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FULLY INFORMED PARTICLE SWARM WITH ADAPTIVE SMALL-WORLD TOPOLOGY

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Abstract. *Population topologies are crucial for the performance of particle swarm optimization (PSO) algorithms. In the traditional fully informed particle swarm (FIPS) algorithm, topologies are completely regular, which may cause the population to lose diversity and to get trapped into local optima. This paper proposes an adaptive small-world topology for the FIPS algorithm. In the proposed small-world topology, some particles communicate with neighborhood nearby while others by chance communicate with some distant particles, so as to promote the balance between global search and local search. The proposed topology is adaptively adjusted based on the stagnation state during the optimization. Moreover, the position update strategy of FIPS is improved by a novel adaptive mechanism, which helps dynamically change the impact degree of source information. A novel algorithm named fully informed particle swarm with adaptive small-world topology (ASWFIPS) is developed by applying the proposed adaptive small-world topology to the improved FIPS. Experimental studies show that the proposed algorithm outperforms classic PSO and FIPS algorithms over 13 benchmark functions in terms of solution accuracy, convergence speed, and algorithm reliability.*

Keywords: Particle swarm optimization, Adaptive small-world topology, Global optimization

1. **Introduction.** Particle swarm optimization (PSO), which was proposed by Kennedy and Eberhart [1], is a representative swarm intelligence (SI) algorithm. PSO emulates the social behaviors in birds flocking and fish schooling. In PSO, particles are updated with the help of the best previous success, the best previous success of neighborhood, the current position, and previous velocity. The particles in the population randomly scatter in the solution space and cooperate to search for the global optimum. As PSO is easy to implement, it has been widely applied to solving various real-world problems in recent years [2, 3, 4].

The traditional PSO algorithms could be classified into two types according to their topologies, namely, global and local PSOs [1, 5, 6]. Global and local PSO algorithms both have their strengths and weaknesses. In the global PSO algorithm, the neighborhood of each particle is set as the entire population. Therefore, the global PSO algorithm converges very fast but may easily get trapped into local optima. Various kinds of population topologies have been designed for the local PSO algorithms, namely, ring, star, etc. The local PSO algorithms have more chances to find the global optimum with a slower convergence speed. Among the local PSO algorithms, fully informed particle swarm (FIPS) [7] algorithm is a representative. The fully informed learning strategy in FIPS could further enhance the population diversity, which is crucial for the search of PSO algorithms.

In the FIPS algorithm, rather than choosing one of the neighbors as the source of influence, all the neighbors are set as the source. Thus, the degree of diversity could be directly controlled by the size of neighborhood. However, in the traditional FIPS algorithm, the population topologies are totally regular. In this way, the neighbors of each particle are unchanged during the optimization, which may cause the source information to lose diversity.

Considering the above issues, an adaptive small-world topology is proposed for the FIPS algorithm in this paper. In this topology, some particles communicate with the neighborhood nearby while some others by chance communicate with the distant particles. Therefore, a better balance between global search and local search can be achieved. During the optimization, the proposed adaptive topology is adjusted according to the stagnation state of the population. In this way, the diversity of source information for each particle is guaranteed. Furthermore, the particle update strategy of FIPS algorithm is improved with an adaptive mechanism, which is also based on the stagnation state. By combining the proposed adaptive small-world topology and the improved FIPS algorithm, this paper proposes a novel algorithm named fully informed particle swarm with an adaptive small-world topology (ASWFIPS). In the experimental studies, the proposed ASWFIPS algorithm is compared with global particle swarm optimization (GPSO) [1], local particle swarm optimization with ring topology (RPSO) [5], and fully informed particle swarm with ring topology (RFIPS) [7] over 13 test functions with various features. Results show that ASWFIPS performs better than the GPSO, RPSO, and RFIPS in terms of solution accuracy, convergence speed, and algorithm reliability.

The remainder of this paper is organized as follows. In Section 2, small-world network and FIPS algorithm are briefly introduced. Subsequently, Section 3 describes the proposed ASWFIPS algorithm in detail. Experiments with discussion are provided in Section 4. Finally, Section 5 draws the conclusion.

