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RECURSIVE LEAST SQUARES IDENTIFICATION METHOD FOR A WIENER NONLINEAR SYSTEM

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Abstract. *This paper investigates a recursive least squares implementation for a Wiener system with an output error linear element and an invertible nonlinear part. It is difficult to parameterize Wiener systems, and get a simple output-input representation for Wiener systems with an output error linear element. This paper presents a new linear-in-parameter output-input expression for a Wiener output error model by using the auxiliary model idea and the invertible expression of the nonlinear part, and implements a recursive least squares algorithm for the Wiener system. The simulation results show that the proposed algorithm is effective.*
Keywords: System identification, Wiener system, The auxiliary model, Least squares

1. **Introduction.** Block-oriented systems are popular in modeling of nonlinear systems due to their simple and useful representation. The commonly used block-oriented nonlinear structures are the Hammerstein structure and the Wiener structure [1-4]. The Hammerstein structure puts a linear element after a nonlinear element, and the Wiener structure is in a reverse arrangement. So the output-input expression of the Wiener systems is more complex than that of the Hammerstein systems, and it is hard to parameterize Wiener systems into a linear-in-parameter form to which the standard least squares or stochastic gradient identification methods can be directly applied.

In order to solve the difficulty of parameterizing Wiener systems, some researchers investigated Wiener systems with a simple piecewise-linear nonlinearity. Figueroa et al. proposed a least squares algorithm for a Wiener model with a Laguerre basis linear model and a piecewise linear representation of the nonlinear static block [4]. Kozek and Sinanović discussed a least squares algorithm for a Wiener system with a deterministic autoregressive moving average linear model and a piecewise linear nonlinearity [5]. In addition, Hagenblad et al. derived a maximum likelihood method for a controlled autoregressive Wiener system [6]. Pelckmans discussed a minimal Lipschitz estimator applied to identification of a monotone finite impulse response Wiener system [7]. Tang et al. used step signals and particle swarm optimization to identify a Wiener system [8]. Wang and Ding presented a least squares based and a gradient based iterative identification for a Wiener like nonlinear system [9].

The motivation of this paper is to find an effective method to estimate the parameters of a Wiener system with a more complex nonlinear part than a piecewise linear nonlinearity. This paper focuses on a Wiener system with an output error linear element and

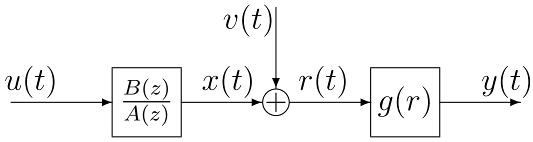

FIGURE 1. The Wiener output error system

an invertible nonlinear part, as shown in Figure 1. The contributions of this paper lie in the following.

- By the invertible expression of the nonlinear part and the virtual system idea, we construct a virtual system expressed in a linear-in-parameter equation instead of a bilinear parameter equation in [9].
- By the auxiliary model idea, we replace the unknown variable in the information vector in the linear-in-parameter equation with the output of an auxiliary model.
- This paper presents a recursive least squares identification algorithm for the virtual linear-in-parameter system.

The paper is organized as follows. Section 2 describes the system formulation related to a Wiener system with an invertible nonlinear part. Section 3 constructs a new linear-in-parameter output-input expression for the Wiener output error model, and implements a recursive least squares algorithm for the Wiener system. Section 4 provides an illustrative example. Finally, concluding remarks are given in Section 5.

