ICIC Express Letters ICIC International ©2016 ISSN 1881-803X
Volume **10**, Number **10**, October **2016** pp. **2333–2340**

TRANSPORTATION PROBLEM WITH FUZZY DEMANDS AND FUZZY SUPPLIES USING TRIANGULAR AND INTERVAL-VALUED FUZZY NUMBERS

Feng-Tse Lin

Department of Applied Mathematics
Chinese Culture University
No. 55, Hwa-Kang Road, Yangminshan, Taipei 11114, Taiwan
ftlin@faculty.pccu.edu.tw

Received March 2016; accepted June 2016

Abstract. *The transportation problem is a special class of the linear programming problem to be used for reducing cost and improving service in logistics and supply chain management over the past years. The fuzzy transportation problem arises when the nature of the related coefficients are uncertain or imprecise. This study examines the classical transportation problem with fuzzy demands and fuzzy supplies and introduces two kinds of fuzzy numbers to characterize the imprecise values. Two methods for ranking triangular and interval-valued fuzzy numbers as well as an approach to solving the fuzzy transportation problem are proposed. Numerical examples are given to illustrate and analyze final solutions obtained when using triangular and interval-valued fuzzy numbers to the fuzzy problem. The proposed approach obtains very good solutions to the transportation problems with fuzzy demands and fuzzy supplies.*

Keywords: Fuzzy transportation problem, Fuzzy demand, Fuzzy supply, Triangular fuzzy number, Interval-valued fuzzy number

1. **Introduction.** The transportation problem is concerned with shipping a commodity between a set of sources (e.g., manufacturers) and a set of destinations (e.g., warehouses). Each source has a capacity dictating the amount it supplies and each destination has a demand dictating the amount it receives. The objective is to determine the amounts shipped from each source to each destination that minimizes the total cost while satisfying both the supply limits and the demand requirements [15]. Several efficient heuristics and algorithms have been developed over the past decades to solve the transportation problem when the cost coefficients and the supply and demand values are known exactly. Some well-known methods include the north-west corner rule, the least-cost, Vogel's approximation and Hungarian algorithm [15]. Nevertheless, in real world applications, the supply and demand quantities in the transportation problem are sometimes hardly specified precisely because of changing weather, social and economic conditions, uncertainty in judgements, or lack of evidence [13]. These imprecise data are not always well represented by random variable selected from a probability distribution. However, fuzzy sets provide a powerful tool to model and solve the problem [8].

One straightforward approach to solving the fuzzy transportation problem is to apply the existing fuzzy linear programming techniques directly. Zimmermann [16] showed that solutions obtained by fuzzy linear programming are always efficient and developed fuzzy optimization methods for solving the fuzzy optimization problem. Heigearataigh [4] introduced a fuzzy transportation algorithm for solving transportation problem with fuzzy constraints and also investigated the relationship between the algebraic of the optimal solution of the deterministic problem. Kaufmann and Gupta [6] examined the transportation problem with fuzzy data and then proposed the stepping method to deal with fuzzy numbers. Chanas et al. [1] investigated the transportation problem with fuzzy supplies and fuzzy demands and solved them via the parametric programming technique in terms of the Bellman-Zadeh criterion. Chanas and Kuchta [2] proposed a concept of the optimal solution of the transportation problem with fuzzy coefficients and an algorithm determining this solution. Tada and Ishii [14] considered an integer fuzzy transportation problem by introducing two kinds of membership functions corresponding to supplies and demands. The objective is to determine an optimal flow that maximizes the smallest value of all membership functions under the constraint that the total transportation cost must not exceed a certain upper limit. Liu and Kao [12] developed a method to find the membership function of the fuzzy total transportation cost when the unit shipping costs, the supply quantities, and the demand quantities are fuzzy numbers. Their method is based on extension principle to transform the fuzzy transportation problem to a pair of mathematical programs. Kumar and Kaur [9] proposed two methods to find the exact fuzzy optimal solution of unbalanced fuzzy transportation problem by representing all the parameters as LR flat fuzzy numbers. Li et al. [10] presented an evolutionary program for solving the fuzzy multicriteria solid transportation problem in which the coefficients of objective functions are represented as fuzzy numbers. Gao et al. [3] proposed a method for minimizing transportation cost when the supply, demand and transportation cost are interval numbers. For this case, an auxiliary problem is obtained in order to find a solution. Kaur et al. [7] presented an application of a modified fuzzy programming technique for the fuzzy optimal solution to the single objective fuzzy transportation problem with fuzzy parameters in terms of triangular fuzzy numbers without defuzzifying the problem.

