the discernibility matrix is a symmetric matrix).*

According to importance degree of a, its weight is defined as:

$$w_D(a) = \frac{D_a}{\sum\limits_{a \in A} D_a} \tag{4}$$

To $\forall a \in A$, the properties are as the following:
(1) $0 \leq \frac{|a \cap d(x_i,x_j)|}{|d(x_i,x_j)|} \leq 1$;
(2) $0 \leq D_a \leq 1$;
(3) $0 \leq w_D(a) \leq 1$;
(4) $\sum\limits_{a \in A} w_D(a) = 1$.

Proof: (1) If $a \cap d(x_i, x_j) = \phi$, then $\frac{|a \cap d(x_i,x_j)|}{|d(x_i,x_j)|} = 0$. If $a \cap d(x_i, x_j) \neq \phi$: Let $d(x_i, x_j) = \{a\}$, and then $\frac{|a \cap d(x_i,x_j)|}{|d(x_i,x_j)|} = 1$. Let at least 2 parameters exist in $d(x_i, x_j)$, and we have $|d(x_i, x_j)| > 1$. Then $0 < \frac{|a \cap d(x_i,x_j)|}{|d(x_i,x_j)|} < 1$. Hence, $0 \leq \frac{|a \cap d(x_i,x_j)|}{|d(x_i,x_j)|} \leq 1$.

(2) To any $a \in A$, let $a \notin d(x_i, x_j)$, and we have $a \cap d(x_i, x_j) = \phi$. Then $D_a = 0$. Thus, a does not appear in the discernibility matrix and has no effect on the distinction of objects. To any $a \in A$, let $a \in d(x_i, x_j)$. If there is only one parameter in every $d(x_i, x_j)$, then $D_a = 1$. If there are at least 2 parameters in every $d(x_i, x_j)$, then $0 < \frac{|a \cap d(x_i,x_j)|}{|d(x_i,x_j)|} < 1$. Thus, $0 < D_a < 1$. Hence, $0 \leq D_a \leq 1$.

(3) According to the principle that parts are smaller than the whole, this property is proved.

(4) $\sum\limits_{a \in A} w_D(a) = \sum\limits_{a \in A} \frac{D_a}{\sum\limits_{a \in A} D_a} = \frac{\sum\limits_{a \in A} D_a}{\sum\limits_{a \in A} D_a} = 1$.

We can see that the corresponding parameters' weights are all 0 when the objects' values are 1 or 0. However, this is unreasonable. So this paper considers not only importance of each parameter but also their overall importance in the fuzzy soft set, and it uses the combined method [14] to determine parameters' weights.

Definition 2.5. *Given $S = (F, A)$ is a fuzzy soft set over U, $|U| = n$, the overall weight of a is defined as:*

$$w_S(a) = \frac{1}{n}\sum_{x\in U}\frac{\mu_{F(a)}(x)}{\sum\limits_{a\in A}\mu_{F(a)}(x)} \tag{5}$$

To $\forall a \in A$, the properties are as the following:
(1) $0 \leq w_S(a) \leq 1$;
(2) $\sum\limits_{a\in A} w_S(a) = 1$.

Proof: (1) Since $0 \leq \frac{\mu_{F(a)}(x)}{\sum\limits_{a\in A}\mu_{F(a)}(x)} \leq 1$, then $0 \leq \sum\limits_{x\in U}\frac{\mu_{F(a)}(x)}{\sum\limits_{a\in A}\mu_{F(a)}(x)} \leq n$. We have $0 \leq \frac{1}{n}\sum\limits_{x\in U}\frac{\mu_{F(a)}(x)}{\sum\limits_{a\in A}\mu_{F(a)}(x)} \leq 1$, then $0 \leq w_S(a) \leq 1$.

(2) $\sum\limits_{a\in A} w_S(a) = \sum\limits_{a\in A}\frac{1}{n}\sum\limits_{x\in U}\frac{\mu_{F(a)}(x)}{\sum\limits_{a\in A}\mu_{F(a)}(x)} == \frac{1}{n}\sum\limits_{a\in A}\sum\limits_{x\in U}\frac{\mu_{F(a)}(x)}{\sum\limits_{a\in A}\mu_{F(a)}(x)} =\frac{1}{n}\sum\limits_{x\in U}\sum\limits_{a\in A}\frac{\mu_{F(a)}(x)}{\sum\limits_{a\in A}\mu_{F(a)}(x)} == \frac{1}{n}\sum\limits_{x\in U} 1 = 1$.

Definition 2.6. *Given $S = (F, A)$ is a fuzzy soft set over U, the combined weight of a is defined as:*

$$w(a) = \frac{w_S(a) + w_D(a)}{\sum\limits_{a\in A}(w_S(a) + w_D(a))} \tag{6}$$

To $\forall a \in A$, the properties are as the following:
(1) $w(a) = \frac{w_S(a)+w_D(a)}{2}$;
(2) $0 \leq w(a) \leq 1$;
(3) $\sum\limits_{a\in A} w(a) = 1$.

