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FORMATION RECONFIGURATION CONTROL OF MULTIPLE UNMANNED AERIAL VEHICLES BASED ON CONSENSUS PROTOCOL

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Abstract. *Flight formation of multiple unmanned aerial vehicles (UAVs) needs to be reconfigured when the environment changes or missions are modified. The collision problem poses a great threat for formation reconfiguration. At the same time, communication delay is inevitable in process of multi-UAV formation flight. Based on the analysis of collision problem during reconfiguration, combined with improved artificial potential field method and mathematical graph theory, a distributed formation reconfiguration control algorithm is proposed based on consensus protocol for multi-UAV formation with time-varying communication delays. The simulation results show that the proposed control algorithm can realize formation reconfiguration and avoid collisions efficiently.*
Keywords: Multi-UAV, Formation reconfiguration, Collision avoidance, Consensus protocol, Communication delay

1. **Introduction.** Formation reconfiguration is one of main research contents of formation control [1]. In reconfiguration process, the location of each UAV in newly organized formation needs to reassign, and generate a track from the original location to a new location. These trajectories must be guaranteed for flight safety.

Consensus protocol was introduced into formation control of multi-agent system by Ren [2] and Liu and Tian [3]. Studies on multi-UAV formation flight control based on consensus protocol had been proposed in literature [4], but the communication delay is not considered. In view of a time-varying multi-UAV formation, a controller of the UAV formation based on consensus protocols was designed in literature [5]. It can maintain and transform UAV formation. However, these methods did not discuss the problems of anti-collision and formation reconfiguration. The UAV formation collision avoidance is controlled by artificial potential field method in literature [6]. However, this method is complicated and cannot be applied to multi-UAV formation reconfiguration.

In this paper, an improved artificial potential field method is introduced for collision avoidance and a distributed formation control algorithm is proposed based on consensus protocol. The reminder of this paper is organized as follows. The model of multi-UAV system is introduced at first. Then collision avoidance control algorithm is described in detail. Formation reconfiguration control algorithm based on consensus protocol is described in Section 4. The simulation analysis is given in Section 5. Finally, some conclusions are presented.

2. Model of Multi-UAV System.

2.1. Model of UAV. In this paper, consider a formation consists of n autonomous UAVs. The engine thrust is along the direction of flight speed. Assume that all UAVs had the same dynamic characteristics and there are no dynamics coupled between UAVs. The motion model of UAV i is described by the differential Equations (1)-(6):

$$\dot{x}_i = V_i \cos\chi_i \cos\gamma_i \tag{1}$$
$$\dot{y}_i = V_i \sin\chi_i \cos\gamma_i \tag{2}$$
$$\dot{z}_i = V_i \sin\gamma_i \tag{3}$$
$$\dot{V}_i = -g\sin\gamma_i + \frac{1}{m}\left(T_i - D_i\right) \tag{4}$$
$$\dot{\chi}_i = \frac{L_i \sin\phi_i}{m_i V_i \cos\gamma_i} \tag{5}$$
$$\dot{\gamma}_i = \frac{1}{m_i V_i}\left(L_i \cos\phi_i - m_i g \cos\gamma_i\right) \tag{6}$$

where (x_i, y_i, z_i) is the inertial position of UAV i. χ_i, γ_i, ϕ_i are azimuth, inclination and roll angle. L_i, T_i, D_i are lift, thrust and drag. m_i, V_i, g are the quality of UAV i, linear velocity and gravity acceleration. Formulae (1)-(6) are the nonlinear relationship between the state vector $\boldsymbol{X}_i = [x_i, y_i, z_i, V_i, \chi_i, \gamma_i]^T$ $(i = 1, 2, \cdots, n)$ and the actual control input vector $\boldsymbol{U}_i = [\phi_i, L_i, T_i]^T$. Let $\boldsymbol{p}_i = [x_i, y_i, z_i]^T \in \mathbb{R}^3$, $\boldsymbol{u}_i = [u_{x_i}, u_{y_i}, u_{z_i}]^T \in \mathbb{R}^3$, and take Equations (4)-(6) into the time derivative of Equations (1)-(3). The UAV model can be simplified to second order integral model:

$$\ddot{\boldsymbol{p}}_i = \boldsymbol{u}_i \tag{7}$$

where $\boldsymbol{p}_i$ is the position of UAV i, and $\boldsymbol{u}_i$ is the virtual control input for UAV. The relationship between $\boldsymbol{u}_i$ and $\boldsymbol{U}_i$ is shown in Formulae (8)-(10):

$$\phi_i = \arctan\frac{u_{y_i}\cos\chi_i - u_{x_i}\sin\chi_i}{\cos\gamma_i\left(u_{z_i} + g\right) - \sin\gamma_i\left(u_{x_i}\cos\chi_i + u_{y_i}\sin\chi_i\right)} \tag{8}$$
$$L_i = m_i\frac{\cos\gamma_i\left(u_{z_i} + g\right) - \sin\gamma_i\left(u_{x_i}\cos\chi_i + u_{y_i}\sin\chi_i\right)}{\cos\phi_i} \tag{9}$$
$$T_i = m_i\left(\sin\gamma_i\left(u_{z_i} + g\right) + \cos\gamma_i\left(u_{x_i}\cos\chi_i + u_{y_i}\sin\chi_i\right)\right) + D_i \tag{10}$$

2.2. Motion control model. In multi-agent, relative motions exist in agents. The cluster has a macro movement relative to external environment. With the rapid development of sensor network, multi-agent can accomplish some complex tasks by forming formation [7]. Therefore, the model of multi-UAV motion control can be realized as follows:

$$\boldsymbol{u}_i = \boldsymbol{u}_i^G + \boldsymbol{u}_i^O + \boldsymbol{u}_i^R \tag{11}$$

where i is UAV i; $\boldsymbol{u}_i^G$ is the control effect of moving to target points; $\boldsymbol{u}_i^O$ is the anti-collision control; $\boldsymbol{u}_i^R$ stands for the formation reconfiguration control.

3. Design of Collision Avoidance Control Algorithm for Multi-UAV Formation.

3.1. Improved artificial potential field method. In two-dimensional space, the potential field of UAV i is:

$$U_i(p) = U_{att}^G(p) + U_{rep}^O(p) \tag{12}$$

where $U_{att}^G(p)$ indicates the gravitational field of the target point, $U_{rep}^O(p)$ indicates the repulsion field caused by obstacles, and p is the position of UAV i. Under the action of two potential fields, UAV i can avoid obstacles and move towards the target point.

For UAV, the direction of the optimal path is the negative gradient direction of artificial potential field. The gravity function of improved artificial potential field method is defined as follows [8]:

$$F_{att}(p) = -\nabla\left[U_{att}^{G}(p)\right] = -\nabla\left[\frac{1}{2}k\rho^2(p,p_g)\right] = k\rho(p,p_g) \tag{13}$$

The repulsion function is:

$$F_{rep}(p) = -\nabla\left[U_{rep}^{O}(p)\right] = \begin{cases} \eta F_{rep1}(p) + \dfrac{n}{2}\eta F_{rep2}(X) & \rho(p,p_0) \le \rho_0 \\ 0 & \rho(p,p_0) > \rho_0 \end{cases} \tag{14}$$

Formula (13) is the target point control law for the control model. That is $\boldsymbol{u}_i^G$, where p_0 is the position of obstacle; p_g is the position of target; $\rho(p,p_g)$ is the shortest distance between UAV and target point; $\rho(p,p_0)$ is the shortest distance between UAV and obstacle; ρ_0 is a constant. k and η are positive proportional.

3.2. **Space obstacles collision avoidance.** In practical application, obstacles are mostly irregular 3D objects. Therefore, the modified artificial potential field method should be introduced into 3D space to ensure flight safety. In this paper, use the expansion process to deal with detected obstacles. Obstacles will be completely surrounded by circumscribed spheres. Then tangent point method is applied to achieving the aim that UAV can avoid obstacles in 3D space [9].

