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SIMPLE ADAPTIVE CONTROL WITH ADAPTIVE PARALLEL FEEDFORWARD COMPENSATOR

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Abstract. *This paper deals with a design problem of a simple adaptive control (SAC) system with a parallel feedforward compensator (PFC). To adopt SAC, the controlled system has to be almost strictly positive real (ASPR). In the proposed method, the ASPR-ness of the augmented system will be maintained by adaptive PFC which is using online input/output data. This enables us to maintain the stability of SAC even if the controlled system is unknown and is changing during the operation. In addition, the boundedness of the signals in the control system will be analyzed.*
Keywords: Simple adaptive control, ASPR, Parallel feedforward compensator

1. **Introduction.** In recent decades, almost strictly positive real (ASPR) based adaptive output feedback controls, which are typified by simple adaptive control (SAC) [1], have received a lot of attention [2, 3, 4, 5]. Under ASPR conditions, the method can constitute a stable adaptive control system only with the output feedback. Unfortunately, however, the ASPR conditions are very severe conditions for actual systems and most of the actual system does not satisfy the conditions. To overcome this problem, the introduction of a parallel feedforward compensator (PFC) has been proposed [5]. By introducing PFC, the augmented system which consists of the plant and the PFC can satisfy the ASPR conditions. With this in mind, several design methods of such a PFC have been proposed [6, 7, 8, 9]. Recently, an adaptive PFC, whose parameter is adaptively adjusted by online data, has been proposed [10]. Although the output of augmented system tracks to the reference signal, the output of the controlled system might not track to the reference signal because of the gain of PFC.

With this in mind, the introduction of adaptive PFC to the SAC will be proposed in this paper. The boundedness of all the signals in the control system will also be analyzed.

This paper is organized as follows. Section 2 shows the problem statements which are considered in this paper. In Section 3, ideal PFC will be shown. In Sections 4 and 5, parameter adjusting laws of adaptive feedback gain and the design of adaptive PFC will be proposed. Then the boundedness of the control system with parameter adjusting laws will be analyzed in Section 6. Numerical simulation is in Section 7. Finally, concluding remarks are presented in Section 8.

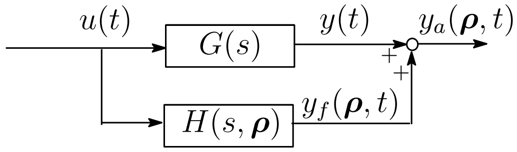


FIGURE 1. Augmented system with PFC

2. **Problem Statements.** Let's consider an augmented system shown in Figure 1. Here, $G(s)$ is unknown but stable single input single output (SISO) controlled system and $H(s, \boldsymbol{\rho})$ is a PFC which is parameterized by $\boldsymbol{\rho}$. Also, the controlled system $G(s)$ can be described by the following nth order SISO state-space representation.

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + \boldsymbol{b}u(t),\ y(t) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(t) \tag{1}$$

Then, for this system, we consider a tracking control to the following n_mth order SISO reference model output $y_m(t)$.

$$\dot{\boldsymbol{x}}_m(t) = A_m\boldsymbol{x}_m(t) + \boldsymbol{b}_m r(t),\ y_m(t) = \boldsymbol{c}_m^{\mathrm{T}}\boldsymbol{x}_m(t) \tag{2}$$

where $r(t)$ is a reference signal. We suppose that $G(s)$, $H(s, \boldsymbol{\rho})$ and the reference model satisfy the following assumptions.

Assumption 2.1. [i]

$$\mathrm{rank}\begin{bmatrix} A & \boldsymbol{b} \\ \boldsymbol{c}^{\mathrm{T}} & 0 \end{bmatrix} = n + 1 \tag{3}$$

[ii] Ω_{ij} is the solution of the following equation.

$$\begin{bmatrix} A & \boldsymbol{b} \\ \boldsymbol{c}^{\mathrm{T}} & 0 \end{bmatrix}\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = I_{n+1} \tag{4}$$

[iii] The eigenvalues of Ω_{11} are not equal to the reciprocal of eigenvalues of A_m.

Assumption 2.2. $H(s, \boldsymbol{\rho}) = 0$ with $\boldsymbol{\rho} = \boldsymbol{0}$.

Assumption 2.3. The reference signal $r(t)$ is bounded and its derivative signal $\dot{r}(t)$ is also bounded.