Proof: (1) Since $\sum\limits_{a\in A} w_D(a) = 1$ and $\sum\limits_{a\in A} w_S(a) = 1$, then $\sum\limits_{a\in A}(w_S(a) + w_D(a)) = \sum\limits_{a\in A} w_S(a) + \sum\limits_{a\in A} w_D(a) = 1 + 1 = 2$. Thus, $w(a) = \frac{w_S(a)+w_D(a)}{\sum\limits_{a\in A}(w_S(a)+w_D(a))} = \frac{w_S(a)+w_D(a)}{2}$.

(2) According to the principle that parts are smaller than the whole, this property is proved.

(3) $\sum\limits_{a\in A} w(a) = \sum\limits_{a\in A}\frac{w_S(a)+w_D(a)}{\sum\limits_{a\in A}(w_S(a)+w_D(a))} = \frac{\sum\limits_{a\in A}(w_S(a)+w_D(a))}{\sum\limits_{a\in A}(w_S(a)+w_D(a))} = 1$.

We can see that the combined weight can avoid the cases that the corresponding parameters' weights are all 0 when the objects' values are 1 or 0 in λ-soft set effectively. Hence, it can reflect importance degree of each parameter better.

In many cases, there are at least two persons from different backgrounds and viewpoints to make the decision. Aiming at this problem, a new multi-person decision-making method based on weighted fuzzy soft set is proposed.

Definition 2.7. *Given $S = (F, A)$ is a fuzzy soft set over U, the weighted value $R(x_i)$ of x_i is defined as:*

$$R(x_i) = \sum_{a\in A}\mu_{F(a)}(x_i) \times w(a) \tag{7}$$

$R(x_i)$ represents the satisfaction degree of all decision makers for x_i. The greater $R(x_i)$ is, the better x_i is.

3. **Example Analysis.** A company wants to recruit a marketing-staff. There are 6 candidates. The company designates 2 responsible persons to recruit 1 candidate who fits with the job best. One responsible person considers the business skill, and other person considers the potentiality of occupation. $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ is the candidates set,

and $E = \{a_1, a_2, a_3, a_4, a_5\}$ is the parameters set. a_1, a_2, a_3, a_4, a_5 stand for "work background", "professional skills", "personality", "communication" and "occupation orientation" respectively. The parameters that one responsible person considers are $A = \{a_1, a_2\}$, and the parameters that other responsible person considers are $B = \{a_3, a_4, a_5\}$.

TABLE 1. Fuzzy soft set from the first responsible person

U	a_1	a_2
h_1	0.6	0.7
h_2	0.5	0.7
h_3	0.8	0.8
h_4	0.9	0.3
h_5	0.1	0.9
h_6	0.7	0.6

TABLE 2. Fuzzy soft set from the second responsible person

U	a_3	a_4	a_5
h_1	0.6	0.6	0.8
h_2	0.1	0.6	0.3
h_3	0.8	0.6	0.9
h_4	0.9	0.9	0.6
h_5	0.3	0.8	0.7
h_6	0.8	0.7	0.9

The concrete steps of multi-person decision-making method based on weighted fuzzy soft set are as follows.

Step 1: According to fuzzy soft sets that two responsible persons give, a new fuzzy soft set is obtained.

U	a_1	a_2	a_3	a_4	a_5
h_1	0.6	0.7	0.6	0.6	0.8
h_2	0.5	0.7	0.1	0.6	0.3
h_3	0.8	0.8	0.8	0.6	0.9
h_4	0.9	0.3	0.9	0.9	0.6
h_5	0.1	0.9	0.3	0.8	0.7
h_6	0.7	0.6	0.8	0.7	0.9

Step 2: If the first responsible person lets $\lambda' = (0.5, 0.6)$, and the other responsible person lets $\lambda'' = (0.6, 0.6, 0.6)$, then λ-soft set can be obtained.

U	a_1	a_2	a_3	a_4	a_5
h_1	1	1	1	1	1
h_2	1	1	0	1	0
h_3	1	1	1	1	1
h_4	1	0	1	1	1
h_5	0	1	0	1	1
h_6	1	1	1	1	1

According to the soft set, the discernibility matrix is constructed.

$$\begin{bmatrix} \phi & a_3, a_5 & \phi & a_2 & a_1, a_3 & \phi \\ & \phi & a_3, a_5 & a_2, a_3, a_5 & a_1, a_5 & a_3, a_5 \\ & & \phi & a_2 & a_1, a_3 & \phi \\ & & & \phi & a_1, a_2, a_3 & a_2 \\ & & & & \phi & a_1, a_3 \\ & & & & & \phi \end{bmatrix}$$

According to the discernibility matrix, weights of all parameters are obtained.

$$w_D(a_1) = 0.194,\ w_D(a_2) = 0.306,\ w_D(a_3) = 0.306,\ w_D(a_4) = 0,\ w_D(a_5) = 0.194$$

Step 3: In the new fuzzy soft set, overall weights of all parameters are obtained.