2. FIPS and Small-World Network.

2.1. PSO.
In the PSO algorithms, a particle is updated based on the best previous success, the best previous success of neighbor, its current position, and previous velocity. The update of velocity and position of the particles are defined as:

$$\vec{v_{t+1}} = \omega \times \vec{v_t} + c_1 \times rand_1 \times \left(p\vec{Best_i} - \vec{X_t}\right) + c_2 \times rand_2 \times \left(g\vec{Best} - \vec{X_t}\right) \quad (1)$$

$$\vec{X_{t+1}} = \vec{X_t} + \vec{v_{t+1}} \quad (2)$$

where ω is the inertia weight, c_1 and c_2 refer to accelerating coefficients, $rand_1$ and $rand_2$ are two random values between 0 and 1, $p\vec{Best_i}$ refers to the best position found by the particle i and $g\vec{Best}$ refers to the position found by the member of its neighborhood that has the best performance so far.

2.2. FIPS.
In FIPS, rather than just choosing the best neighbor as the source, particles utilize the information from all the neighbors. The velocity and position of particles in FIPS are updated as follows:

$$\vec{v_{t+1}} = \chi\left(v_t + \varphi\left(\vec{P_m} - \vec{X_t}\right)\right) \quad (3)$$

$$\vec{X_{t+1}} = \vec{X_t} + \vec{v_{t+1}} \quad (4)$$

where χ indicates the constriction coefficient, φ is the acceleration coefficient limit, $\vec{P_m}$ is the comprehensive information from neighborhood and defined as:

$$\vec{P_m} = \frac{\sum_{k\in N} \omega_k \vec{\varphi_k} \otimes \vec{P_k}}{\sum_{k\in N} \omega_k \vec{\varphi_k}} \quad (5)$$

$$\vec{\varphi_k} = \vec{U}\left[0, \frac{\varphi}{|N|}\right], \quad \forall k \in N \quad (6)$$

where N is the set of neighbors, $\vec{P}_k$ is the best position found by neighbor k, and φ_k is the accelerating coefficient of the particle k. The parameter ω describes any aspect of the particle that is hypothesized to be relevant. In general, the fitness of the best position found by the particle and the distance from the particle to the current individual are adopted. In this way, the particle is fully informed and thus the population diversity can be enhanced.

2.3. **Small-world network.** According to the network theory [8], networks can be classified into three classes. In the first class, networks are completely regular, such as the ring and star. In contrast, networks in the second class are randomly generated by some specific probabilistic model. However, researchers have found that most of the real-world networks do not belong to one of these two extremes. Instead, they lie between the regular and random networks and involve both regular and random features. These networks are of the third class. Among these networks, small-world network has attracted lots of attention.

A ring network and a small-world network are shown in Figure 1. In the ring network of N vertices, each vertex is connected to its successors. Specifically, the successors of vertex k are vertices $k-1$ and $k+1$. Starting with the ring network, a small-world network could be constructed. With a probability P, each edge is reconnected to a vertex selected by random. The number of edges or vertices is not changed during the rewriting procedure. Some "long-range links" are introduced through the random reconnecting. With a small number of long-range links, the diameter of the network is reduced while the clustering coefficient stays large. By applying the small-world network as the population topology to PSO, the balance between exploration and exploitation abilities of the algorithm can be promoted.