2. **Problem Formulation.** The Weiner system with an output error linear part in Figure 1 can be expressed as

$$r(t) = \frac{B(z)}{A(z)}u(t) + v(t), \tag{1}$$

$$y(t) = g[r(t)], \tag{2}$$

where $u(t)$ and $y(t)$ are the system input and output, respectively; $x(t)$ is the noise free output of the linear block; $r(t)$ is the true output with noises of the linear block; $v(t)$ is a stochastic white noise with zero mean and variance σ^2; the linear block is an output error model; $A(z)$ and $B(z)$ are polynomials in the unit backward shift operator z^{-1} ($z^{-1}y(t) = y(t-1)$), and defined by

$$A(z) := 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{n_a} z^{-n_a},$$
$$B(z) := b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{n_b} z^{-n_b}.$$

Assume that the orders n_a and n_b are known and $y(t) = 0$, $u(t) = 0$, $x(t) = 0$, $r(t) = 0$ and $v(t) = 0$ for $t \leqslant 0$. The nonlinear block is a basis function.

The output of the linear block can be expressed as

$$x(t) = \frac{B(z)}{A(z)}u(t). \tag{3}$$

Then we can get

$$x(t) = -\sum_{i=1}^{n_a} a_i x(t-i) + \sum_{j=1}^{n_b} b_j u(t-j),$$

$$r(t) = -\sum_{i=1}^{n_a} a_i x(t-i) + \sum_{j=1}^{n_b} b_j u(t-j) + v(t). \tag{4}$$

Assume that the output nonlinearity g is considered to be invertible, and g^{-1} can be written as a linear combination of basis functions g_l:

$$r(t) = g^{-1}[y(t)] = \sum_{l=1}^{p} d_l g_l[y(t)]. \tag{5}$$

3. **The Recursive Least Squares Algorithm.** Notice that parameterization of the linear block in (1) and the nonlinear block in (5) is not unique. Without loss of generality, the first coefficient of the nonlinear part is unity, i.e., $d_1 = 1$ [10].

Combining Equations (4) and (5) gives

$$g_1[y(t)] + \sum_{l=2}^{p} d_l g_l[y(t)] = -\sum_{i=1}^{n_a} a_i x(t-i) + \sum_{j=1}^{n_b} b_j u(t-j) + v(t). \tag{6}$$

Define

$$\psi(t) = [-g_2[y(t)], \cdots, -g_p[y(t)], -x(t-1), \cdots, -x(t-n_a), u(t-1), \cdots, u(t-n_b)]^T \in R^{p+n_a+n_b-1},$$
$$\vartheta = [d_2, \cdots, d_p, a_1, \cdots, a_{n_a}, b_1, \cdots, b_{n_b}]^T \in R^{p+n_a+n_b-1},$$
$$\varphi(t) = [-x(t-1), \cdots, -x(t-n_a), u(t-1), \cdots, u(t-n_b)]^T \in R^{n_a+n_b},$$
$$\theta = [a_1, a_2, \cdots, a_{n_a}, b_1, b_2, \cdots, b_{n_b}]^T \in R^{n_a+n_b}.$$

Then we can construct a virtual system in the following linear-in-parameter expression

$$\begin{aligned} g_1[y(t)] &= -\sum_{l=2}^{p} d_l g_l[y(t)] - \sum_{i=1}^{n_a} a_i x(t-i) + \sum_{j=1}^{n_b} b_j u(t-j) + v(t), \\ &=: \psi^T(t)\vartheta + v(t), \end{aligned} \tag{7}$$

and we also have

$$x(t) =: \varphi^T(t)\theta. \tag{8}$$

The information vectors $\psi(t)$ and $\varphi(t)$ on the right-hand sides of (7) and (8) contain unknown internal variables $x(t-i)$, here we use the auxiliary model idea, to replace the unknown $x(t-i)$ with the outputs $x_a(t-i)$ of an auxiliary model [11], and it can be expressed as

$$x_a(t) = \varphi_a^T(t)\theta_a. \tag{9}$$

We take $\hat{\varphi}(t)$ to be the information vector $\varphi_a(t)$ of the auxiliary model, and $\hat{\theta}(t)$ to be the parameter vector θ_a of the auxiliary model, and we have

$$x_a(t) = \hat{\varphi}^T(t)\hat{\theta}(t). \tag{10}$$

Thus the estimates $\hat{\psi}(t)$ and $\hat{\varphi}(t)$ of $\psi(t)$ and $\varphi(t)$ can be written as

$$\hat{\psi}(t) = [-g_2[y(t)], \cdots, -g_p[y(t)], -x_a(t-1), \cdots, -x_a(t-n_a), u(t-1), \cdots, u(t-n_b)]^T,$$
$$\hat{\varphi}(t) = [-x_a(t-1), \cdots, -x_a(t-n_a), u(t-1), \cdots, u(t-n_b)]^T.$$

