

## AN IMPROVED $\alpha$ - $\beta$ FILTER FOR MANEUVERING TARGET TRACKING

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**ABSTRACT.** *To improve the precision of maneuvering target tracking, an improved  $\alpha$ - $\beta$  filter is proposed in this paper. First, we define the relationship between available measurements and predictions. Afterwards, the ratio of motion stability is put forward to adaptively adjust filtering gains and position deviation in the predicted wave-gate. Finally, a typical scheme of maneuvering target tracking is performed. Simulation results have been carried out to confirm the validity performance of the proposed filter.*

**Keywords:** Maneuvering target, Filtering gain, Motion state, Target tracking

1. **Introduction.** The Kalman filter (KF) can predict and update position of targets with available measurements. As an optimal filtering algorithm, it minimizes state error as the target dynamics are modeled. However, it cannot well represent the rapid maneuverability, which leads to the velocity and position estimates deviated from the true motion state of targets [1]. In contrast, the  $\alpha$ - $\beta$  filter is a basic discrete-time algorithm based on the Kalman prediction-update structure and can obtain the steady-state solution of the KF with exponentially reduced computation [2,3].

Recently, the scholars have been dedicated themselves to this field with a great deal of achievements, and many papers are one after another published in important international journals [4-7]. In [4], an adaptive  $\alpha$ - $\beta$  filter was introduced to compare with the KF, and then the effectiveness was validated. Subsequently, a novel  $\alpha$ - $\beta$  filter based on the fuzzy logic was proposed for determining filtering parameters in [5]. In [6], an adaptive  $\alpha$ - $\beta$  filtering algorithm in the rectangular coordinate system was proposed for the automatic radar plotting aid (ARPA) system. Vinaykumar and Jatoth [7] designed a new  $\alpha$ - $\beta$  filter for target tracking under the noisy and noiseless condition and obtained the expected results. Based on the adaptive theory and the generalized pseudo Bayesian (GPB) theory, the above filters have high accuracy for non-maneuvering target tracking. Nevertheless, they are not applicable to maneuvering target tracking owing to the constant gains  $\alpha$  and  $\beta$ .

To solve this problem, we present an improved  $\alpha$ - $\beta$  filter for maneuvering target tracking in this paper. The proposed filter mainly focuses on the ratio of motion stability, which can adaptively adjust the velocity and position estimates when the motion state changes. The remainder of this note is organized as follows. In Section 2, the standard  $\alpha$ - $\beta$  filter is analyzed and its limitations are discussed. Section 3 presents the principle of the proposed  $\alpha$ - $\beta$  filter. In Section 4, the implementation of the proposed  $\alpha$ - $\beta$  filter is explored. In Section 5, the numerical simulations are presented with results to validate the tracking

performance for maneuvering target. Finally, Section 6 draws the conclusions and future work.

**2. Problem Statements.** Considering a 2-dimension state vector  $\mathbf{X}_k = [x_k \ \dot{x}_k \ y_k \ \dot{y}_k]^\top$  with the planar position  $(x_k, y_k)$  and velocity  $(\dot{x}_k, \dot{y}_k)$  at scan  $k$ , we have the dynamic equations of the standard  $\alpha$ - $\beta$  filter as follows:

$$\hat{\mathbf{X}}_k = \mathbf{X}_{k|k-1} + \mathbf{K} (\mathbf{X}_k - \mathbf{X}_{k|k-1}) \quad (1)$$

$$\mathbf{X}_{k+1|k} = \Phi \hat{\mathbf{X}}_k \quad (2)$$

where  $\hat{\mathbf{X}}_k$  and  $\mathbf{X}_{k|k-1}$  are the estimated state and the predicted state,  $\Phi$  is the state transition matrix,  $[\cdot]^\top$  denotes the transpose of matrix, and  $\mathbf{K}$  is the filtering gain, i.e.,

$$\mathbf{K} = \text{diag}[\alpha \ \beta/T \ \alpha \ \beta/T] \quad (3)$$

In (3),  $T$  is the sampling period,  $\text{diag}[\cdot]$  denotes the diagonal matrix, and  $\alpha$  represents the level of position difference between the measurement and the predicted value. Similarly,  $\beta$  reflects the level of velocity difference. Usually,  $\alpha$  and  $\beta$  are given by

$$\alpha = \frac{2(2k-1)}{k(k+1)} \quad (4)$$

$$\beta = \frac{6}{k(k+1)} \quad (5)$$

In (4) and (5), we note that  $\alpha$  and  $\beta$  would gradually decrease with the increase of the scan  $k$ . When  $k$  tends to infinity, both  $\alpha$  and  $\beta$  go to zero [8,9]. At this time, if the change of motion state is fast or the measurement noise is great, the maneuvering state cannot be effectively represented. Therefore, the standard  $\alpha$ - $\beta$  filter can hardly estimate the velocity and position of maneuvering targets.