When Assumption 2.1 holds, for the command generator tracker (CGT) problem, the following theorem and lemma stand [1].

Theorem 2.1. *The ideal input $u^*(t)$ and the ideal state $\boldsymbol{x}^*(t)$, which can achieve perfect tracking to the reference model output, that is,*

$$y(t) \equiv y_m(t) \tag{5}$$

can be given by the following equation

$$\begin{bmatrix} \boldsymbol{x}^*(t) \\ u^*(t) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}\begin{bmatrix} \boldsymbol{x}_m(t) \\ u_m(t) \end{bmatrix} + \begin{bmatrix} \Omega_{11} \\ \Omega_{21} \end{bmatrix}\boldsymbol{v}(t) \tag{6}$$

with

$$\begin{aligned} S_{11} &= \Omega_{11}S_{11}A_m + \Omega_{12}\boldsymbol{c}_m^{\mathrm{T}}, & S_{12} &= \Omega_{11}S_{11}\boldsymbol{b}_m \\ S_{21} &= \Omega_{21}S_{11}A_m + \Omega_{22}\boldsymbol{c}_m^{\mathrm{T}}, & S_{22} &= \Omega_{21}S_{11}\boldsymbol{b}_m \end{aligned}$$

$$\Omega_{11}\dot{\boldsymbol{v}}(t) = \boldsymbol{v}(t) - S_{12}\dot{u}_m(t),\ \boldsymbol{v}(0) = \boldsymbol{0} \tag{7}$$

Lemma 2.1. *When the signals $r(t)$ and $\dot{r}(t)$ are bounded, then the signal $\boldsymbol{v}(t)$ in Equation (7) is bounded.*

Now, from Figure 1, the open-loop system from the input $u(t)$ to the output $y_a(t, \boldsymbol{\rho})$ of the augmented system with a PFC can be expressed by $G_a(s, \boldsymbol{\rho}) = G(s) + H(s, \boldsymbol{\rho})$.

Under these statements, the objective here is to adjust the parameters of the PFC adaptively so that the augmented system $G_a(s, \boldsymbol{\rho})$ becomes close to a given ideal ASPR model $G_a^*(s)$ and estimates the parameters of the gains k^*, $S_{21} = \boldsymbol{k}_x^*$ and $S_{22} = k_r^*$ for the tracking control.

3. **Ideal PFC.** Define the ideal output $y_a^*(t)$ of the ideal augmented system $G_a^*(s)$ as follows.

$$y_a^*(t) = G_a^*(s)\,[u(t)] \tag{8}$$

Then, we consider finding parameters which make the output $y_h(t, \boldsymbol{\rho})$ of adaptive PFC as ideal output $y_h^*(t)$.

Here, the ideal output of PFC $y_h^*(t)$ can be described by using the output $y(t)$ of controlled system as follows.

$$y_h^*(t) = y_a^*(t) - y(t) \tag{9}$$

Now, suppose that the ideal PFC $H^*(s)$ given by the following n_hth compensator:

$$H^*(s) = \frac{N_H^*(s)}{D_H^*(s)} = \frac{b_1^* s^{n_h-1} + b_2^* s^{n_h-2} + \cdots + b_{n_h}^*}{s^{n_h} + a_1^* s^{n_h-1} + \cdots + a_{n_h}^*} \tag{10}$$

then, the ideal output of the PFC can also be described as follows.

$$y_h^*(s) = H^*(s)\,[u(t)] \tag{11}$$

By introducing the following stable n_hth filter

$$\frac{1}{F(s)} = \frac{1}{s^{n_h} + f_1 s^{n_h-1} + \cdots + f_{n_h}} \tag{12}$$

the ideal output of the PFC can be represented by

$$y_h^*(t) = \frac{Z(s)}{F(s)}\,[y_h^*(t)] + \frac{N_H^*(s)}{F(s)}\,[u(t)] = \boldsymbol{\rho}^{*\mathrm{T}} \boldsymbol{z}(t) \tag{13}$$

where $\boldsymbol{\rho}^* = [z_1\ z_2\ \cdots\ z_{n_h}\ b_1\ b_2\ \cdots\ b_{n_h}]^{\mathrm{T}}$ ($z_i = f_i - a_i$) and $\boldsymbol{z}(t) = \Big[\frac{s^{n_h-1}}{F(s)} y_h^* \ \cdots \frac{1}{F(s)} y_h^* \ \frac{s^{n_h-1}}{F(s)} u \ \cdots \frac{1}{F(s)} u\Big]^{\mathrm{T}}$. Unfortunately, the ideal parameter $\boldsymbol{\rho}$ is unknown, and we consider adjusting the PFC adaptively.

