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A DESIGN METHOD FOR STABILIZING MODIFIED SMITH PREDICTOR FOR NON-SQUARE MULTIPLE TIME-DELAY PLANTS

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Abstract. *In this paper, we examine a design method for stabilizing modified Smith predictors for non-square multiple time-delay plants. The modified Smith predictor is well known as an effective time-delay compensator for a plant with large time-delay, and several papers considered the problem to design modified Smith predictors for time-delay plants. At the same time, another important control problem is the parameterization problem, the problem of finding all stabilizing controllers for a plant. By several studies, the parameterization of all stabilizing modified Smith predictors for unstable time-delay plants was clarified. In some cases, the plant includes multiple time-delays, but those parameterizations cannot be applied to multiple time-delay plants. After that, the parameterization for multiple-input/multiple-output multiple time-delay plants was proposed. Since it is still not including the parameterization for non-square multiple-input/multiple-output multiple time-delay plant, it needs further study. In this paper, we propose the parameterization of all stabilizing modified Smith predictors for non-square multiple time-delay plants. And we show the characteristics of control system using the parameterization of all stabilizing modified Smith predictors.*
Keywords: Smith predictor, Parameterization, Non-square plant, Multiple time-delay

1. **Introduction.** In this paper, we examine a design method for modified Smith predictors for non-square multiple time-delay plants. The Smith predictor was proposed by Smith in order to overcome time-delay [1]. And it is known as an effective time-delay compensator for a stable plant with large time-delay [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. However, the Smith predictor in [1] cannot be used for plants having an integral mode, because a step disturbance will result in a steady state error [2, 3, 4]. In order to overcome this problem, Watanabe and Ito [4], Astrom et al. [9], and Matusek and Micic [10] proposed a design method for a modified Smith predictor for time-delay plants with an integrator. Watanabe and Sato expanded the result in [4] and proposed a design method for modified Smith predictors for multivariable systems with multiple time-delays in inputs and outputs [5].

Because the modified Smith predictor cannot be used for unstable plants [2, 3, 4, 5, 6, 8, 7, 9, 10, 11], De Paor [6], De Paor and Egan [8] and Kwak et al. [12] proposed a design method for modified Smith predictors for unstable plants. Thus, several design methods of modified Smith predictors have been published.

On the other hand, another important control issue is the parameterization problem, which is the problem of finding all stabilizing controllers for a plant [13, 14, 15, 16, 17, 18, 19, 20, 21]. The parameterization of all stabilizing controllers for time-delay plants was considered in [20, 21], but that of all stabilizing modified Smith predictors was not obtained. After that, Yamada and Matsushima [22] gave a resolution. They showed the parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants. Then, Yamada et al. [23] expanded the result in [22] and clarified the parametrization of all stabilizing modified Smith predictors for non-minimum-phase systems. And then, Yamada et al. [24] clarified the parameterization of all stabilizing modified Smith predictors for multiple-input/multiple-output non-minimum-phase time-delay plants. Since the parametrization of all stabilizing modified Smith predictor is obtained, we can express previous studies of modified Smith predictors in a uniform manner. In addition, modified Smith predictors could be designed systematically. In some cases, the system includes multiple time-delays. And, the parameterizations in [22, 23, 24] cannot be applied to multiple time-delay plants. After that, Mai and Yamada [25] expanded that parameterizations [22, 23, 24], and proposed the parameterization of all stabilizing modified Smith predictors for multiple-input/multiple-output non-minimum phase multiple time-delay plants. However, there still remains the problem that it is not including the parameterization for non-square multiple-input/multiple-output plant. Since there are many actual plants which are non-square multiple-input/multiple-output, it is important to obtain the parameterization of all stabilizing modified Smith predictors for non-square multiple time-delay plants.

The purpose of this paper is to propose the parameterization of all stabilizing modified Smith predictors for non-square stable multiple time-delay plants. First, the structure and necessary characteristics of modified Smith predictors are defined. Next, the parameterization of all stabilizing modified Smith predictors for non-square multiple time-delay stable plants is proposed.