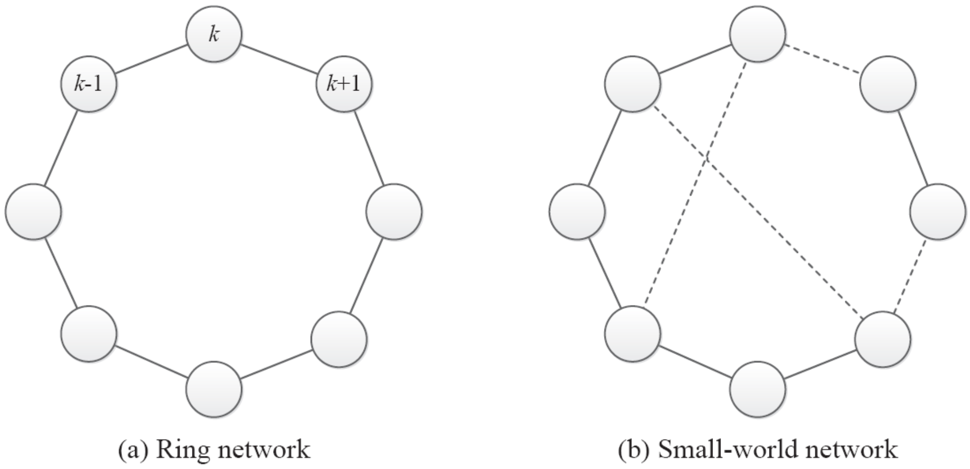

FIGURE 1. Ring network and small-world network

3. **ASWFIPS.** In this section, an adaptive small-world topology is designed for FIPS algorithm. During the optimization, the proposed topology is adaptively adjusted according to the requirements of different evolution stages so as to balance the diversity and convergence of the algorithm. Moreover, an adaptive mechanism based on the stagnation state is designed for the particle update strategy of FIPS algorithm. A novel ASWFIPS algorithm is developed by combining the proposed adaptive small-world topology and the improved FIPS. The proposed algorithm could possess both good global search ability and fast convergence speed, which is efficient in tacking various kinds of problems.

In what follows, the approach to constructing and updating the proposed small-world topology is firstly introduced. Subsequently, the improved particle update of FIPS is introduced in detail. Finally, the fully informed particle swam with an adaptive small-world topology (ASWFIPS) is algorithmically illustrated.

3.1. **The proposed adaptive small-world topology.** The proposed topology is initialized as a small-world topology. As shown in Figure 1, at first, each particle i is connected with its successor particles. Then, with a probability P_1, each edge is reconnected to a vertex chosen by random. Note that each dimension of particles is assigned with a small-world network.

The proposed topology is adaptively adjusted according to the stagnation state of the population. A stagnation coefficient S_{avg} is defined as:

$$S_{avg} = \frac{\sum_{i \in P} S_i}{NP} \tag{7}$$

where S_i indicates the number of generations particle i stagnates, P is the set of population and NP indicates the population size. The average value of S_i, namely, S_{avg}, could help indicate the stagnation state of the entire population. As shown in Algorithm 1, the small-world topology is adaptively adjusted during optimization process. The topology is updated once the value of S_{avg} is larger than the threshold value S_k.

Algorithm 1 Topology update of particle i

if $S_{avg} \geq S_k$ **then**
    for each dimension j of particle i **do**
        r = random(0,1);
        if $r \leq P_2$ **then**
            Randomly assign two neighbors to particle i;
        else
            Set two successors of particle i as its neighbors;
        end if
    end for
end if

3.2. **Particle update.** In the proposed topology, each particle has two neighbors. Therefore, the parameter $\vec{P_m}$ could be simplified as:

$$\vec{P_m} = \frac{\vec{\varphi_1}\vec{P_1}\omega_1 + \vec{\varphi_2}\vec{P_2}\omega_2}{\vec{\varphi_1}\omega_1 + \vec{\varphi_2}\omega_2} \tag{8}$$

where $\vec{P_1}$ and $\vec{P_2}$ represent the first and second neighbors of current particle, respectively. Accordingly, ω_1 and ω_2 are their weights. The weight of the neighbor k is related to its fitness and defined as:

$$\omega_k = \frac{fit_k + 1}{\sum_{i \in N} fit_i + 1} \tag{9}$$

where fit_i is the fitness value of neighbor i, and N is the set of neighborhood. The position of particles is updated as follows:

$$\vec{X_{t+1}} = \vec{X_t} + Kc \times \vec{v_{t+1}} \tag{10}$$

where parameter Kc is the weight of velocity. Its value is dynamic and could help dynamically adjust the weight of source information. Kc is adjusted as follows:

$$Kc = \begin{cases} 1, & S_i > 8 \\ (0.5, 1), & \text{otherwise} \end{cases} \tag{11}$$

where S_i denotes the number of stagnating generations for particle i. On the one hand, a high value of S_i indicates that the particle stagnates and thus the value of Kc is increased to maintain a high weight of source information. On the other hand, a low value of S_i

indicates that the current particle has a good search state and thus Kc is decreased to reduce the impact of source information. Based on this position update strategy, the impact degree of neighbors for each particle could be adaptively controlled.