4. **Adaptive Control Input.** Let us consider adjusting the parameters, which can realize the ideal input by CGT, adaptively.

Here, the feedforward input will not be applied to PFC. Thus, the control system will be configured as Figure 2. Then consider designing the input by

$$u(t) = u_e(t) + u_f(t) \tag{14}$$

$$u_e(t) = -k(t)\bar{e}_a(t) - \rho_z \left\|\bar{\boldsymbol{z}}_f(t)\right\|^2 \bar{e}_a(t),\ u_f(t) = \boldsymbol{k}_x(t)^{\mathrm{T}} \boldsymbol{x}_m(t) + k_r(t) r(t) \tag{15}$$

where $\bar{\boldsymbol{z}}_f(t)$ will be given later in Equation (25).

The gains $k(t)$, $\boldsymbol{k}_x(t)$ and $k_r(t)$ are adaptively adjusted by the adjusting laws:

$$\dot{k}(t) = \gamma \bar{e}_a(t)^2 - \sigma k(t) \tag{16}$$

$$\dot{\boldsymbol{k}}_x(t) = -\Gamma_x \boldsymbol{x}_m(t) \bar{e}_a(t) - \sigma_x \boldsymbol{k}_x(t) \tag{17}$$

$$\dot{k}_r(t) = -\gamma_r r(t) \bar{e}_a(t) - \sigma_r k_r(t) \tag{18}$$

where,

$$\bar{e}_a(t) = \bar{y}_a(t) - y_m(t),\ \bar{y}_a(t) = y(t) + \bar{y}_h(t) \tag{19}$$

and $\bar{y}_h(t)$ is output of the PFC generated by the input $u_e(t)$.

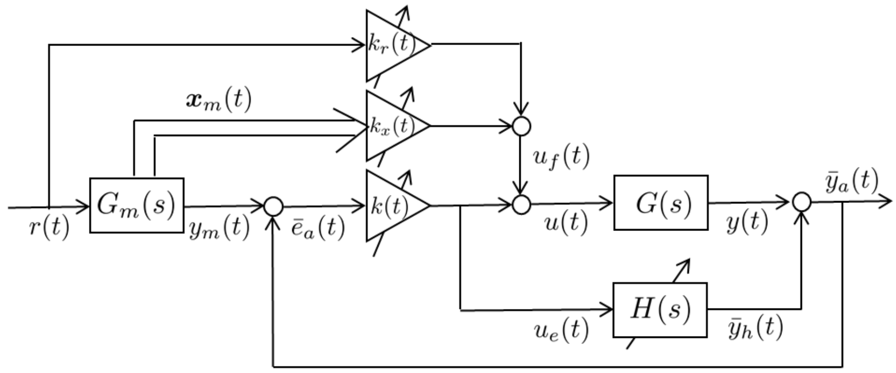


FIGURE 2. SAC with adaptive PFC

5. **Adaptive PFC.** From Equation (13), the output of the PFC generated by using estimated value of parameter $\boldsymbol{\rho}$ and input u can be described as follows:

$$y_h(t, \boldsymbol{\rho}) = \boldsymbol{\rho}(t)^{\mathrm{T}} \boldsymbol{z}(t), \;\; \boldsymbol{z}(t) = \boldsymbol{z}(y_h^*, u) \tag{20}$$

Also, the output with the parameter $\boldsymbol{\rho}$ and the input u_f can be described as follows:

$$y_{hf}(t, \boldsymbol{\rho}) = \boldsymbol{\rho}^{\mathrm{T}} \boldsymbol{z}_f(t), \;\; \boldsymbol{z}_f(t) = \left[\frac{s^{n_h-1}}{F(s)} y_{hf} \cdots \frac{1}{F(s)} y_{hf} \;\; \frac{s^{n_h-1}}{F(s)} u_f \;\; \cdots \frac{1}{F(s)} u_f \right]^{\mathrm{T}} \tag{21}$$