2. **Problem Formulation.** Consider the control system in Figure 1. Here, $G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}$ is the multiple time-delay plant, $T_i > 0$ $(i = 1, 2)$, $G_i(s) \in R^{m\times p}(s)$ $(i = 1, 2)$, $m \le p$, $C(s)$ is the controller, $y \in R^m$ is the output, $u \in R^p$ is the control input, $d \in R^m$ is the disturbance and $r \in R^m$ is the reference input. We assume that $G_i(s) \in R^{m\times p}(s)$ $(i = 1, 2)$ is coprime, that is, $G_i(s)$ $(i = 1, 2)$ is controllable and observable. In addition, $G_i(s) \in R^{p\times p}(s)$ $(i = 1, 2)$ is assumed to satisfy rank $G_i(s) = m$ $(i = 1, 2)$. Without loss of generality, $T_1 \ne T_2$ is satisfied. Note that for easy explanation, the plant is assumed to have only 2 time-delays.

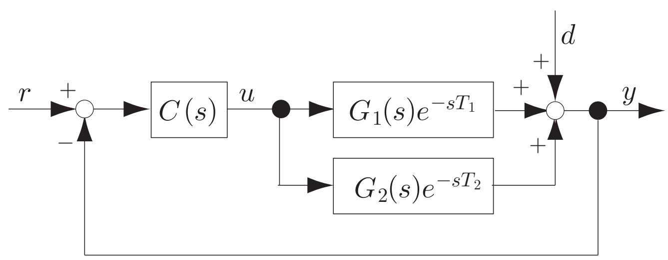

FIGURE 1. Feedback control system for $G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}$

According to past studies, the modified Smith predictor $C(s)$ for $G_1(s)e^{-sT_1}+G_2(s)e^{-sT_2}$ is decided by the form:

$$C(s) = C_1(s)\left\{I + C_2(s)e^{-sT_1} + C_3(s)e^{-sT_2}\right\}^{-1}, \quad (1)$$

where $C_1(s) \in R^{p\times m}(s)$, $C_2(s) \in R^{m\times m}(s)$ and $C_3(s) \in R^{m\times m}(s)$. In addition, using the modified Smith predictor in [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12], the transfer function from r to y of the control system in Figure 1, written as

$$y = \left\{I + \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right)C(s)\right\}^{-1}\left\{\left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right)C(s)\right\}r \quad (2)$$

has a finite number of poles. That is, the transfer function from r to y of the control system in Figure 1 is written as

$$y = \left(\bar{G}_1(s)e^{-sT_1} + \bar{G}_2(s)e^{-sT_2}\right) r, \tag{3}$$

where $\bar{G}_i(s) \in RH_\infty^{m\times m}$ $(i = 1, 2)$. Therefore, we call $C(s)$ the modified Smith predictor if $C(s)$ takes the form of (1) and the transfer function from r to y of the control system in Figure 1 has a finite number of poles.

The problem considered in this paper is to obtain the parameterization of all modified Smith predictors $C(s)$ that make the control system in Figure 1 stable. In Section 3, we propose the parameterization of all stabilizing modified Smith predictors $C(s)$ for stable plants.

3. **The Parameterization for Stable Plants.** The parameterization of all stabilizing modified Smith predictors for stable plant $G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}$ is summarized in the following theorem.

Theorem 3.1. *$G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}$ is assumed to be stable. The parameterization of all stabilizing modified Smith predictors $C(s)$ takes the form*

$$C(s) = Q(s)\left\{I - \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) Q(s)\right\}^{-1}, \tag{4}$$

where $Q(s) \in RH_\infty^{p\times m}$ is any function.

Proof: First, the necessity is shown. That is, we show that if the controller $C(s)$ in (1) makes the control system in Figure 1 stable and makes the transfer function from r to y of the control system in Figure 1 have a finite number of poles, then $C(s)$ takes the form of (4). From the assumption that the controller $C(s)$ in (1) makes the transfer function from r to y of the control system in Figure 1 have a finite number of poles,

$$\begin{aligned} &\left\{I + \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s)\right\}^{-1} \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s) \\ &= \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C_1(s)\left\{I + \left(C_2(s) + G_1(s)C_1(s)\right)e^{-sT_1} \right. \\ &\quad \left. + \left(C_3(s) + G_2(s)C_1(s)\right)e^{-sT_2}\right\}^{-1} \end{aligned} \tag{5}$$

has a finite number of poles. This implies that

$$C_2(s) = -G_1(s)C_1(s) \tag{6}$$

and

$$C_3(s) = -G_2(s)C_1(s) \tag{7}$$

are necessary, that is:

$$C(s) = C_1(s)\left\{I - \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C_1(s)\right\}^{-1}. \tag{8}$$