The vector $\boldsymbol{e}_i$ from obstacle to UAV can be calculated by the UAV's location $p_j(x_j,y_j,z_j)$ of the jth step and the circumscribed sphere i's center $p_{0i}(x_{0i},y_{0i},z_{0i})$. With radius r_i of the ith circumscribed sphere, the closest point $p_{i,j}(x_{i,j},y_{i,j},z_{i,j})$ from circumscribed sphere of the ith obstacle to UAV can be got. That is the point of tangency, as shown in Figure 1.

$$\boldsymbol{e}_i = \frac{(x_j - x_{0i}, y_j - y_{0i}, z_j - z_{0i})}{\sqrt{(x_j - x_{0i})^2 + (y_j - y_{0i}) + (z_j - z_{0i})}} \tag{15}$$

$$p_{i,j}(x_{i,j},y_{i,j},z_{i,j}) = (x_{0i} + r_i a_i, y_{0i} + r_i b_i, z_{0i} + r_i c_i) \tag{16}$$

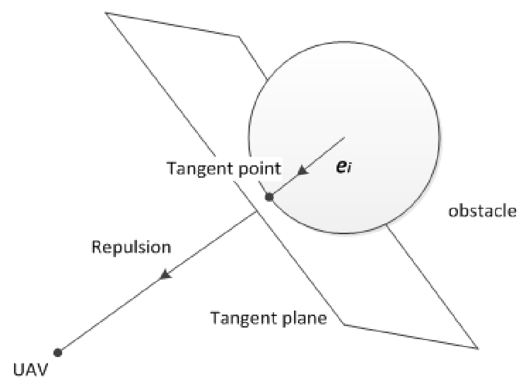

Figure 1. The repulsion schematic diagram of obstacle to UAV in 3D space

By Formulae (15) and (16), when the mobile UAV is in path point $p_j(x_j,y_j,z_j)$ of the jth step, the closest point $p_{i,j}(x_{i,j},y_{i,j},z_{i,j})$ $(i = 1,2,\ldots,n)$ from each obstacle to mobile UAV can be got. Through these points, the attraction and repulsion of obstacle to mobile UAV can be calculated. By Formulae (17)-(19), the next step of UAV can be determined. It is $\boldsymbol{u}_i^O$.

$$x_{j+1} = x_j + \gamma \times \cos\frac{\boldsymbol{F}_{attx}(p_j,p_g) + \sum_i \boldsymbol{F}_{repx}(p_j,p_{i,j})}{\boldsymbol{F}_t} \tag{17}$$

$$y_{j+1} = y_j + \gamma \times \cos \frac{\boldsymbol{F}_{atty}(p_j, p_g) + \sum_i \boldsymbol{F}_{repy}(p_j, p_{i,j})}{\boldsymbol{F}_t} \tag{18}$$

$$z_{j+1} = z_j + \gamma \times \cos \frac{\boldsymbol{F}_{attz}(p_j, p_g) + \sum_i \boldsymbol{F}_{repz}(p_j, p_{i,j})}{\boldsymbol{F}_t} \tag{19}$$

4. Design of Formation Reconfiguration Control Algorithm for Multi-UAV Based on Consensus Protocol.

4.1. Communication network topology. In multi-UAV formation, the state information is transmitted in communication. Suppose that a node in algebraic diagram represents a UAV, and an edge represents the communication relationship. So a graph can be used to represent a multi-UAV formation. In this paper, the definition of communication network topology is presented.

Definition 4.1. *Communication network topology is a weighted undirected graph, and it is expressed in* $G = \{C, E, A\}$. *Vertex set* $C = \{c_1, c_2, \cdots, c_n\}$ *expresses* n *UAVs; edge set* $E = \{(c_i, c_j) \in C \times C | c_i \sim c_j\}$ *expresses communication link between UAVs; weighted adjacency matrix* $\boldsymbol{A} = [a_{ij}], a_{ij}$ *expresses communication relationship between UAV* i *and UAV* j, $a_{ij} = a_{ji}$ *and:*

$$a_{ij} = \begin{cases} 1, & (c_i, c_j) \in E \\ 0, & (c_i, c_j) \notin E \end{cases} \tag{20}$$

4.2. UAV collision avoidance. In order to avoid multi-UAV collision, assume that the expected distance between two related UAVs is d_{ij}. When the distance between two UAVs is more than d_{ij}, it is reflected in gravitational field; when the distance between two UAVs is less than d_{ij}, it is reflected in repulsion field. According to the above scenario, the potential field control of the two related UAVs is defined as follows:

$$-\nabla \left[U_i^R(p)\right] = \begin{cases} \sum_{j \in N_i} \frac{-k_1 (p_{ij} - d_{ij})(p_{ij} - \sigma) X_i}{p_{ij}^2}, & 0 < p_{ij} \leq \sigma \\ 0, & p_{ij} \geq \sigma \end{cases} \tag{21}$$

where N_i is the adjacency set of UAV i; p_{ij} is the relative distance between UAV i and j; σ is the distance of interact relation; $k_1 > 0$ is adjustable parameter.