Then the output of the PFC with ideal parameter $\boldsymbol{\rho}^*$ and input u_e can be described as

$$\bar{y}_h(t, \boldsymbol{\rho}^*) = H(s, \boldsymbol{\rho}^*)[u_e(t)] = H(s, \boldsymbol{\rho}^*)[u(t) - u_f(t)] = y_h(t, \boldsymbol{\rho}^*) - y_{hf}(t, \boldsymbol{\rho}^*) \tag{22}$$

Here, define the output of adaptive PFC as

$$\bar{y}_h(t) = y_h(t) - y_{hf}(t) \tag{23}$$

where,

$$y_h(t) \;=\; G_a^*(s) \left[\boldsymbol{\rho}(t)^{\mathrm{T}} \bar{\boldsymbol{z}}(t) \right], \;\; \bar{\boldsymbol{z}}(t) = G_a^{*-1}(s)[\boldsymbol{z}(t)] \tag{24}$$

$$y_{hf}(t) = G_a^*(s) \left[\boldsymbol{\rho}(t)^{\mathrm{T}} \bar{\boldsymbol{z}}_f(t) \right], \;\; \bar{\boldsymbol{z}}_f(t) = G_a^{*-1}(s)[\boldsymbol{z}_f(t)] \tag{25}$$

with estimated parameter $\boldsymbol{\rho}(t)$. $\boldsymbol{\rho}(t)$ is adaptively adjusted by the following parameter adjusting law.

$$\dot{\boldsymbol{\rho}}(t) = -\Gamma_h \left(\bar{\boldsymbol{z}}(t) - \bar{\boldsymbol{z}}_f(t) \right) \bar{e}_a(t) - \sigma_h \boldsymbol{\rho}(t) \tag{26}$$

6. **Boundedness Analysis.** In this section, the boundedness of all the signals in the control system, which is configured with (14) to (19), (23) and (26), is shown.

First, from the relational expression of $\bar{y}_h^*(t) = y_h^*(t) - y_{hf}^*(t)$, the output of PFC with u_e can be represented as

$$\begin{aligned} \bar{y}_h(t) \;&=\; \bar{y}_h(t) - \bar{y}_h^*(t) + \bar{y}_h^*(t) = (y_h(t) - y_{hf}(t)) - (y_h^*(t) - y_{hf}^*(t)) + \bar{y}_h^*(t) \\ &=\; G_a^* \left[\Delta\boldsymbol{\rho}(t)^{\mathrm{T}} \left(\bar{\boldsymbol{z}}(t) - \bar{\boldsymbol{z}}_f(t) \right) \right] + y_h^*(t) - \boldsymbol{\rho}^{*\mathrm{T}} \boldsymbol{z}_f(t) \end{aligned} \tag{27}$$

where $\Delta\boldsymbol{\rho} = \boldsymbol{\rho}(t) - \boldsymbol{\rho}^*$ is estimation error of PFC parameter vector. Also, the output $\bar{y}_a(t)$ of augmented system, can be represented from Equations (8), (19) and (27) as

$$\begin{aligned} \bar{y}_a(t) \;&=\; y_a^*(t) - y_h^*(t) + \bar{y}_h(t) \\ &=\; G_a^*(s)[u_e(t) + u_f(t)] + G_a^* \left[\Delta\boldsymbol{\rho}(t)^{\mathrm{T}} \bar{\boldsymbol{z}}(t) \right] - G_a^* \left[\Delta\boldsymbol{\rho}(t)^{\mathrm{T}} \bar{\boldsymbol{z}}_f(t) \right] - \boldsymbol{\rho}^{*\mathrm{T}} \boldsymbol{z}_f(t) \\ &=\; G_a^*(s)[u_e(t)] + G_a^*(s) \left[u_f(t) - u_f^*(t) \right] + G(s)[u_f^*(t)] + \boldsymbol{\rho}^{*\mathrm{T}} \left(\boldsymbol{z}_f^\star(t) - \boldsymbol{z}_f(t) \right) \\ &\quad + G_a^* \left[\Delta\boldsymbol{\rho}(t)^{\mathrm{T}} \bar{\boldsymbol{z}}(t) \right] - G_a^* \left[\Delta\boldsymbol{\rho}(t)^{\mathrm{T}} \bar{\boldsymbol{z}}_f(t) \right] \end{aligned} \tag{28}$$

with

$$\boldsymbol{z}_f^\star(t) = \left[\frac{s^{n_h-1}}{F(s)} y_{hf}^\star \cdots \frac{1}{F(s)} y_{hf}^\star \ \frac{s^{n_h-1}}{F(s)} u_f^* \ \cdots \frac{1}{F(s)} u_f^* \right]^{\mathrm{T}}, \ y_{hf}^\star(t) = H^*(s)\left[u_f^*(t)\right] \tag{29}$$