This study, however, examines the classical transportation problem with fuzzy demands and fuzzy supplies and then uses triangular and interval-valued fuzzy numbers to characterize the imprecise data. When the coefficients are represented with fuzzy numbers, the values of objective functions also become fuzzy numbers. Obviously, when the supply and demand quantities are uncertain, the total transportation cost will vary within an interval. Since a fuzzy number represents many possible real numbers, it is not easy to compare among solutions to determine the Pareto optimal solution. Subsequently, two methods for ranking triangular and interval-valued fuzzy numbers as well as an approach to solving the fuzzy transportation problem are then proposed. Numerical examples are given to illustrate and analyze final solutions obtained when using triangular and interval-valued fuzzy numbers to the fuzzy problem. The proposed approach obtains very good solutions to the transportation problems with fuzzy demands and fuzzy supplies.

The paper is organized as follows. Section 2 states the preliminaries where two proposed distance methods for ranking triangular and interval-valued fuzzy numbers are defined. Section 3 formulates the fuzzy transportation problem with fuzzy demands and fuzzy supplies. Two numerical examples are given to illustrate the effectiveness of the proposed approach to solving fuzzy transportation problems in Section 4. Section 5 concludes the paper with future work.

2. Preliminaries.

Definition 2.1. *A fuzzy set $\tilde{P}_\alpha$, $0 < \alpha \leq 1$, defined on $R = (-\infty, \infty)$ is called a level α fuzzy point which has the following membership function*

$$\mu_{\tilde{p}_\alpha}(x) = \begin{cases} \alpha, & x = p \\ 0, & otherwise \end{cases} \tag{1}$$

Definition 2.2. *A fuzzy set $[p_\alpha, q_\alpha]$, $0 \leq \alpha \leq 1$, defined on R is called a level α fuzzy interval which has the following membership function*

$$\mu_{[p_\alpha, q_\alpha]}(x) = \begin{cases} \alpha, & p \leq x \leq q \\ 0, & otherwise \end{cases} \tag{2}$$

Definition 2.3. *The level α triangular fuzzy number $\tilde{B}$, $0 < \alpha \le 1$, is a fuzzy set defined on R with the membership function as*

$$\mu_{\tilde{B}}(x) = \begin{cases} \frac{\alpha(x-p)}{q-p}, & p \le x \le q \\ \frac{\alpha(r-x)}{r-q}, & q \le x \le r \\ 0, & otherwise \end{cases} \tag{3}$$

Note that a level α triangular fuzzy number is denoted by $\tilde{B} = (p, q, r; \alpha)$ and the family of all level α fuzzy numbers is denoted by $F_N(\alpha) = \{(p, q, r; \alpha) | \forall p < q < r, p, q, r \in R\}$, $0 < \alpha \le 1$. When $p = q = r$, $(p, q, r; \alpha) = (r, r, r; \alpha) = \tilde{r}_\alpha$, the level α fuzzy number is the level α fuzzy point, and $\tilde{r}_\alpha$ is a member of $F_N(\alpha)$. In particular, $(p, q, r; 1)$ is called triangular fuzzy number and denoted by (p, q, r).