From the assumption that $C(s)$ in (1) makes the control system in Figure 1 stable, $\left\{I + \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s)\right\}^{-1} \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s)$, $C(s)\left\{I+\left(G_1(s)e^{-sT_1}+G_2(s)e^{-sT_2}\right) C(s)\right\}^{-1}$, $\left\{I+\left(G_1(s)e^{-sT_1}+G_2(s)e^{-sT_2}\right) C(s)\right\}^{-1} \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right)$ and $\left\{I + \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s)\right\}^{-1}$ are stable. From simple manipulation, (6) and (7), we have

$$\begin{aligned} &\left\{I + \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s)\right\}^{-1} \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s) \\ &= \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C_1(s), \end{aligned} \tag{9}$$

$$C(s)\left\{I + \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s)\right\}^{-1} = C_1(s), \tag{10}$$

$$\begin{aligned} &\left\{I + \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s)\right\}^{-1} \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) \\ &= \left\{I - \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C_1(s)\right\} \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) \end{aligned} \tag{11}$$

and

$$\left\{I + \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s)\right\}^{-1} = I - \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C_1(s). \quad (12)$$

It is obvious that the necessary condition for all the transfer functions in (9), (10), (11) and (12) to be stable is $C_1(s) \in RH_\infty^{p\times m}$. Using $Q(s) \in RH_\infty^{p\times m}$, let $C_1(s)$ be

$$C_1(s) = Q(s), \quad (13)$$

we find that $C(s)$ takes the form of (4). Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, if $C(s)$ takes the form of (4) and $Q(s) \in RH_\infty$, then the controller $C(s)$ makes the control system in Figure 1 stable and makes the transfer function from r to y of the control system in Figure 1 have a finite number of poles. From simple manipulation, we have

$$\begin{aligned} &\left\{I + \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s)\right\}^{-1} \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s) \\ &= \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) Q(s), \end{aligned} \quad (14)$$

$$C(s)\left\{I + \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s)\right\}^{-1} = Q(s), \quad (15)$$

$$\begin{aligned} &\left\{I + \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s)\right\}^{-1} \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) \\ &= \left\{I - \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) Q(s)\right\} \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) \end{aligned} \quad (16)$$

and

$$\left\{I + \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) C(s)\right\}^{-1} = I - \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) Q(s). \quad (17)$$

From the assumption that $G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}$ is stable and $Q(s) \in RH_\infty^{p\times m}$, (14), (15), (16) and (17) are all stable. In addition, because the transfer function from r to y of the control system in Figure 1 takes the form (14) and $Q(s) \in RH_\infty$, the transfer function from r to y of the control system in Figure 1 has a finite number of poles.

We have thus proved Theorem 3.1.

Next, we explain the control characteristics of the control system using the parameterization of all stabilizing modified Smith predictors in (4). Due to (4), the transfer function from the reference input r to the output y of the control system in Figure 1 takes the form

$$y = \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) Q(s)r. \quad (18)$$

Therefore, for the output y to follow the step reference input r without steady state error,

$$\left(G_1(0) + G_2(0)\right) Q(0) = I \quad (19)$$

must be satisfied.

The disturbance attenuation characteristic is as follows. The transfer function from the disturbance d to the output y of the control system in Figure 1 is given by

$$y = \left\{I - \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) Q(s)\right\} d. \quad (20)$$

Therefore, to attenuate the step disturbance d effectively, $Q(s)$ must satisfy

$$\left(G_1(0) + G_2(0)\right) Q(0) = I. \quad (21)$$

4. **Conclusions.** In this paper, we proposed the parameterization of all stabilizing modified Smith predictors for non-square multiple time-delay stable plants. The control characteristics of the control system using the parameterization of all stabilizing modified Smith predictors were also given.

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