Since $G(s)[u_f^*(t)] = y_m(t)$ stands, the error equation can be represented as follows.

$$\bar{e}_a(t) = G_a^*(s)\left[u_e(t) + \Delta\boldsymbol{\rho}(t)^{\mathrm{T}}\left(\bar{\boldsymbol{z}}(t) - \bar{\boldsymbol{z}}_f(t)\right) + u_f(t) - u_f^*(t) + \boldsymbol{\rho}^{*\mathrm{T}}(\bar{\boldsymbol{z}}_f^\star(t) - \bar{\boldsymbol{z}}_f(t))\right] \tag{30}$$

Then, it can be represented as the following state equation.

$$\left\{ \begin{array}{rcl} \dot{\boldsymbol{x}}_a(t) &=& A_a\boldsymbol{x}_a(t) + \boldsymbol{b}_a\left\{u_e(t) + \Delta\boldsymbol{\rho}(t)^{\mathrm{T}}\left(\bar{\boldsymbol{z}}(t) - \bar{\boldsymbol{z}}_f(t)\right) + \Delta\boldsymbol{k}_x(t)^{\mathrm{T}}\boldsymbol{x}_m(t)\right. \\ && \left. +\Delta k_r(t)r(t) - \Omega_{21}\boldsymbol{v}(t) + \boldsymbol{\rho}^{*\mathrm{T}}(\bar{\boldsymbol{z}}_f^\star(t) - \bar{\boldsymbol{z}}_f(t))\right\} \\ \bar{e}_a(t) &=& \boldsymbol{c}_a^{\mathrm{T}}\boldsymbol{x}_a(t) \end{array} \right. \tag{31}$$

where $\Delta\boldsymbol{k}_x(t) = \boldsymbol{k}_x(t) - \boldsymbol{k}_x^*$ and $\Delta k_r(t) = k_r(t) - k_r^*$ are estimation error of gains. Finally, because the augmented system is ASPR and thus it has relative degree of 1, the augmented system can be described as follows.

$$\left\{ \begin{array}{rcl} \dot{\bar{e}}_a(t) &=& a\bar{e}_a(t) + b\left\{u_e(t) + \Delta\boldsymbol{\rho}(t)^{\mathrm{T}}(\bar{\boldsymbol{z}}(t) - \bar{\boldsymbol{z}}_f(t)) + \Delta\boldsymbol{k}_x(t)^{\mathrm{T}}\boldsymbol{x}_m(t)\right. \\ && \left. +\Delta k_r(t)r(t) - \Omega_{21}\boldsymbol{v}(t) + \boldsymbol{\rho}^{*\mathrm{T}}(\bar{\boldsymbol{z}}_f^\star(t) - \bar{\boldsymbol{z}}_f(t))\right\} + \boldsymbol{c}_\eta^{\mathrm{T}}\boldsymbol{\eta}_a(t) \\ \dot{\boldsymbol{\eta}}_a(t) &=& A_\eta\boldsymbol{\eta}_a(t) + \boldsymbol{b}_\eta\bar{e}_a(t) \end{array} \right. \tag{32}$$

Here, we consider positive definite function:

$$\begin{array}{rcl} V(t) &=& V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) \\ V_1(t) &=& \bar{e}_a(t)^2, \ V_2(t) = \boldsymbol{\eta}_a(t)^{\mathrm{T}}P_\eta\boldsymbol{\eta}_a(t), \ V_3(t) = \dfrac{b}{\gamma}\Delta k(t)^2 \\ V_4(t) &=& b\Delta\boldsymbol{k}_x(t)^{\mathrm{T}}\Gamma_x^{-1}\Delta\boldsymbol{k}_x(t) + \dfrac{b}{\gamma_r}\Delta k_r(t)^2, \ V_5(t) = b\Delta\boldsymbol{\rho}(t)^{\mathrm{T}}\Gamma_h^{-1}\Delta\boldsymbol{\rho}(t) \end{array} \tag{33}$$

where $\Delta k(t) = k(t) - k^*$ is the estimation error of feedback gain.