Definition 2.4. *A fuzzy set $\tilde{A}$ defined on R, where $\tilde{A} = \{(x, [\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)])\}$, $x \in R$ and $0 \le \mu_{\tilde{A}^L}(x) \le \mu_{\tilde{A}^U}(x) \le 1$, is called an interval-valued fuzzy set. Symbolically, $\tilde{A}$ is denoted by $\left[\tilde{A}^L, \tilde{A}^U\right]$. Let $\tilde{A}^L = (a, b, c; \lambda)$ and $\tilde{A}^U = (p, b, r; \rho)$, $0 < \lambda \le \rho \le 1$. Then the level (λ, ρ) interval-valued fuzzy number is defined by $\tilde{A} = \left[\tilde{A}^L, \tilde{A}^U\right] = [((a, b, c; \lambda), (p, b, r; \rho))]$, $p < a < b < c < r$. The membership function of $\tilde{A}$ can be expressed as*

$$\mu_{\tilde{A}^L}(x) = \begin{cases} \frac{\lambda(x-a)}{b-a}, & a \le x \le b \\ \frac{\lambda(c-x)}{c-b}, & b \le x \le c \\ 0, & otherwise \end{cases} \tag{4}$$

$$\mu_{\tilde{A}^U}(x) = \begin{cases} \frac{\rho(x-p)}{b-p}, & p \le x \le b \\ \frac{\rho(r-x)}{r-b}, & b \le x \le r \\ 0, & otherwise \end{cases} \tag{5}$$

The family of all level (λ, ρ) interval-valued fuzzy numbers is denoted by $F_{IV}\ (\lambda, \rho) = \{[(a, b, c; \lambda), (p, b, r; \rho)] | \forall p < a < b < c < r, a, b, c, p, r \in R\}$, $0 < \lambda < \rho \le 1$. In particular, if $\tilde{A}^L = (a, b, c; \lambda)$ and $\tilde{A}^U = (p, b, r; \rho)$, then $\tilde{A}$ is called $(\lambda, 1)$ interval-valued fuzzy number (see Figure 1).

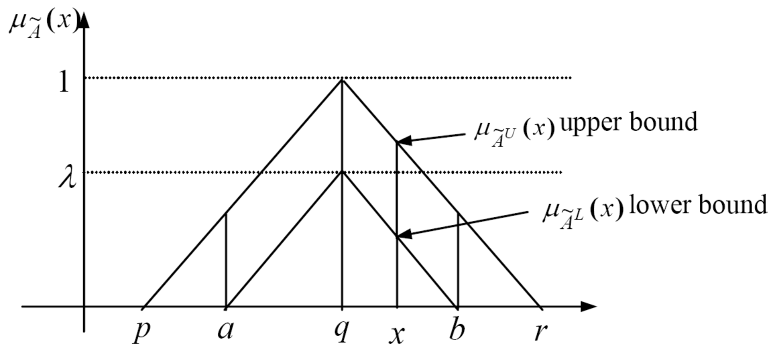

FIGURE 1. The level $(\lambda, 1)$ interval-valued fuzzy number

Let $\tilde{B} = (a, b, c; \lambda) \in F_N(\lambda)$. The α-cut of $\tilde{B}$ is $B(\alpha) = [B_L(\alpha), B_R(\alpha)]$, $0 \le \alpha \le \lambda$, where $B_L(\alpha) = a + (b-a)\frac{\alpha}{\lambda}$ is the left endpoint of the α-cut and $B_R(\alpha) = c + (c-b)\frac{\alpha}{\lambda}$ is the right endpoint of the α-cut. From Definition 2.4, the distance of $B_L(\alpha)$ is $d(B_L(\alpha), 0) = B_L(\alpha)$ and the distance of $B_R(\alpha)$ is $d(B_R(\alpha), 0) = B_R(\alpha)$. Then, the distance of interval $[B_L(\alpha), B_R(\alpha)]$ measured from the origin is defined by $d([B_L(\alpha), B_R(\alpha)], 0) = \frac{1}{2}(d(B_L(\alpha), 0) + d(B_R(\alpha), 0)) = \frac{1}{2}(a + c + (2b - a - c)\frac{\alpha}{\lambda})$, $0 \le \alpha \le \lambda$. Because the intervals

