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FINANCIAL RISK PROPAGATION MODEL BASED ON PATTERN BASED SPECTRAL CLUSTERING IN DIRECTED WEIGHTED NETWORK

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Abstract. *In this paper, we proposed a financial risk propagation model based on pattern based spectral clustering in directed weighted network. In the first step, the univariate GARCH model was used to characterize daily volatility of financial markets. In the second step, the Granger causality test model was used to calculate pair-wise volatility spillover between each two financial markets. In the third step, the pattern based spectral clustering in directed weighted network was used to describe risk propagation property of all the considered financial markets. At last, the proposed model was used to analyze the financial risk propagation among the main European countries during the beginning period of the European sovereign debt crisis.*

Keywords: Financial risk propagation model, GARCH model, Granger causality test model, Directed weighted network, Spectral clustering

1. **Introduction.** Over the past years, international financial markets have become increasingly volatile after the financial liberalization and opening of economies, and this phenomenon has generated increasing interest in the analysis of financial risk propagation [1]. There is a lot of literature devoted to this topic. Meng and Liang attempted to make use of a Copula-based GARCH Model to find out the relationships between the volatility of rubber futures returns in the Agricultural Futures Exchange of Thailand and other four main markets [2]. Lean and Teng examined the financial integration of two world leaders and two emerging powers into the Malaysian stock market. A DCC-MGARCH approach was employed to examine the correlations among these countries in a time-variant manner to indicate the degree of financial integration among the countries [3]. Niu and Wang investigated the statistical behaviors of long-range dependence phenomena and volatility clustering of logarithmic returns for a financial price model and two real financial market indexes [4]. Fernandez studied the existence of feedback effects between volatility and institutional investor holdings by investigating this issue for Pension Fund Administrators in Chile [5]. Hafner and Herwartz introduced a new concept of impulse response functions tracing the effects of independent shocks on volatility through time while avoiding typical orthogonalization and ordering problems [6]. Sharifi-Renani and Mirfatah used the Johansen cointegration system approach model to evaluate the determinants of inward FDI particularly volatility of exchange rate in Iran [7]. Kuttu used multivariate VAR-EGARCH to examine the returns and volatility dynamics between thin-traded adjusted equity returns from Ghana, Kenya, Nigeria and South Africa [8]. Amado and Teräsvirta proposed two parametric alternatives to the standard GJR-GARCH model of Glosten et al., based on additive and multiplicative decompositions of the variance [9]. Bastos and Caiado introduced a new distance measure for clustering financial time series based on variance ratio test statistics [10]. D'Urso et al. addressed the topic of classifying financial

time series in a fuzzy framework proposing two fuzzy clustering models both based on GARCH models [11].

The literature mentioned above mainly used traditional financial time series models, namely, ARCH series models, VAR model (including Granger causality analysis, variance decomposition and impulse response), Copula method, Johansen cointegration test model, and DCC model to analyze the propagation of the financial risk. The others used clustering methods to analyze comovement of the financial markets. Main merit of the traditional financial time series models is the ability to characterize volatility spillover among the different financial markets, but shortcoming of the models is that they could not capture the nature of the whole markets. Clustering methods can be used to analyze comovement of the whole markets, but it cannot characterize volatility spillover among the different financial markets. Meanwhile, most of the existing clustering methods neglect asymmetry of the edges of networks which represent similarities between data sets. However, in real applications, similarities among data sets are not always symmetric, for example, studies showed that volatility spillover from the American stock markets to the China's stock markets is not always the same with the volatility spillover from the China's stock markets to the American stock markets. The presence of directed edges implies more sophisticated types of clusters that do not exist in undirected networks and cannot be captured using only traditional density and edge concentration characteristics. More precisely, the nodes of a directed network can be naturally clustered together according to similar connectivity patterns that may exist and are not captured completely applying only density criteria. Actually, in some cases, two or more nodes can belong to the same cluster even though they are not directly connected by common edges. We refer to this category of clusters as pattern-based clusters, since they represent structures with interesting connectivity properties in directed networks [12]. Citation based spectral clustering method in directed weighted network is one of the mostly used pattern based clusters discovering methods [12].

In view of the studies mentioned above, we propose a financial risk propagation model based on pattern based spectral clustering in directed weighted network. In the first step, the univariate GARCH model is used to characterize daily volatility of financial markets. In the second step, the Granger causality test model is used to calculate pair-wise volatility spillover between each two financial markets. In the third step, the pattern based spectral clustering in directed weighted network is used to analyze risk propagation property of all the considered financial markets.