**3. Principle of the Proposed Filter.** First, we rewrite (1) using the matrix element.

$$\begin{cases} \hat{x}_k = x_{k|k-1} + \alpha (x_k - x_{k|k-1}) \\ \hat{\dot{x}}_k = \dot{x}_{k|k-1} + \beta (x_k - x_{k|k-1})/T \\ \hat{y}_k = y_{k|k-1} + \alpha (y_k - y_{k|k-1}) \\ \hat{\dot{y}}_k = \dot{y}_{k|k-1} + \beta (y_k - y_{k|k-1})/T \end{cases} \quad (6)$$

where  $(x_{k|k-1}, y_{k|k-1})$  and  $(\hat{x}_k, \hat{y}_k)$  are the predicted position and the updated position, and  $(\dot{x}_{k|k-1}, \dot{y}_{k|k-1})$  and  $(\hat{\dot{x}}_k, \hat{\dot{y}}_k)$  are the predicted velocity and the updated velocity.

Then, we define  $\beta$  as follows:

$$\beta = 2(2 - \alpha) - 4\sqrt{1 - \alpha} = 2(1 - \sqrt{1 - \alpha})^2 \quad (7)$$

Substituting (4) into (7), we have

$$\beta = 2 \left( 1 - \sqrt{\frac{(k-1)(k-2)}{k(k+1)}} \right)^2 \quad (8)$$

In (8), there is  $\beta = 2$  when  $k = 1, 2$ . If  $k \geq 3$ , the value of  $\beta$  starts to convergence, which means the  $\alpha$ - $\beta$  filter can regularly work from the 3rd scan.

Based on (2), the updated state can be rewritten as

$$\begin{cases} x_{k+1|k} = \hat{x}_k + \hat{x}_k T \\ \dot{x}_{k+1|k} = \hat{\dot{x}}_k \\ y_{k+1|k} = \hat{y}_k + \hat{y}_k T \\ \dot{y}_{k+1|k} = \hat{\dot{y}}_k \end{cases} \quad (9)$$

Define the relative distance between the measurement and the prediction as follows:

$$d = \sqrt{(x_k - x_{k|k-1})^2 + (y_k - y_{k|k-1})^2} \quad (10)$$

For the single sensor system, assume  $\rho$  is the radius of the predicted position, and we have the distance error.

$$\Delta d = \lambda \sqrt{(\Delta \rho)^2 + \rho^2 (\Delta \theta)^2} \quad (11)$$

where  $\Delta \rho$  and  $\Delta \theta$  are the radius error and the bearing error respectively, and  $\lambda$  is the position deviation between the measurement and the prediction.

$$\lambda = \sqrt{\alpha} \quad (12)$$

Figure 1 shows the above parameters in  $x$ - $y$  coordinate. As seen, the predicted position  $(x_{k|k-1}, y_{k|k-1})$  is as the central location of the predicted wave-gate  $\Delta x \times \Delta y$  (length:  $\Delta x$ , width:  $\Delta y$ ), and then the size of the updated wave-gate is scaled by  $\lambda$ .