3.3. **Overall process.** The proposed adaptive small-world topology is embedded in the improved FIPS algorithm to develop the ASWFIPS. The flowchart of ASWFIPS is shown in Figure 2 and the overcall process contains the following steps:
(1) Initialize the positions and velocities of the population;
(2) Evaluate the position and select the best position;
(3) Update the state of stagnation and the small-world topology;
(4) Update the positions and velocities of the population;
(5) If the stopping criterion is not satisfied, return to Step 2.

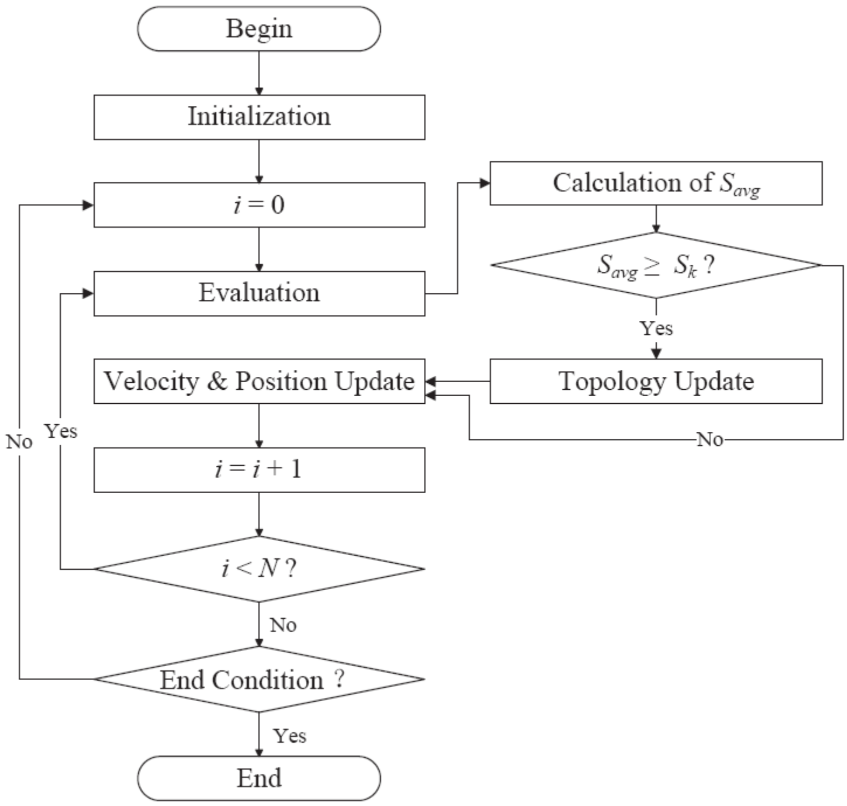

Figure 2. Flowchart of the proposed ASWFIPS algorithm

4. Experimental Results.

4.1. **Experimental setup.** In the experiments, 13 benchmark functions with different features are used to test the performance of ASWFIPS [9]. These functions are listed in Table 1, where F_1 to F_5 are unimodal functions, F_6 is a step function, F_7 is a noisy function, and F_8 to F_{13} are multimodal functions with a great number of local optima.

Table 1. Characteristics of test functions

Functions	**Characteristics**	**Dimension**	**MNFEs**
F_1, F_6, F_7	Unimodal, separable	30	3.00E+05
F_2, F_3, F_4, F_5	Unimodal, nonseparable	30	3.00E+05
F_9	Multimodal, separable	30	3.00E+05
F_8, F_{10}, F_{11}, F_{12}, F_{13}	Multimodal, nonseparable	30	3.00E+05

In what follows, experimental studies are carried out over these 13 test functions to compare ASWFIPS algorithm with two PSO algorithms and one FIPS algorithm, namely, GPSO, RPSO, and RFIPS. These algorithms are compared in terms of solution accuracy, convergence speed, and algorithm reliability.

In ASWFIPS, P_1 is set as 0.07, P_2 linearly increases from 0.17 to 0.27 during the optimization and S_k is set as 10. According to [7], χ and C are set as 0.7298 and 2.05, respectively. All the algorithms are tested on 30 dimensions functions with population size 30 and maximum number of function evaluations (MNFEs) 300000. For each function, 30 independent trials are carried out by applying GPSO, RPSO, RFIPS, and ASWFIPS under the same circumstances.