$$w_S(a_1) = 0.1815,\ w_S(a_2) = 0.2171,\ w_S(a_3) = 0.1676,\ w_S(a_4) = 0.2222,\ w_S(a_5) = 0.2116$$

Step 4: Combined weights of all parameters are obtained.

$$w(a_1) = 0.19,\ w(a_2) = 0.26,\ w(a_3) = 0.24,\ w(a_4) = 0.11,\ w(a_5) = 0.20$$

Step 5: In the new fuzzy soft set, the weighted values are obtained.

$$R(h_1) = 0.666,\ R(h_2) = 0.427,\ R(h_3) = 0.798,\ R(h_4) = 0.684,\ R(h_5) = 0.553,$$
$$R(h_6) = 0.738$$

The sorting result is obtained: $R(h_3) > R(h_6) > R(h_4) > R(h_1) > R(h_5) > R(h_2)$
Hence, h_3 is the best candidate.

This result is compared with other multi-person decision-making methods. The results are as follows.

	The sorting results	The decision results
The method in the paper [8]		h_1, h_3, h_6
The method in the paper [11]	$R(h_3) > R(h_6) > R(h_1) > R(h_4) > R(h_5) > R(h_2)$	h_3
The method in this paper	$R(h_3) > R(h_6) > R(h_4) > R(h_1) > R(h_5) > R(h_2)$	h_3

The analyses of three multi-person decision-making methods are as follows.

From the view of the decision results, the method in the paper [8] is unreasonable and it cannot get the best decision result, but the method in this paper can get best decision result. Hence, the method in this paper is more effective. Although the decision result in this paper is the same as the result in the paper [11], their sorting results are different. Hence, the method in this paper is feasible.

From the view of the methods, the method in the paper [8] not only is too complex, but also has strong subjectivity. The method in the paper [11] not only does not consider the influence that parameters' weights bring for the decision result, but also has large calculation. However, the method in this paper considers not only required standards of different decision makers for different parameters but also the influence that the parameters' weights bring for the decision result. Hence, the method in this paper can make the decision result more reasonable.

4. **Conclusions.** In order to solve the problem that multi-person decision-making methods exist, this paper gives the method of determining parameter weight. Based on it, a multi-person decision-making method based on weighted fuzzy soft set is proposed. Finally, the example can verify that this new multi-person decision-making method is feasible. Moreover, compared with other multi-person decision-making methods, this multi-person decision-making method is simpler, more reasonable and effective.

This paper focuses on the study of multi-person decision-making method based on weighted fuzzy soft set. The decision-making problems based on interval-valued and incomplete fuzzy soft set are the focus of next research.

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REFERENCES

[1] Z. Pawlak, Rough sets, *International Journal of Computer and Information Sciences*, vol.11, no.5, pp.341-356, 1982.
[2] W. X. Zhang, *Rough Set Theory and Method*, Science Press, Beijing, 2001.
[3] L. A. Zadeh, Fuzzy sets, *Information and Control*, vol.8, no.3, pp.338-353, 1965.
[4] L. Shui, G. Jing and G. Xiang, *Fuzzy Set Theory and Its Application*, Science Press, Beijing, 2005.
[5] D. Molodtsov, Soft set theory – First results, *Computers & Mathematics with Applications*, vol.37, no.4, pp.19-31, 1999.
[6] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, *Journal of Fuzzy Mathematics*, vol.9, no.3, pp.589-602, 2001.
[7] A. R. Roy and P. K. Maji, A fuzzy soft set theoretic approach to decision making problems, *Journal of Computational and Applied Mathematics*, vol.203, no.2, pp.412-418, 2007.
[8] B. Miu, *Research of Uncertain Multi-attribute Decision-making Method Based on Soft Set Theory*, Kunming University of Science and Technology, 2014.
[9] Q. H. He and D. B. Yao, Multi-experts group decision-making method based on fuzzy soft matrix, *Journal of Hefei University (Natural Sciences)*, vol.22, no.3, pp.29-34, 2012.
[10] C. C. Wang, *Researches about Group Decision-making Methods and Granular Analysis of Fuzzy Soft Set Information*, Anhui University, 2013.
[11] F. Yue, Z. P. Su, Y. Lu and G. F. Zhang, Comprehensive evaluation of software quality based on fuzzy soft sets, *Systems Engineering and Electronics*, vol.35, no.7, pp.1460-1466, 2013.
[12] Y. B. Shao, X. X. Zhang and Y. L. Xu, Group decision-making method based on fuzzy soft set and weighted fuzzy soft set, *Statistics and Decision*, no.19, pp.46-49, 2014.
[13] Z. W. Li, N. H. Gao and S. G. Zhang, New methods on parameter reduction of soft sets, *Control and Decision*, vol.29, no.7, pp.1285-1290, 2014.
[14] J. Ye and L. Wang, Approach of ascertaining combinatorial attribute weight based on discernibility matrix, *Computer Science*, vol.41, no.11, pp.273-277, 2014.