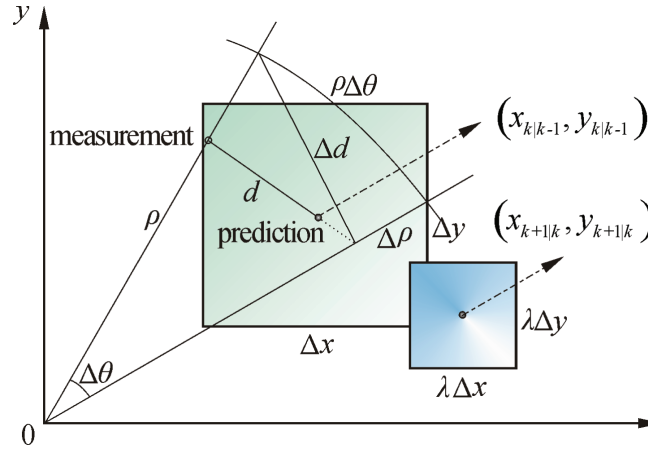


FIGURE 1. Related parameters in  $x$ - $y$  coordinate

Subsequently, we determine the motion stability with the ratio  $r_k$  between  $d$  and  $\Delta d$ .

$$r_k = d / \Delta d \quad (13)$$

Since the target maneuvers increase with the decreasing  $r_k$ , we can get the values of  $\alpha$ ,  $\beta$  and  $\lambda$  according to the index  $r_k$ . Assuming  $R$  is the maximum value of  $r_k$ , the motion state can be regarded as stable in the case of  $r_k \leq R$ , and then  $r_k + 1$ . Otherwise, the target is maneuvering, and we should decrease the value of  $r_k$  by the difference  $\Delta r_k$ .

$$\Delta r_k = \text{round}((r_k - R) r_k) \quad (14)$$

where  $\text{round}(\cdot)$  denotes the nearest integer.

Then, the updated ratio of distance is

$$r_{k+1} = r_k - \Delta r_k \quad (15)$$

**4. Implementation of the Proposed Filter.** The process of the proposed  $\alpha$ - $\beta$  filter is described as follows:

- Compute the filtering gains  $\alpha$  and  $\beta$  using (4) and (8);

- Compute the position deviation  $\lambda$  using (12);
- Within the predicted wave-gate  $\Delta x \times \Delta y$  and its center  $(x_{k|k-1}, y_{k|k-1})$ , compute the time-updated position  $(\hat{x}_k, \hat{y}_k)$  and velocity  $(\dot{\hat{x}}_k, \dot{\hat{y}}_k)$  using (6);
- Compute the relative distance  $d$  and the distance error  $\Delta d$  using (10) and (11);
- Compute the ratio of motion stability  $r_k$  using (13);
- If the motion state is stable, i.e.,  $r_k \leq R$ , then  $r_k = r_{k-1} + 1$ ; otherwise, compute the difference  $\Delta r_k$  and the updated  $r_{k+1}$  using (14) and (15);
- Compute the measurement-updated position  $(x_{k+1|k}, y_{k+1|k})$  and velocity  $(\dot{x}_{k+1|k}, \dot{y}_{k+1|k})$  using (9); redefine the size of the updated wave-gate  $\lambda \Delta x \times \lambda \Delta y$  and its center  $(x_{k+1|k}, y_{k+1|k})$ .

**5. Experimental Results and Discussions.** The numerical study is presented to evaluate the proposed  $\alpha$ - $\beta$  filter for maneuvering target tracking. During the surveillance period of 100 scans, the target has the constant velocity (CV) motion state and the constant acceleration (CA) motion state. From the initial position (0, 0) m, the target moves with the velocity of (10, 5) m/s. Especially, the maneuvering stages are as follows: the acceleration is (2, 0) m/s<sup>2</sup> during scans 12 ~ 16; the acceleration is (0, 2) m/s<sup>2</sup> during scans 32 ~ 36; the acceleration is (0, -1.5) m/s<sup>2</sup> during scans 53 ~ 73; the acceleration is (-1.5, 0) m/s<sup>2</sup> during scans 82 ~ 86. In the scenario, 100 Monte Carlo runs are done to obtain the comparison results. Further, we define the sampling period  $T$  is 1 s and the covariance of noise is 1.