4.2. **Results and comparisons.** In Table 2, the mean and standard deviations of the error values achieved by GPSO, RPSO, RFIPS, and the proposed ASWFIPS over 30 independent runs are presented. To show the advantage of proposed migration topology in a statistical sense, single-problem Wilcoxon signed-rank test at a significance level 0.05 is performed, where the significantly better results are highlighted in boldface. It can be observed that ASWFIPS comprehensively outperforms the compared algorithms. On the one hand, for unimodal functions, the proposed algorithm could achieve higher solution accuracy on F_4 and F_7 and have similar performance to the compared algorithms on the other functions. On the other side, in optimizing multimodal functions, the proposed ASWFIPS exhibits much stronger global search ability than the other three algorithms.

In order to compare the search speed and reliability, tolerance of each functions is set and presented in Table 3 [9]. We record the success rates at which tolerances are found over 30 runs and the mean NFEs required to find the tolerances. The best results are

TABLE 2. Comparison between ASWFIPS and other three PSO algorithms

Approaches	ASWFIPS		GPSO		RPSO		RFIPS	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
F_1	1.19E-59	4.00E-61	1.11E-59	6.27E-65	**1.51E-83**	1.80E-86	3.86E-30	6.31E-31
F_2	3.03E-31	1.04E-31	6.48E-39	3.75E-42	**2.43E-50**	1.25E-51	1.82E-17	7.92E-18
F_3	1.86E-03	1.56E-04	1.12E-02	1.03E-03	**1.83E-05**	6.34E-07	9.76E-01	3.20E-01
F_4	**4.29E-13**	2.69E-14	3.51E-02	5.27E-02	2.59E-05	1.32E-06	2.30E-05	9.47E-06
F_5	2.26E+01	1.79E+01	2.68E+01	6.24E-04	**1.02E+01**	6.03E-03	2.29E+01	2.22E+01
F_6	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00
F_7	**7.52E-04**	2.57E-04	5.24E-03	1.80E-03	3.52E-03	1.69E-03	2.20E-03	1.23E-03
F_8	–9.26E+03	–1.04E+04	–9.94E+03	–1.07E+04	–8.02E+03	–9.10E+03	**–1.08E+04**	–1.16E+04
F_9	**8.06E+00**	2.98E+00	1.69E+01	1.09E+01	4.70E+01	3.08E+01	2.65E+01	4.51E+00
F_{10}	**4.00E-15**	4.00E-15	1.05E-14	7.55E-15	**7.43E-15**	4.00E-15	**7.43E-15**	4.00E-15
F_{11}	**0.00E+00**	0.00E+00	7.40E-03	7.40E-03	2.05E-03	0.00E+00	**0.00E+00**	0.00E+00
F_{12}	**1.58E-32**	1.57E-32	1.04E-02	1.57E-32	3.46E-03	1.57E-32	1.64E-31	9.96E-32
F_{13}	**6.54E-32**	1.35E-32	2.20E-03	1.35E-32	3.66E-04	1.35E-32	1.98E-30	7.51E-31

TABLE 3. Success rate and convergence speed comparisons

Approaches	Tolerance	ASWFIPS		GPSO		RPSO		RFIPS	
		NFEs	SR %	NFEs	SR %	NFEs	SR %	NFEs	SR %
F_1	200	**169.46**	**100**	3238.09	**100**	172.62	**100**	333.38	**100**
F_2	10	128.05	**100**	2215.16	**100**	**122.29**	**100**	210.92	**100**
F_3	10000	**45.58**	**100**	817.17	98	89.78	**100**	74.92	**100**
F_4	20	**74.66**	**100**	1219.45	**100**	135.62	**100**	133.04	**100**
F_5	2000	**245.59**	**100**	3939.8	93	262.91	**100**	453.52	**100**
F_6	200	184.21	**100**	3297.62	**100**	**177.37**	**100**	330.37	**100**
F_7	0.2	**79.83**	**100**	2171.77	99	142.65	**100**	148.21	**100**
F_8	–8000	1206.52	**100**	2470.84	**100**	**371.66**	41	2980.66	**100**
F_9	20	**1449.65**	**100**	6208.47	23	–	0	7551.81	36
F_{10}	4	234.66	**100**	3626.11	**100**	**212.76**	**100**	422.58	**100**
F_{11}	4	153.54	**100**	2968.52	**100**	**150.75**	**100**	264.58	**100**
F_{12}	4	**191.81**	**100**	3445.31	**100**	264.37	**100**	32582	**100**
F_{13}	20	**181.54**	**100**	3399.07	**100**	262.12	**100**	293.53	**100**