$[B_L(\alpha), B_R(\alpha)]$ and $[B_L(\alpha)_\alpha, B_R(\alpha)_\alpha]$ have a one-to-one mapping for each $\alpha \in [0, \lambda]$, the distance of $[B_L(\alpha)_\alpha, B_R(\alpha)_\alpha]$ measured from $\tilde{0}_1$ (y-axis) is defined by $d([B_L(\alpha)_\alpha, B_R(\alpha)_\alpha], \tilde{0}_1) = \frac{1}{2}\left(a + c + (2b - a - c)\frac{\alpha}{\lambda}\right)$. Because the function is continuous over the interval $0 \le \alpha \le \lambda$, the integral method can be applied to obtaining the mean value of the distance. That is,

$$\frac{1}{\lambda}\int_0^\lambda d\left([B_L(\alpha)_\alpha, B_R(\alpha)_\alpha], \tilde{0}_1\right) d\alpha = \frac{1}{2\lambda}\int_0^\lambda \left(a + c + (2b - a - c)\frac{\alpha}{\lambda}\right) d\alpha = \frac{1}{4}(2b + a + c).$$

Definition 2.5. *For each $\lambda \in (0, 1]$ and $\tilde{B} = (a, b, c; \lambda) \in F_N(\lambda)$, the ranking distance from $\tilde{0}_1$ to $\tilde{B}$ is defined by $d\left(\tilde{B}, \tilde{0}_1\right) = \frac{1}{4}(2b + a + c)$.*

Definition 2.6. *Let $\tilde{A} = [(a, b, c; \lambda), (p, b, r; \rho)] \in F_{IV}(\lambda, \rho)$, $0 < \lambda < \rho \le 1$. The ranking distance from $\tilde{0}_1$ to $\tilde{A}$ is defined by $d\left(\tilde{A}, \tilde{0}_1\right) = \frac{1}{8}\left(6b + a + c + 4p + 4r + 3(2b - p - r)\frac{\lambda}{\rho}\right)$.*

Definition 2.7. *Let $\tilde{C} = (a, b, c; \lambda)$ and $\tilde{D} = (p, q, r; \lambda) \in F_N(\lambda)$. For $\lambda \in (0, 1]$, the ranking of level λ fuzzy numbers on $F_N(\lambda)$ is defined by*

$$\tilde{D} \prec \tilde{C} \text{ iff } d\left(\tilde{D}, \tilde{0}_1\right) < d\left(\tilde{C}, \tilde{0}_1\right)$$
$$\tilde{D} \approx \tilde{C} \text{ iff } d\left(\hat{D}, \tilde{0}_1\right) = d\left(\tilde{C}, \tilde{0}_1\right)$$

3. **Problem Formulation.** Given m origins and n destinations, the transportation problem can be formulated as the following linear programming model [11]:

$$\text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{6}$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \ldots, m \tag{7}$$

$$\begin{aligned} &\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \ldots, n \\ &x_{ij} \ge 0, \text{ for all } i \text{ and } j \end{aligned} \tag{8}$$

where x_{ij} is the amount of units shipped from origin i to destination j and c_{ij} is the cost of shipping one unit from origin i to destination j. The amount of supply at origin i is a_i and the amount of demand at destination j is b_j. The objective is to determine the unknown x_{ij} that will minimize the total transportation cost while satisfying all the supply and demand constraints. The above formulation assumes that total supply and total demand are equal to one another, that is,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_i \tag{9}$$