Figure 2 shows the target trajectory and estimates. In this figure, we note that the proposed  $\alpha$ - $\beta$  filter can effectively estimate the target in  $x$  and  $y$  positions. The estimated position is near the true trajectory. For comparison, the most serious deviation comes from the standard  $\alpha$ - $\beta$  filter. Figure 3 demonstrates the root of mean square error (RMSE) of  $x$  and  $y$  velocities against time. Since the RMSE of velocity is dominated by target maneuvers, the proposed  $\alpha$ - $\beta$  filter can adjust the velocity estimates when the motion state changes. We can conclude that the velocity estimates are more accurate in the proposed  $\alpha$ - $\beta$  filter from the 3rd scan. Especially, it represents the change of  $x$  and  $y$  velocities rapidly when the motion dynamics are switching. In Figure 4, the obtained results for the RMSE of  $x$  and  $y$  positions are presented. It can be observed that the performance of the standard  $\alpha$ - $\beta$  filter is worse because it exaggerates the biased position as the target is maneuvering. In contrast, it can be verified that the proposed  $\alpha$ - $\beta$  filter

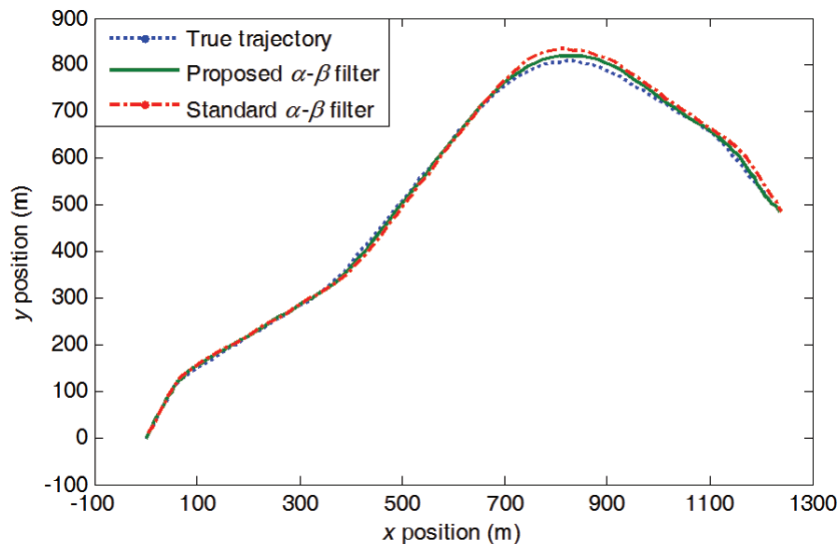


FIGURE 2. Target trajectory and estimates

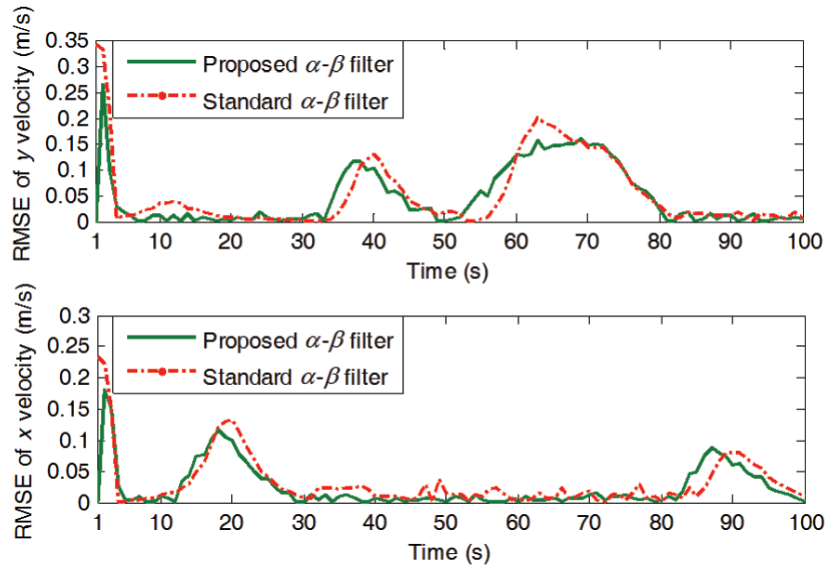


FIGURE 3. RMSE of  $x$  and  $y$  velocities

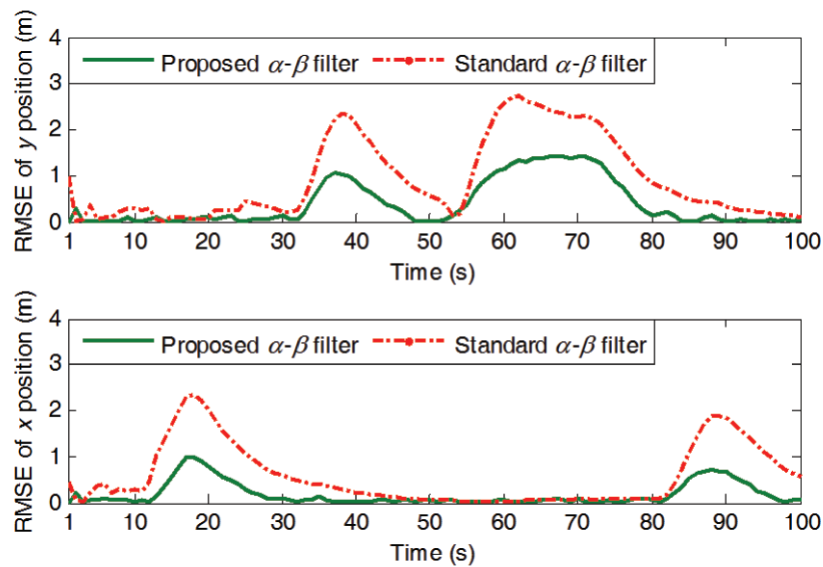


FIGURE 4. RMSE of  $x$  and  $y$  positions

achieves better performance whether the target is maneuvering or non-maneuvering. As a result, the RMSE of position performs a minimum distance permutation between the target position and prediction.

In accordance with the strategy, we further analyze the computational cost during 100 Monte Carlo runs. It is reported that the running time of the proposed  $\alpha$ - $\beta$  filter is 2.386 s, which is more than that of the standard filter (2.071 s). In other words, the proposed filter expends 15.21% extra cost associated with computing the ratio of motion stability, which the standard  $\alpha$ - $\beta$  filter does not. As for the overall tracking performance, the proposed filter is acceptable. It has slightly more computational expense while estimating the target state more accurately.