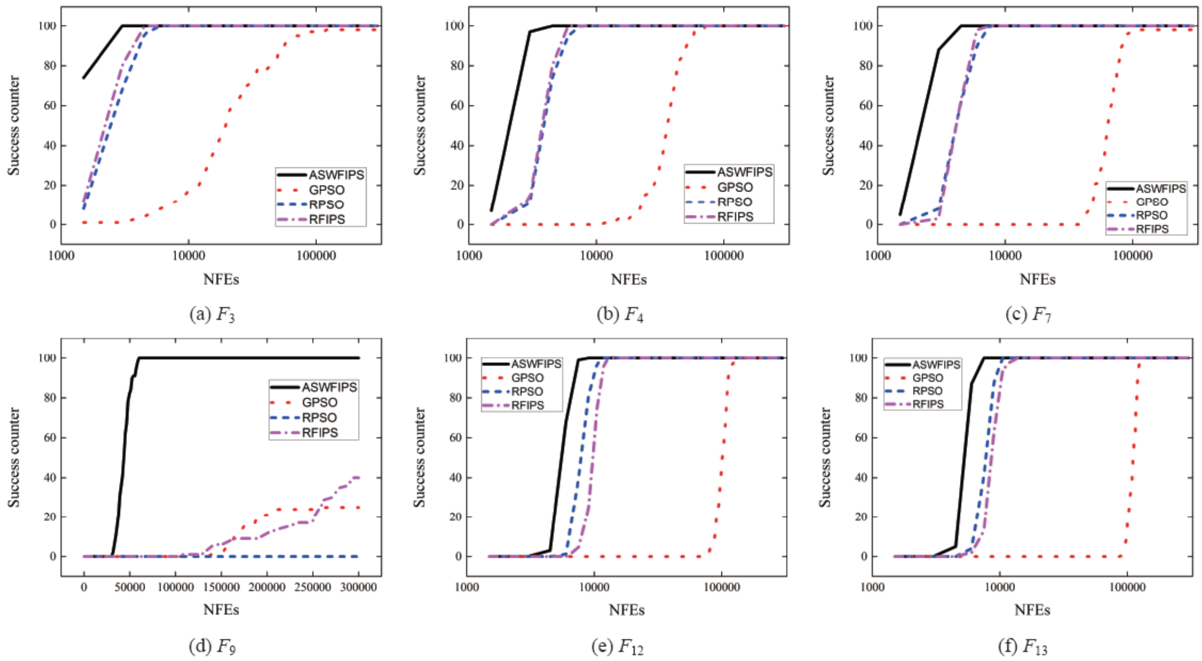

FIGURE 3. Comparisons of convergence speed on six typical functions

highlighted in boldface. The results show that the proposed ASWFIPS algorithm converges fastest on majority of the test functions. In Figure 3, the growth curves of success counter over the number of function evaluations are plotted for six typical functions. It can be observed that the proposed ASWFIPS algorithm shows the highest search efficiency. Moreover, the ASWFIPS is the only algorithm that could guarantee the success rate of 100% on all the test functions. Overall, the proposed ASWFIPS algorithm outperforms the compared PSO algorithms in terms of convergence speed and reliability.

5. **Conclusion.** An adaptive small-world topology for FIPS algorithm is proposed in this paper. During the optimization, the proposed topology is dynamically adjusted according to the requirements of different evolution states so as to balance the diversity and convergence of the algorithm. In addition, an adaptive mechanism based stagnation state of population is designed for improving the position update strategy of FIPS. By embedding the proposed adaptive small-world topology in the improved FIPS, a novel ASWFIPS algorithm is developed. Experimental results show that the proposed ASWFIPS outperforms the compared GPSO, RPSO, and RFIPS algorithms in terms of solution accuracy, convergence speed, and algorithm reliability. For future work, we will extend the ASWFIPS algorithm to solve other complex optimization problems.

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