This balanced condition is treated as a necessary and sufficient condition for the existence of a feasible solution to the problem [10]. The simplex method, the least-cost method, north-west corner rule or Vogel's approximation method [15] can be used to obtain the optimal solution for the transportation problem defined in (6)-(8) if c_{ij}, a_i and b_j are known, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. In this study, however, we consider the supply values, the demand values and the equality constraint are all fuzzy. In other words, the coefficients a_i and b_j are uncertain or lack of precision. The uncertainties can be represented using fuzzy numbers and hence the equations in (6)-(9) can be formulated as the following fuzzy transportation problem:

$$\text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{10}$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} \approx \tilde{a}_i, \quad i = 1, 2, \ldots, m \tag{11}$$

$$\sum_{i=1}^{m} x_{ij} \approx \tilde{b}_j, \quad j = 1, 2, \ldots, n, \quad x_{ij} \geq 0, \text{ for all } i \text{ and } j \tag{12}$$

Then the balanced condition becomes

$$\sum_{i=1}^{m} a_i \approx \sum_{j=1}^{n} b_i \tag{13}$$

If we assume the coefficients a_i and b_j in (11) and (12) are triangular fuzzy numbers, denoted by $\tilde{p}_i = (p_i - \Delta_{i1}, p_i, p_i + \Delta_{i2})$, $0 \leq \Delta_{i1} \leq p_i$, $0 \leq \Delta_{i2} \leq p_i$, $1 \leq i \leq m$, and $\tilde{q}_j = (q_j - \Delta_{j1}, q_j, q_j + \Delta_{j2})$, $0 \leq \Delta_{j1} \leq q_j$, $0 \leq \Delta_{j2} \leq q_j$, $1 \leq j \leq n$, the membership function of $\tilde{p}_i$ and $\tilde{q}_j$ is as defined in (3). Generally, we use $\tilde{B} = (p, q, r)$, $p < q < r$, as the triangular fuzzy number. Similarly, if we assume that the coefficients a_i and b_j are interval-valued fuzzy numbers, denoted by $\tilde{A} = \left[\tilde{A}^L, \tilde{A}^U\right] = [((a, b, c; \lambda), (p, b, r; \rho))]$, $p < a < b < c < r$, the membership function of $\tilde{A}$ is then as defined in (4) and (5). When the coefficients a_i and b_j in (11) and (12) are fuzzy numbers, i.e., fuzzy supplies and fuzzy demands, the transportation problem becomes a fuzzy transportation problem. Because a fuzzy number may represent many possible real numbers, it is not easy to compare among solutions to determine the Pareto optimal solution. Therefore, ranking methods for fuzzy numbers help us to compare fuzzy numbers and then Pareto optimal solutions can be determined based on the ranked values of the objective function. Several methods of ranking fuzzy numbers have been proposed over the past decades [5,8]. Here, we propose two useful and flexible ranking methods as defined in Definitions 2.5 and 2.6.

4. **Numerical Examples.**

Two numerical examples are given here to illustrate the effectiveness of the proposed approach to solving transportation problem with fuzzy supplies and fuzzy demands.