**6. Conclusions.** This paper introduces an improved  $\alpha$ - $\beta$  filter for maneuvering target tracking. Employing the ratio of motion stability, we enhance the tracking precision, particularly when the motion state is maneuvering. The simulation results suggest that the proposed filter can effectively represent the motion state of maneuvering target with

a remarkable improvement. In the future work, how to reduce the computational cost should be further considered.

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#### REFERENCES

- [1] L. W. He, Y. P. Li and B. Fan, An improved adaptive  $\alpha$ - $\beta$  tracking filter algorithm, *Modern Electronics Technique*, vol.35, no.21, pp.28-30, 2012.
- [2] P. F. Li, J. P. Yu and L. Q. Li, Maneuvering target tracking based on fuzzy adaptive  $\alpha$ - $\beta$  filter, *Systems Engineering and Electronics*, vol.30, no.11, pp.2138-2141, 2008.
- [3] L. P. Xu, An adaptive  $\alpha$ - $\beta$  filter algorithm for tracking the maneuvering target, *Journal of Xidian University*, vol.25, no.3, pp.314-317, 1998.
- [4] W. Zhu and C. H. Xia, Adaptive  $\alpha$ - $\beta$  filter algorithm, *Computer Applications*, vol.27, no.8, pp.2053-2055, 2007.
- [5] T. E. Lee, K. H. Hsia, K. W. Yu and C. C. Wang, Design of an  $\alpha$ - $\beta$  filter by combining fuzzy logic with evolutionary methods, *Proc. of International Symposium on Computer, Communication, Control and Automation*, pp.270-273, 2010.
- [6] Y. J. Ran, H. Li and H. L. Li, Study on adaptive  $\alpha$ - $\beta$  filtering used in ARPA system of the marine radar, *Fire Control Radar Technology*, vol.41, no.4, pp.39-42, 2012.
- [7] M. Vinaykumar and R. K. Jatoth, Performance evaluation of  $\alpha$ - $\beta$  and Kalman filter for object tracking, *Proc. of IEEE International Conference on Advanced Communication Control and Computing Technologies*, pp.1369-1372, 2014.
- [8] P. R. Kalata, The tracking index: A generalized parameter for at  $\alpha$ - $\beta$  and  $\alpha$ - $\beta$ - $\gamma$  target trackers, *IEEE Trans. Aerospace and Electronic Systems*, vol.20, no.2, pp.174-182, 1984.
- [9] Y. He, *Radar Data Processing with Applications*, Publishing House of Electronic Industry, 2009.