4.3. Formation reconfiguration control algorithm based on consensus protocol. On the premise of avoiding collision, the control target of formation reconfiguration is to reconstruct the formation from A to B. Each UAV has the fixed capability of obstacle avoidance. Consider a formation with n UAVs, the ith UAV's dynamic equation:

$$\begin{cases} \dot{\boldsymbol{p}}_i = \boldsymbol{v}_i \\ \dot{\boldsymbol{v}}_i = \boldsymbol{u}_i \end{cases} \tag{22}$$

where $\boldsymbol{p}_i \in \mathbb{R}^3$ is the position of UAV i, $\boldsymbol{v}_i \in \mathbb{R}^3$ is the speed of the UAV i, and $\boldsymbol{u}_i = \mathbb{R}^3$ is the control input for UAV i.

In this paper, the time varying communication delay is considered. In order to make multi-UAV to a fixed formation and fly at the expected speed, suppose that $\boldsymbol{p}_0 \in \mathbb{R}^3$ is the position of geometrical center of formation, $\boldsymbol{v}_0 \in \mathbb{R}^3$ is the expected speed, it is known to each UAV. Based on the consensus protocol, the distributed formation reconfiguration control algorithm is designed as follows:

$$\begin{aligned} \boldsymbol{u}_i^R = & \sum_{j \in N_i} \frac{-k_1 (p_{ij} - d_{ij})(p_{ij} - \sigma) p_i}{p_{ij}^2} + \sum_{j \in N_i} a_{ij} \left((\boldsymbol{p}_j (t - \tau(t)) - \boldsymbol{p}_i(t)) - (d_j - d_i) \right. \\ & \left. + \gamma (\boldsymbol{v}_j (t - \tau(t)) - \boldsymbol{v}_i(t)) \right) + \boldsymbol{u}_0 - \alpha \left((\boldsymbol{p}_i(t) - d_i) - \boldsymbol{p}_0(t) + \gamma (\boldsymbol{v}_i(t) - \boldsymbol{v}_0(t)) \right) \end{aligned} \tag{23}$$

In Formula (23), $\alpha > 0$, $\gamma > 0$ is gain coefficient, $d_i \in \mathbb{R}^3$ is the expected position, and $d_{ij} = \|d_i - d_j\|$ is the desired distance between two UAVs. The influence of time varying communication delay is $\tau(t)$ and d_i is time invariant. The formed formation of multi-UAV is fixed. According to the consensus protocol analysis, if $t \to \infty$, $\boldsymbol{v}_i \to \boldsymbol{v}_j \to \boldsymbol{v}_0$, $\boldsymbol{p}_i - d_i \to \boldsymbol{p}_j - d_j \to \boldsymbol{p}_0$, the UAVs will fly at the desired formation and excepted speed.

5. **Simulation Analysis.** Consider a formation with three UAVs, it took off from disorderly initial points. The communication network topology is shown in Figure 2.

Adjacency matrix of communication network topology is $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Two simulation examples of multi-UAV formation flight are given below. Among them, communication delay $\tau = 0.5$, $\rho = 0.5$, gain coefficient $\alpha = \gamma = 1$.

Multi-UAV formation reconfiguration simulation under the environment without space obstacles: assume that the initial positions of three UAVs are $(2000, 1000, 3000)$, $(1000, 400, 3000)$, $(2200, 1100, 3000)$ (m). All initial velocity is 100 m/s and the direction is arbitrary. The purpose of controlling is making three UAVs to a triangle formation first, and then reconstructing the formation as a "–". Finally, the formation flight is at a uniform speed of 160 m/s at a height.