Example 4.1. *Consider a transportation problem with two supplies* ($m = 2$) *and three demands* ($n = 3$). *The transportation costs are in the following:* $c_{11} = 16$, $c_{12} = 15$, $c_{13} = 25$, $c_{21} = 19$, $c_{22} = 24$ *and* $c_{23} = 12$. *The amount of supplies are* $a_1 = 10$ *and* $a_2 = 8$. *The amount of demands are* $b_1 = 5$, $b_2 = 6$ *and* $b_3 = 7$ (*see Figure 2*). *The northwest corner rule is applied to solving the problem obtaining* $x_{11} = 4$, $x_{12} = 6$, $x_{13} = 0$, $x_{21} = 1$, $x_{22} = 0$, $x_{23} = 7$ *and the minimum cost* $Z = 257$. *Consider the fuzzy supplies and fuzzy demands based on using triangular fuzzy numbers. The fuzzy supplies are* $\tilde{a}_1 = (7.2, 10, 14.5)$ *and* $\tilde{a}_2 = (5.6, 8, 10.5)$. *The fuzzy demands are* $\tilde{b}_1 = (2.8, 5, 7.4)$, $\tilde{b}_2 = (4.5, 6, 9.5)$ *and* $\tilde{b}_3 = (4.6, 7, 10.6)$. *Using the distance ranking method for triangular fuzzy numbers as defined in Definition 2.5, the estimated values obtained are* $a_1^* = 10.425$, $a_2^* = 7.9$, $b_1^* = 5.05$, $b_2^* = 6.5$ *and* $b_3^* = 7.3$. *The final solution obtained from the proposed approach is* $x_{11} = 4.45$, $x_{12} = 6.5$, $x_{13} = 0$, $x_{21} = 0.6$, $x_{22} = 0$ *and* $x_{23} = 7.3$ *and the minimal transportation cost* $Z^* = 267.7$. *Next, consider the fuzzy supplies and fuzzy demands using interval-valued fuzzy numbers* $[(a, b, c; \lambda), (p, b, r; \rho)]$ *with* $\lambda = 0.9$ *and* $\rho = 1$. *The coefficients are* $\tilde{a}_1 = [(9, 10, 11; 0.9), (6, 10, 19; 1)]$, $\tilde{a}_2 = [(7, 8, 13; 0.9), (6, 8, 14; 1)]$, $\tilde{b}_1 = [(4, 5, 7; 0.9), (3, 5, 11; 1)]$, $\tilde{b}_2 = [(5, 6, 13; 0.9), (4, 6, 16; 1)]$

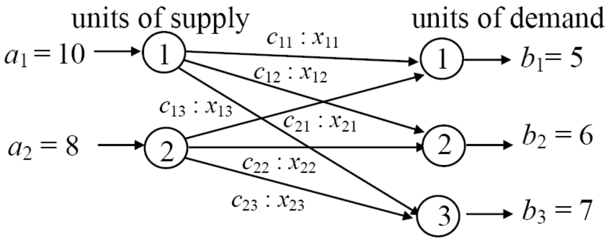

FIGURE 2. A transportation example with two supplies and three demands

and $\tilde{b}_3 = [(3, 7, 8; 0.9), (1, 7, 9; 1)]$. *Using the distance ranking method for interval-valued fuzzy numbers as defined in Definition 2.6, the estimated values obtained are* $a_1^* = 10.41$, $a_2^* = 8.57$, $b_1^* = 5.39$, $b_2^* = 7.02$ *and* $b_3^* = 6.49$. *The final solution obtained is* $x_{11} = 3.39$, $x_{12} = 7.02$, $x_{13} = 0.0$, $x_{21} = 2.0$, $x_{22} = 0.0$ *and* $x_{23} = 6.57$ *and the minimal transportation cost* $Z^* = 276.38$. *Comparing two fuzzy numbers obtained with that of the crisp problem, it shows that the interval-valued fuzzy numbers obtained have a higher cost than those of the triangular fuzzy numbers.*

Example 4.2. *Consider another two supplies* ($m = 2$) *and three demands* ($n = 3$) *transportation problem. The problem has the following form:*

$$\begin{aligned} \text{Min } Z &= 10x_{11} + 50x_{12} + 80x_{13} + 75x_{21} + 60x_{22} + 20x_{23} \\ \text{s.t. } & x_{11} + x_{12} + x_{13} \cong \tilde{a}_1 \\ & x_{21} + x_{22} + x_{23} \cong \tilde{a}_2 \\ & x_{11} + x_{12} \cong \tilde{b}_1 \\ & x_{12} + x_{22} \cong \tilde{b}_2 \\ & x_{13} + x_{23} \cong \tilde{b}_3 \\ & x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geq 0 \end{aligned}$$