The paper is organized as follows. In Section 2, we first introduce the models which will be used in the following discussions. Then propose the financial risk propagation model based on pattern based spectral clustering in directed weighted network. In Section 3, the proposed model is used to analyze financial risk propagation problem among the main European countries during the beginning period of the European sovereign debt crisis. After discussions in Section 4, final results follow in Section 5.

2. **Methodology.**

In this section, we first introduce the citation based spectral clustering method in directed weighted network. Then, propose the financial risk propagation model based on pattern based spectral clustering in directed weighted network.

2.1. **Citation based spectral clustering method in directed weighted network.**

Citation based cluster is one kind of pattern based clusters, for example, the clusters in Figure 1 are two citation based clusters [12-14]. Take the left graph in Figure 1 as an example. In this case, the two nodes in the shadowed region form a cluster, since they have out links to the same nodes, while at the same time having in links from the same group of nodes. This structure constitutes a common situation in the context of directed graphs. For example, it can be used to simulate financial risk propagation, which started

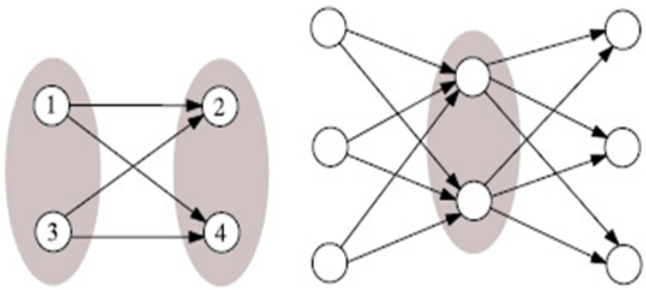

Figure 1. Citation based clusters [12-14]

from markets 1 and 3, and then ended in markets 2 and 4. The nodes in the same cluster are linked by a common group of nodes, but actually they do not have links among them.

Informative egienvectros of spectral clustering matrix were used to cluster data sets in spectral clustering method. The informative eigenvectors are the ones which are composed of eigenvectors corresponding to the largest K eigenvalues. Therefore, by clustering the informative eigenvectors, we can obtain the citation based clustering structure of the original data set.

2.2. **Financial risk propagation model based on citation based spectral clustering in directed weighted network and financial econometric models.**

The first step: For the selected countries, daily close prices of the corresponding main stock indices will be used to analyze propagation of the financial risk in the following steps;

The second step: For each close price P_t, calculate logarithmic return rate $R_t = \ln(P_t/P_{t-1})$;

The third step: For each logarithmic return rate R_t, characterize daily volatility of financial markets by univariate GARCH model;

The fourth step: The Granger causality test model is used to calculate pair-wise volatility spillovers between each two financial markets;

The fifth step: For selected confidence level, construct similarity matrix. Smaller probability given in the fourth step means volatility spillover between corresponding two financial markets is more possible to happen. So the similarity between two financial markets is set as 1-(probability given in the second step). Because the volatility spillovers between each two financial markets calculated in the second step are asymmetric, the constructed similarity matrix is also asymmetric;

The sixth step: In the following discussion, denote by A the asymmetric similarity matrix constructed in the fifth step. Construct spectral clustering matrix $L_A = (A * A' + A' * A)/2$;

The seventh step: Calculate eigenvalues and eigenvectors of spectral clustering matrix L_A. For given cluster number K, eigenvectors corresponding to the largest K eigenvalues are chosen as informative eigenvectors;

The eighth step: Partition results based on the informative eigenvectors.

Figure 2 is flow chart of the model.

Note: For GARCH model and Granger causality test model, please refer to the references [15,16] or any other materials which introduce financial econometric models.

3. **Data and Descriptive Statistics.** The Euro-zone crisis is an ongoing crisis that has been affecting the countries of the Euro-zone since early 2009, when a group of 10 central and eastern European banks asked for a bailout. The crisis made it difficult or impossible for some countries in the Euro-zone to repay or refinance their government debt without the assistance of third parties like the ECB or IMF.