When the control input is (8-10), the simulation results are shown in Figures 3 to 5. Figure 3 shows that three UAVs form a triangle formation at first, and then reconstruct the formation as a "–". Figure 4 shows the velocity curves of three UAVs, and the formation will fly at expected speed of 160 m/s. Figure 5 shows the relative distance

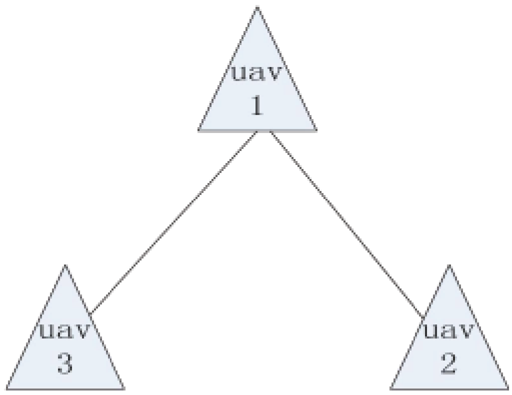

FIGURE 2. Communication topology graph of three UAVs

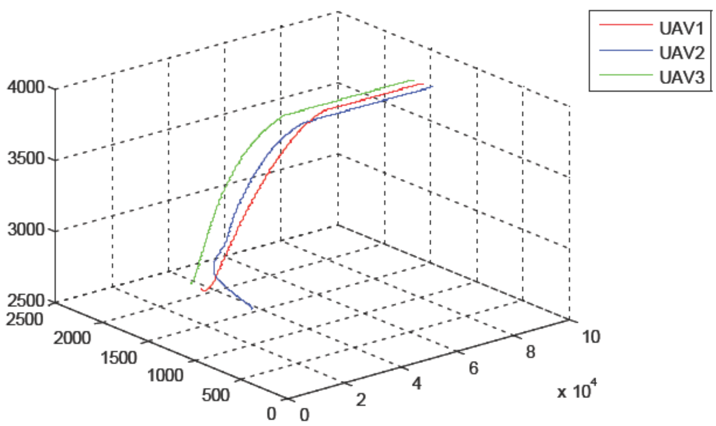

FIGURE 3. Flight trajectories of three UAVs

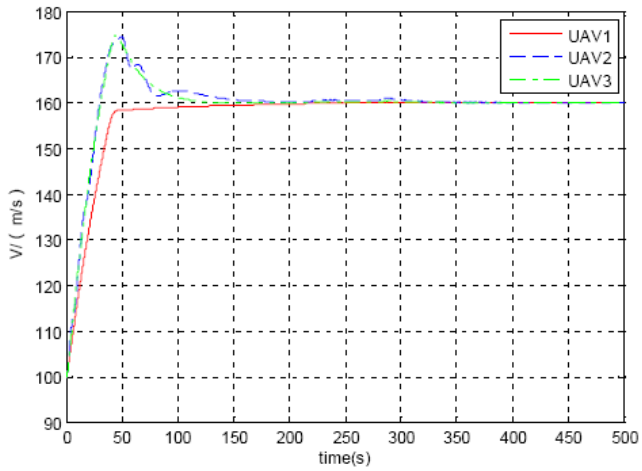

FIGURE 4. Velocity curves of three UAVs

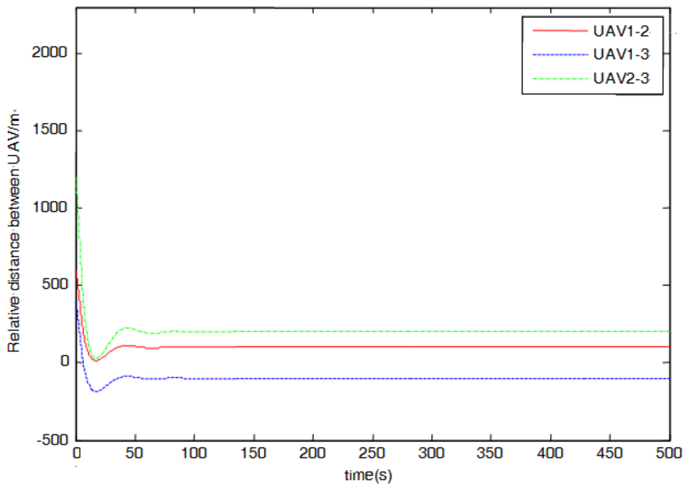

FIGURE 5. Relative distance curves between pairs of UAVs

variation of three UAVs in formation. After the formation is stable, the relative distance between UAV2 and UAV1 is 100 m, the relative distance between UAV3 and UAV1 is −100 m, the distance between UAV2 and UAV3 is 200 m.