The time derivative of $V_1(t)$ to $V_5(t)$ can be obtained as follows.

$$\begin{array}{rcl} \dot{V}_1(t) &=& -2(bk^* - a)\bar{e}_a(t)^2 - 2b\Delta k(t)\bar{e}_a(t)^2 - 2b\rho_z\left\|\bar{\boldsymbol{z}}_f(t)\right\|^2\bar{e}_a(t)^2 + 2b\boldsymbol{c}_\eta^{\mathrm{T}}\boldsymbol{\eta}_a(t)\bar{e}_a(t) \\ && +2b\Delta\boldsymbol{\rho}(t)^{\mathrm{T}}(\bar{\boldsymbol{z}}(t) - \bar{\boldsymbol{z}}_f(t))\bar{e}_a(t) + 2b\Delta\boldsymbol{k}_x(t)^{\mathrm{T}}\boldsymbol{x}_m(t)\bar{e}_a(t) + 2b\Delta k_r(t)r(t)\bar{e}_a(t) \\ && -2b\Omega_{21}\boldsymbol{v}(t)\bar{e}_a(t) + 2b\boldsymbol{\rho}^{*\mathrm{T}}(\bar{\boldsymbol{z}}_f^\star(t) - \bar{\boldsymbol{z}}_f(t))\bar{e}_a(t) \end{array} \tag{34}$$

$$\dot{V}_2(t) = \boldsymbol{\eta}_a(t)^{\mathrm{T}}\left(A_\eta^{\mathrm{T}}P_\eta + P_\eta A_\eta\right)\boldsymbol{\eta}_a(t) + \boldsymbol{b}_\eta^{\mathrm{T}}P_\eta\boldsymbol{\eta}_a(t)\bar{e}_a(t) + \boldsymbol{\eta}_a(t)^{\mathrm{T}}P_\eta\boldsymbol{b}_\eta\bar{e}_a(t) \tag{35}$$

$$\dot{V}_3(t) = 2b\Delta k(t)\bar{e}_a(t)^2 - \frac{2b\sigma}{\gamma}\Delta k(t)k(t) \tag{36}$$

$$\begin{array}{rcl} \dot{V}_4(t) &=& -2b\Delta\boldsymbol{k}_x(t)^{\mathrm{T}}\boldsymbol{x}_m(t)\bar{e}_a(t) - 2b\sigma_x\Delta\boldsymbol{k}_x(t)^{\mathrm{T}}\Gamma_x^{-1}\boldsymbol{k}_x(t) \\ && -2b\Delta k_r(t)r(t)\bar{e}_a(t) - \dfrac{2b\sigma_r}{\gamma_r}\Delta k_r(t)k_r(t) \end{array} \tag{37}$$

$$\dot{V}_5(t) = -2b\Delta\boldsymbol{\rho}(t)^{\mathrm{T}}(\bar{\boldsymbol{z}}(t) - \bar{\boldsymbol{z}}_f(t))\bar{e}_a(t) - 2b\sigma_h\Delta\boldsymbol{\rho}(t)^{\mathrm{T}}\Gamma_h^{-1}\boldsymbol{\rho}(t) \tag{38}$$

Then from Assumption 2.3 and Lemma 2.1, there exists positive constant v_m such that $\|\boldsymbol{v}(t)\| \leq v_m$, and $\dot{V}(t)$ can be evaluated as follows.

$$\begin{array}{rcl} \dot{V}(t) &\leq& -\left(2bk^* - 2a - \dfrac{1}{\delta_1} - \dfrac{1}{\delta_2} - \dfrac{1}{\delta_3} - \dfrac{1}{\delta_4}\right)\bar{e}_a(t)^2 \\ && -\left(\lambda_{\min}\left[Q_\eta\right] - \delta_1\|\boldsymbol{c}_\eta\|^2 - \delta_4\|\boldsymbol{b}_\eta^{\mathrm{T}}P_\eta\|^2\right)\|\boldsymbol{\eta}_a(t)\|^2 \\ && -\left(\dfrac{2b\sigma}{\gamma} - \delta_5\right)\Delta k(t)^2 - \left(2b\sigma_x\lambda_{\min}\left[\Gamma_x^{-1}\right] - \delta_6\right)\|\Delta\boldsymbol{k}_x(t)\|^2 \\ && -\left(\dfrac{2b\sigma_r}{\gamma_r} - \delta_7\right)\Delta k_r(t)^2 - \left(2b\sigma_h\lambda_{\min}\left[\Gamma_h^{-1}\right] - \delta_8\right)\|\Delta\boldsymbol{\rho}(t)\|^2 \end{array}$$