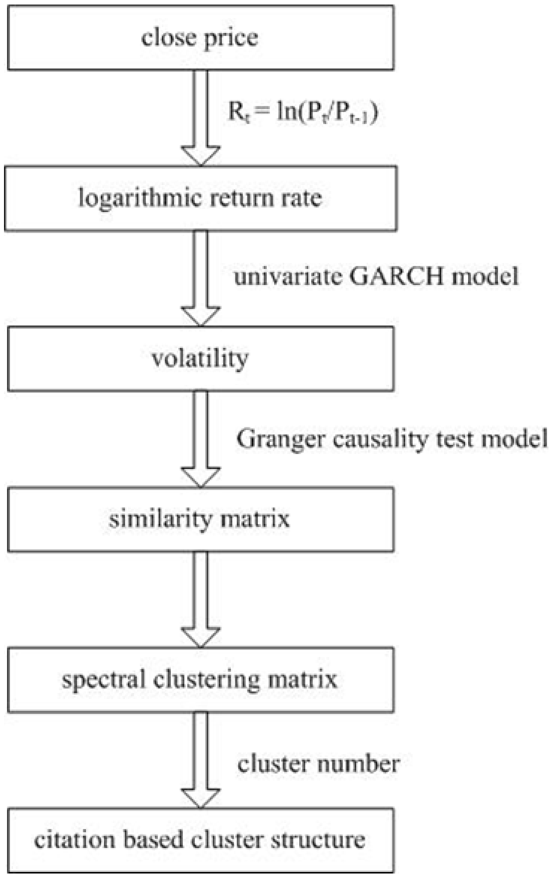

FIGURE 2. Flow chart of the model

In this paper, we study the citation based cluster structure of the financial risk among main European countries during beginning period of the European sovereign debt crisis. According to Fitch's rating, we set the beginning period of European sovereign debt crisis from 2009-04-08 to 2010-10-05. The selected stock indices are FTSE 100 index of the United Kingdom, CAC40 index of France, Frankfurt DAX index of Germany, BEL20 index of Belgium, FTSE MIB index of Italy, IBEX index of Spain, Finland index, KFX index of Denmark, OSE index of Norway, Irish index of Ireland, ATX index of Austria, OMX index of Sweden, Zurich Swiss market index, ASE composite index of Greece, PSI20 index of Portugal and Budapest index of Hungary.

Use the financial risk propagation model based on pattern based spectral clustering in directed weighted network to cluster the 16 financial time series. We noticed that in the fifth step of the model, the cluster number K is a given parameter. However, in the most of real applications, K is an unknown parameter. In this study, the proper cluster number is chosen based on prior knowledge. As we know, the European sovereign debt crisis started from five European countries, namely, Portugal, Italy, Ireland, Greece and Spain, and we will call them PIIGS for short in the following discussion. We choose the proper cluster number K as the one, under which the PIIGS were partitioned in the same cluster, and meanwhile, other 11 indices were partitioned into remaining clusters. In this study, the proper cluster number is chosen as 4. Clustering result is as follows:

Cluster 1: OMX index, Finland index;

Cluster 2: CAC40 index, Zurich Swiss market index, BEL20 index, KFX index, ATX index;

Cluster 3: PSI20 index, FTSE MIB index, Irish index, ASE composite index, IBEX index;

Cluster 4: FTSE 100 index, Frankfurt DAX index, OSE index, Budapest index.

Table 1 is normalized similarities between clusters and Table 2 is simplified similarities between clusters. We take the normalized similarity (Swe/Fin→Fr/Bel/Den/Aus/Swi) as an example, total similarity (Swe/Fin→Fr/Bel/Den/Aus/Swi) is 7.9205, the first cluster contains 2 indices, the second cluster contains 5 indices, so normalized similarity (Swe/Fin→Fr/Bel/Den/Aus/Swi) = 0.79205 was obtained as $7.9205/(2 \times 5)$. Similarly, we can get other similarities in Table 1.

Table 2 is constructed as follows. We take the simplified similarity (Swe/Fin→Fr/Bel/Den/Aus/Swi) and the simplified similarity (Fr/Bel/Den/Aus/Swi→Swe/Fin) as an example. Normalized similarity (Swe/Fin→Fr/Bel/Den/Aus/Swi) = 0.79205 > Normalized similarity (Fr/Bel/Den/Aus/Swi→Swe/Fin) = 0.69521, so the simplified similarity (Swe/Fin→Fr/Bel/Den/Aus/Swi) was set to $0.79205 - 0.69521 = 0.09684$ and the simplified similarity (Fr/Bel/Den/Aus/Swi→Swe/Fin) was set to 0. Similarly, we can get other similarities in Table 2.