Multi-UAV formation reconfiguration simulation under the environment with space obstacles: assume that the initial positions of three UAVs are $(2000, 1000, 3000)$, $(1000, 800, 2500)$, $(2000, 1200, 2500)$ (m). The initial positions of obstacles are $(50000, 1050, 3850)$, $(28000, 350, 3800)$, $(1000, 400, 3250)$, $(28000, 950, 3750)$ (m).

The simulation results of multi-UAV formation reconfiguration are shown in Figures 6 to 8. Figure 6 shows the flight trajectories of three UAVs , it can be seen that three UAVs can avoid collision in obstacles space, and the final formation is "–". Figure 7 shows the velocity curves of three UAVs, the formation will fly at the expected speed of 160 m/s. Figure 8 shows the relative distance variation between pairs UAVs in the formation, and UAV will not crash.

6. **Conclusions.** In this paper, the artificial potential field algorithm is improved by using tangential method to realize anti-collision in three-dimensional space. Combined with this algorithm and the knowledge of graph theory, a distributed formation reconfiguration control algorithm is proposed based on consensus protocol for multi-UAV formation. Two simulation experiments show the effectiveness and feasibility of the proposed algorithm. UAVs' locations and speeds can be uniformly convergent. In future, the system with

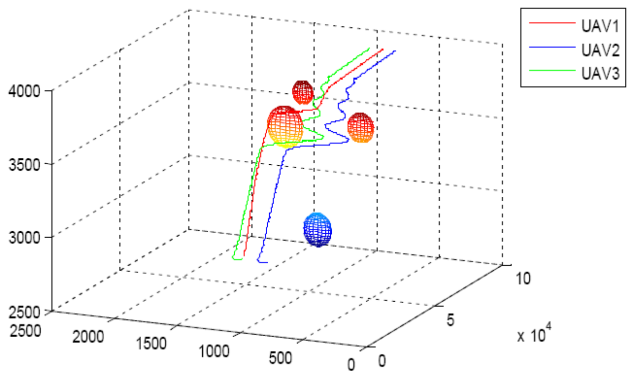

FIGURE 6. Flight trajectories of three UAVs in obstacles space

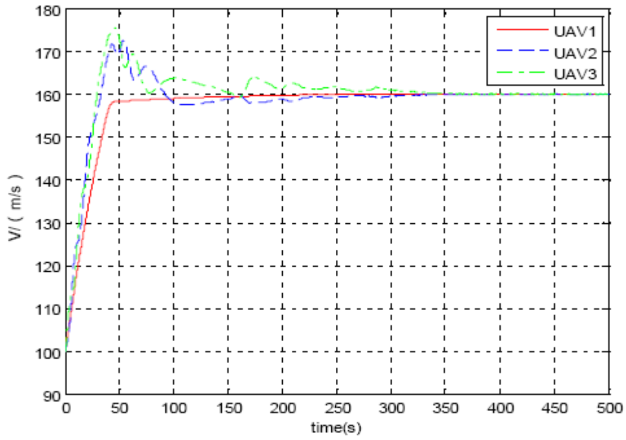

FIGURE 7. Velocity curves of three UAVs

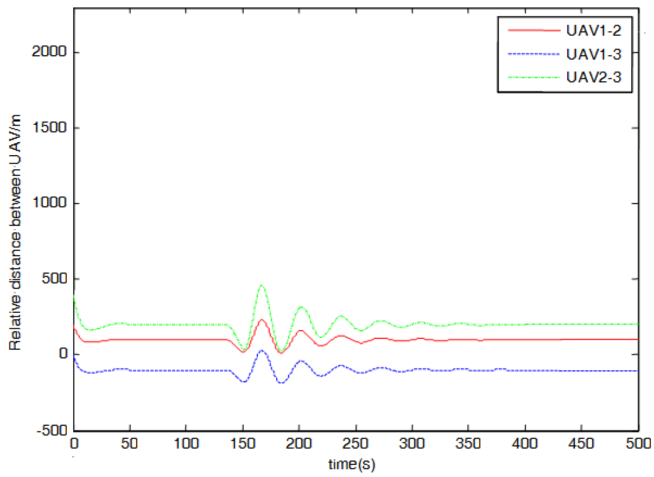

FIGURE 8. Relative distance curves between pairs of UAVs

complex obstacles and the change rate of communication delay is unknown need to be further studied.

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