For the crisp problem, the amount of supplies are $a_1 = 85$ *and* $a_2 = 60$ *and the amount of demands are* $b_1 = 40$, $b_2 = 30$ *and* $b_3 = 55$, *as listed in Table 1. The optimal solution obtained from the least-cost method is* $x_{11} = 55$, $x_{12} = 30$, $x_{13} = 0$, $x_{21} = 0$, $x_{22} = 0$, $x_{23} = 60$ *and the minimal transportation cost* $Z = 3250$. *Assume that the fuzzy supplies and fuzzy demands based on using triangular fuzzy numbers are given as* $\tilde{a}_1 = (74, 85, 90)$ *and* $\tilde{a}_2 = (48, 60, 68)$, $\tilde{b}_1 = (32, 40, 56)$, $\tilde{b}_2 = (20, 30, 46)$ *and* $\tilde{b}_3 = (42, 55, 80)$. *The estimated values obtained are* $a_1^* = 83.5$, $a_2^* = 59$, $b_1^* = 42$, $b_2^* = 31.5$ *and* $b_3^* = 58$. *The final solution obtained from the proposed approach is* $x_{11} = 52.0$, $x_{12} = 31.5$, $x_{13} = 0.0$, $x_{21} = 0.0$, $x_{22} = 0.0$ *and* $x_{23} = 59.0$ *and the minimal transportation cost* $Z^* = 3275.0$. *Consider the fuzzy supplies and fuzzy demands using interval-valued fuzzy numbers* $[(a, b, c; \lambda), (p, b, r; \rho)]$ *with* $\lambda = 0.9$ *and* $\rho = 1$. *Let* $\tilde{a}_1 = [(75, 85, 94; 0.9), (65, 85, 99; 1)]$, $\tilde{a}_2 = [(48, 60, 68; 0.9), (40, 60, 74; 1)]$, $\tilde{b}_1 = [(32, 40, 54; 0.9), (30, 40, 60; 1)]$, $\tilde{b}_2 = [(22, 30, 45; 0.9), (18, 30, 50; 1)]$ *and* $\tilde{b}_3 = [(42, 55, 70; 0.9), (38, 55, 78; 1)]$. *The estimated values of interval-valued fuzzy numbers obtained are* $a_1^* = 84.45$, $a_2^* = 59.26$, $b_1^* = 41.19$, $b_2^* = 31.09$

TABLE 1. Shipping costs, supplies and demands

m \ n	b_1	b_2	b_3	amount of supplies
a_1	10	50	80	85
a_2	75	60	20	60
amount of demands	40	30	55	

and $b_3^* = 55.55$*. The final solution obtained is* $x_{11} = 53.36$*,* $x_{12} = 31.09$*,* $x_{13} = 0.0$*,* $x_{21} = 2.0$*,* $x_{22} = 0.0$ *and* $x_{23} = 59.26$ *and the minimal transportation cost* $Z^* = 3273.3$*. On the other hand, if* $\lambda = 0.5$ *is used instead of 0.9 obtaining* $\tilde{a}_1 = [(75, 85, 94; 0.5), (65, 85, 99; 1)]$*,* $\tilde{a}_2 = [(48, 60, 68; 0.9), (40, 60, 74; 1)]$*,* $\tilde{b}_1 = [(32, 40, 54; 0.5), (30, 40, 60; 1)]$*,* $\tilde{b}_2 = [(22, 30, 45; 0.5), (18, 30, 50; 1)]$ *and* $\tilde{b}_3 = [(42, 55, 70; 0.5), (38, 55, 78; 1)]$*, then, the estimated values become* $a_1^* = 84$*,* $a_2^* = 58.81$*,* $b_1^* = 41.94$*,* $b_2^* = 31.69$ *and* $b_3^* = 56$*. The final solution obtained is* $x_{11} = 52.31$*,* $x_{12} = 31.68$*,* $x_{13} = 0.0$*,* $x_{21} = 2.0$*,* $x_{22} = 0.0$ *and* $x_{23} = 58.81$ *and the minimal transportation cost* $Z^* = 3283.8$*. The minimal cost obtained from* $\lambda = 0.5$ *is higher than that of* $\lambda = 0.9$*. We concluded that when the fluctuation becomes larger in the interval-valued fuzzy numbers, the transportation cost obtained also becomes higher.*