Figure 3 is citation based cluster structure of financial risk based on the simplified between clusters similarities in Table 2 and the asymmetric similarity matrix A.

Table 1. Normalized similarities between clusters

	Swe/Fin	Fr/Bel/Den/Aus/Swi	PIIGS	UK/Ger/Nor/Hun
Swe/Fin	0	0.79205	0	0.9969125
Fr/Bel/Den/Aus/Swi	0.69521	0	0	0.53086
PIIGS	0.49921	0.55136	0	0.68237
UK/Ger/Nor/Hun	0.860375	0.24161	0	0

Table 2. Simplified similarities between clusters

	Swe/Fin	Fr/Bel/Den/Aus/Swi	PIIGS	UK/Ger/Nor/Hun
Swe/Fin	0	0.09864	0	0.1365375
Fr/Bel/Den/Aus/Swi	0	0	0	0.28925
PIIGS	0.49921	0.55136	0	0.68237
UK/Ger/Nor/Hun	0	0	0	0

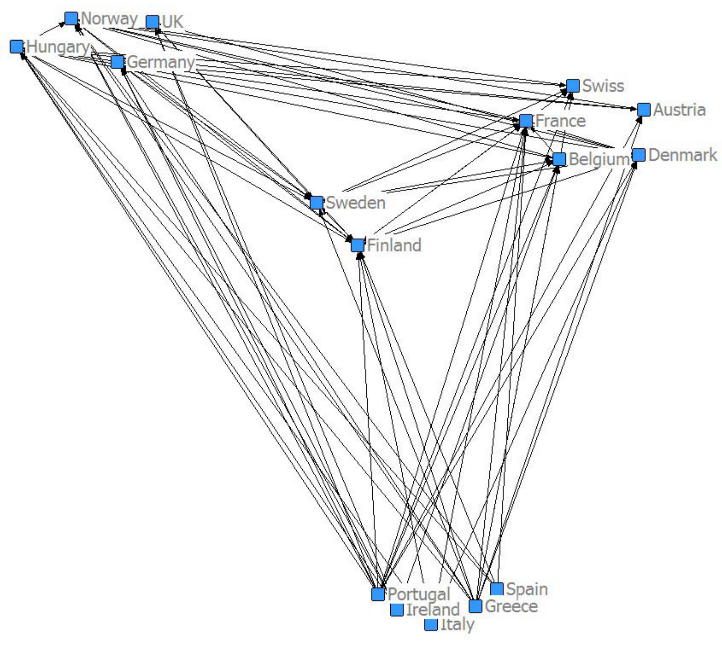

Figure 3. Citation based cluster structure of financial risk

4. **Discussions.** The empirical results of this study show that, the European sovereign debt crisis starting from PIIGS and ending at UK/Ger/Nor/Hun presented three kinds of citation based cluster structures during the beginning period. The crisis starting from PIIGS propagated to UK/Ger/Nor/Hun in several ways. The simplest way was propagated to UK/Ger/Nor/Hun directly. The simpler way was propagated to UK/Ger/Nor/Hun in two steps, one was propagated to Swe/Fin at first and then to UK/Ger/Nor/Hun, and another was propagated to Fr/Bel/Den/Aus/Swi at first and then to UK/Ger/Nor/Hun. The most complex way was composed of three steps, the crisis propagated to Swe/Fin at first, then to Fr/Bel/Den/Aus/Swi, and at last ended at UK/Ger/Nor/Hun. We note that Germany and the UK are two huge economic powers. Therefore, among the main European countries, the European sovereign debt crisis became weaker in huge economic powers.

5. **Conclusions.** In the present paper, a financial risk propagation model based on pattern based spectral clustering in directed weighted network is developed. The model constructs asymmetric similarity matrix based on traditional financial time series analysis models, then, using citation based spectral clustering method to detect the propagation feature of financial risk. The financial risk propagation model based on pattern based spectral clustering in directed weighted network is used to analyze the European sovereign debt crisis in main European countries at the beginning period of the crisis.

The financial risk propagation model was used to depict financial risk propagation among the main European countries during the beginning period of the European sovereign debt crisis better. Therefore, the model provides a tool to analyze financial risk propagation problems. However, pattern based clustering problem in directed networks is more challenging compared to the undirected version. So the future research directions include formal definition of the pattern based clustering, algorithm's parameters selection, algorithm's scalability, algorithm's evaluation and further application in finance.

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