$$+\delta_2 b^2\|\Omega_{21}\|^2 {v_m}^2 + \left(\delta_3 b^2 \left\|\bar{\boldsymbol{z}}_f^\star(t)\right\|^2 + \frac{b}{2\rho_z} + \frac{b^2\sigma_h^2}{\delta_8}\left\|\Gamma_h^{-1}\right\|\right)\|\boldsymbol{\rho}^*\|^2$$
$$+\frac{b^2\sigma^2}{\delta_5\gamma^2}k^{*2} + \frac{b^2\sigma_x^2}{\delta_6}\left\|\Gamma_x^{-1}\right\|^2\|\boldsymbol{k}_x^*\|^2 + \frac{b^2\sigma_r^2}{\delta_7\gamma_r^2}k_r^{*2} \tag{39}$$

Here, by considering sufficiently small positive constants δ_1, δ_4, δ_5, δ_6, δ_7 and δ_8 such that

$$\begin{aligned} \lambda_{\min}\left[Q_\eta\right] - \delta_1\|\boldsymbol{c}_\eta\|^2 - \delta_4\|\boldsymbol{b}_\eta^{\mathrm{T}} P_\eta\|^2 > \alpha_2 > 0,\ \tfrac{2b\sigma}{\gamma} - \delta_5 > \alpha_3 > 0 \\ 2b\sigma_x\lambda_{\min}\left[\Gamma_x^{-1}\right] - \delta_6 > \alpha_4 > 0,\ \tfrac{2b\sigma_r}{\gamma_r} - \delta_7 > \alpha_5 > 0 \\ 2b\sigma_h\lambda_{\min}\left[\Gamma_h^{-1}\right] - \delta_8 > \alpha_6 > 0 \end{aligned} \tag{40}$$

and sufficiently large ideal feedback gain k^* such that

$$2bk^* - 2a - \frac{1}{\delta_1} - \frac{1}{\delta_2} - \frac{1}{\delta_3} - \frac{1}{\delta_4} > \alpha_1 > 0 \tag{41}$$

then $\dot{V}(t)$ can be evaluated as

$$\begin{aligned} \dot{V}(t) \ \leq\ & -\alpha_1\bar{e}_a(t)^2 - \alpha_2\|\boldsymbol{\eta}_a(t)\|^2 - \alpha_3\Delta k(t)^2 \\ & -\alpha_4\|\Delta\boldsymbol{k}_x(t)\|^2 - \alpha_5\Delta k_r(t)^2 - \alpha_6\|\Delta\boldsymbol{\rho}(t)\| + R \end{aligned} \tag{42}$$

with

$$\begin{aligned} R \ =\ & \left(\delta_3 b^2\left\|\bar{z}_{f\max}^\star\right\|^2 + \frac{b}{2\rho_z} + \frac{b^2\sigma_h^2}{\delta_8}\left\|\Gamma_h^{-1}\right\|\right)\|\boldsymbol{\rho}^*\|^2 \\ & +\frac{b^2\sigma^2}{\delta_5\gamma^2}k^{*2} + \frac{b^2\sigma_x^2}{\delta_6}\left\|\Gamma_x^{-1}\right\|^2\|\boldsymbol{k}_x^*\|^2 + \frac{b^2\sigma_r^2}{\delta_7\gamma_r^2}k_r^{*2} \end{aligned} \tag{43}$$

From the above, $\dot{V}(t)$ can be evaluated as

$$\dot{V}(t) \leq -\alpha V(t) + R,\ \alpha = \min\{\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4,\ \alpha_5,\ \alpha_6\} \tag{44}$$

and it shows that all the signals in the control system are bounded.

7. Numerical Simulation.

In this section the effectiveness of the proposed method is shown.

Here, we consider the following system.

$$G_p(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2},\ a_1 = 15,\ a_2 = 5,\ b_1 = 0.5,\ b_2 = 1.0$$

The parameters b_1 and b_2 change during the operation as follows.

$$b_1 = 0.5 - 0.006(t - 50),\ 50 \leq t \leq 140$$
$$b_2 = 1.0 - 0.006(t - 30),\ 30 \leq t \leq 130$$

We assume that these are unknown.