5. **Conclusions.** This study has investigated the classical transportation problem with fuzzy demands and fuzzy supplies. When the coefficients are represented using fuzzy numbers, the values of objective functions also become fuzzy numbers. Obviously, when the supply and demand quantities are uncertain or imprecise, the total transportation cost will vary within an interval. Two methods for ranking triangular and interval-valued fuzzy numbers as well as an approach to solving the fuzzy transportation problem are proposed. Numerical examples are given to illustrate and analyze final solutions obtained when using triangular and interval-valued fuzzy numbers for the fuzzy problem. The proposed approach obtains very good solutions to the fuzzy transportation problems. Future study will focus on using other approaches to solving the fuzzy problem.

REFERENCES

[1] S. Chanas, W. Kolodziejczyk and A. Machaj, A fuzzy approach to the transportation problem, *Fuzzy Sets and Systems*, vol.13, pp.211-221, 1984.

[2] S. Chanas and D. Kuchta, A concept of the optimal solution of the transportation problem with fuzzy cost coefficients, *Fuzzy Sets and Systems*, vol.82, pp.299-305, 1996.

[3] X. Gao, Q. Liu, L. Zhen, Y. Wang and Y. Gao, A new approach for solving fuzzy transportation problem, *The 5th International Conference on Intelligent Systems Design and Engineering Applications*, pp.37-39, 2014.

[4] M. Heigearataigh, A fuzzy transportation algorithm, *Fuzzy Sets and Systems*, vol.8, pp.235-243, 1982.

[5] F. Jimenez and J. L. Verdegay, Obtaining fuzzy solutions to the fuzzy solid transportation problem with genetic algorithms, *Proc. of the 6th IEEE International Conference on Fuzzy Systems*, Barcelona, Spain, pp.1657-1663, 1997.

[6] A. Kaufmann and M. M. Gupta, *Fuzzy Mathematical Models in Engineering and Management Science*, Van Nostrand Reinhold, 1988.

[7] D. Kaur, S. Mukherjee and K. Basu, A new fuzzy programming technique approach to solve fuzzy transportation problem, *The 2nd International Conference on Business and Information Management*, pp.144-150, 2014.

[8] S. Kikuchi, A method to defuzzify the fuzzy number: Transportation problem application, *Fuzzy Sets and Systems*, vol.116, pp.3-9, 2000.

[9] A. Kumar and A. Kaur, Application of classical transportation methods to find the fuzzy optimal solution of fuzzy transportation problem, *Fuzzy Information and Engineering*, vol.3, no.1, pp.81-99, 2011.

[10] Y. Li, K. Ida and M. Gen, Evolutionary program for multicriteria solid transportation problem with fuzzy numbers, *Proc. of the 3rd IEEE Conference on Evolutionary Computation*, Nagoya, Japan, 1996.

[11] F.-T. Lin and T.-R. Tsai, A two-stage genetic algorithm for solving the transportation problem with fuzzy demands and fuzzy supplies, *International Journal of Innovative Computing, Information and Control*, vol.5, no.12(B), pp.4775-4785, 2009.

[12] S. T. Liu and C. Kao, Solving fuzzy transportation problems based on extension principle, *European Journal of Operational Research*, vol.153, pp.661-674, 2004.

[13] S. T. Liu, The total cost bounds of the transportation problem with varying demand and supply, *Omega*, vol.31, pp.247-251, 2003.

[14] M. Tada and H. Ishii, An integer fuzzy transportation problem, *Computers Mathematical Applications*, vol.31, no.9, pp.71-87, 1996.
[15] H. A. Taha, *Operation Research*, 5th Edition, Macmillan Publishing Company, New York, 1992.
[16] H.-J. Zimmermann, Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets and Systems*, vol.1, pp.45-55, 1978.