The reference signal $y_m(t)$ is given by

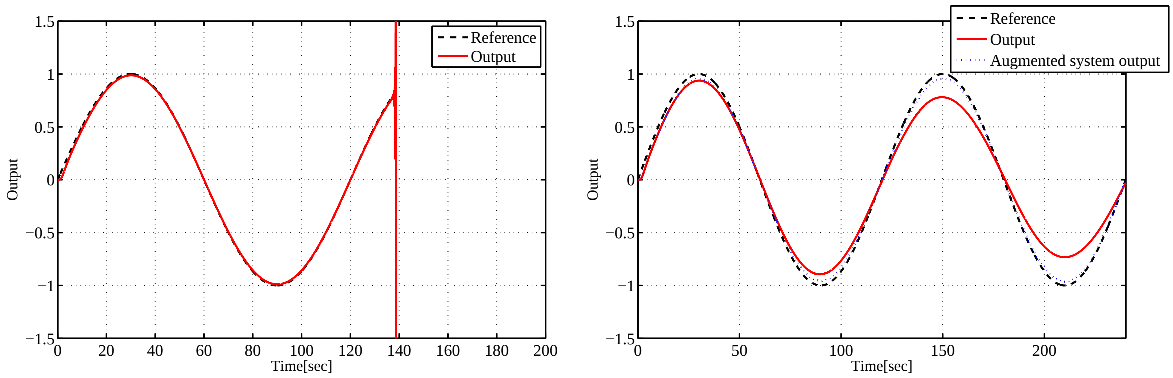
$$y_m(t) = \sin\left(\frac{\pi}{60}t\right) \tag{45}$$

Second order PFC is considered and the design parameters are set as follows.

$$G_a^*(s) = \frac{1}{s+1},\ \frac{1}{F(s)} = \frac{1}{s^2 + 25s + 50}$$

$$\Gamma_h = \mathrm{diag}[10,\ 1.0,\ 10,\ 1.0],\ \sigma_h = 1.0\times10^{-3},\ \rho_z = 100,\ \gamma = 1.0\times10^3,\ \sigma = 1.0\times10^{-3},$$
$$\Gamma_x = 5.0\times10^2,\ \sigma_x = 1.0\times10^{-3},\ \gamma_r = 5.0\times10^2,\ \sigma_r = 1.0\times10^{-3}$$

Figure 3(a) shows the result by only SAC. It can be seen that the control system became unstable after around 140 [sec]. This is because the plant became non-minimum phase system.



(a) Output by only SAC (b) Output by adaptive PFC

FIGURE 3. Simulation results by conventional method

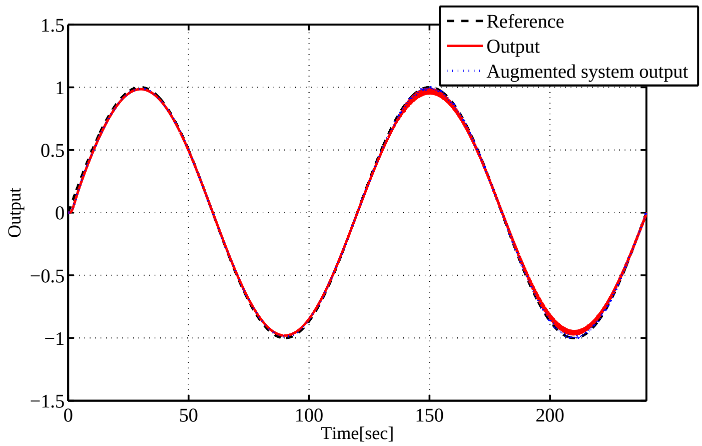


FIGURE 4. Simulation result by the proposed method

Figure 3(b) shows the result by adaptive P control with adaptive PFC [10]. It can be seen that the output of augmented system shows good performance but the output of the plant is getting worse. This is because the gain of the plant became lower.

Figure 4 shows the result by the proposed method. It can be seen that even if the plant has changed during the operation, the proposed method can maintain a high control performance.

8. **Conclusions.** In this paper, an SAC system with an adaptive PFC has been proposed. The proposed method makes it possible to hold the ASPRness of the augmented system by adjusting the parameters of a PFC adaptively. Therefore, even if the controlled system is changing during the operation, SAC system can maintain the stability. The boundedness of all the signals in the control system has also been analyzed. In addition, the effectiveness of the proposed method has been confirmed through a numerical simulation. In the future work, we will try to develop the method for nonlinear systems.

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