Suppose the data length $L \gg p + n_a + n_b$. Define a quadratic cost function,

$$J(\theta) = \sum_{t=1}^{L} \left[g_1[y(t)] - \psi^T(t)\vartheta\right]^2. \tag{11}$$

Minimizing the cost function, and replacing $\psi(t)$ with $\hat{\psi}(t)$ and $\varphi(t)$ with $\hat{\varphi}(t)$, we can obtain the following recursive least squares algorithm of estimating ϑ for the Wiener output error system (the W-RLS algorithm for short) [12]:

$$\hat{\vartheta}(t) = \hat{\vartheta}(t-1) + L(t)\left[g_1[y(t)] - \hat{\psi}^T(t)\hat{\vartheta}(t-1)\right], \tag{12}$$

$$\hat{\theta}(t) = \hat{\vartheta}(t)(p : p + n_a + n_b - 1), \tag{13}$$

$$L(t) = P(t-1)\hat{\psi}(t)\left[1 + \hat{\psi}^T(t)P(t-1)\hat{\psi}(t)\right]^{-1}, \tag{14}$$

$$P(t) = \left[I - L(t)\hat{\psi}^T(t)\right]P(t-1),\ P(0) = p_0 I, \tag{15}$$

$$\hat{\psi}(t) = [-g_2[y(t)], \cdots, -g_p[y(t)], -x_a(t-1), \cdots, -x_a(t-n_a), u(t-1), \cdots, u(t-n_b)]^T, \tag{16}$$

$$\hat{\varphi}(t) = [-x_a(t-1), \cdots, -x_a(t-n_a), u(t-1), \cdots, u(t-n_b)]^T, \tag{17}$$

$$x_a(t) = \hat{\varphi}^T(t)\hat{\theta}(t), \tag{18}$$

$$\hat{\vartheta}(t) = \left[\hat{d}_2(t), \cdots, \hat{d}_p(t), \hat{a}_1(t), \cdots, \hat{a}_{n_a}(t), \hat{b}_1(t), \cdots, \hat{b}_{n_b}(t)\right]^T, \tag{19}$$

$$\hat{\theta}(t) = \left[\hat{a}_1(t), \hat{a}_2(t), \cdots, \hat{a}_{n_a}(t), \hat{b}_1(t), \hat{b}_2(t), \cdots, \hat{b}_{n_b}(t)\right]^T. \tag{20}$$

The procedures of computing $\hat{\vartheta}(t)$ in the W-RLS algorithm are listed as follows.

1. Let $t = 1$, $\hat{\vartheta}(0) = 1_{p+n_a+n_c-1}/p_0$, $P(0) = p_0 I$, $x_a(i) = 1/p_0$, $u(i) = 0$, $y(i) = 0$ as $i \leqslant 0$, $p_0 = 10^2$.
2. Collect the data $u(t)$ and $y(t)$, and form $\hat{\psi}(t)$ and $\hat{\varphi}(t)$ by (16) and (17), respectively.
3. Compute $L(t)$ and $P(t)$ by (14) and (15), respectively; and compute $g_l[y(t)]$.
4. Update the parameter estimates $\hat{\vartheta}(t)$ and $\hat{\theta}(t)$ by (12) and (13).
5. Compute $x_a(t)$ by (18).
6. Increase t by 1 and go to Step 2.

The flowchart of computing the parameter estimate $\hat{\vartheta}(t)$ by the W-RLS algorithm in (12)-(20) is shown in Figure 2.

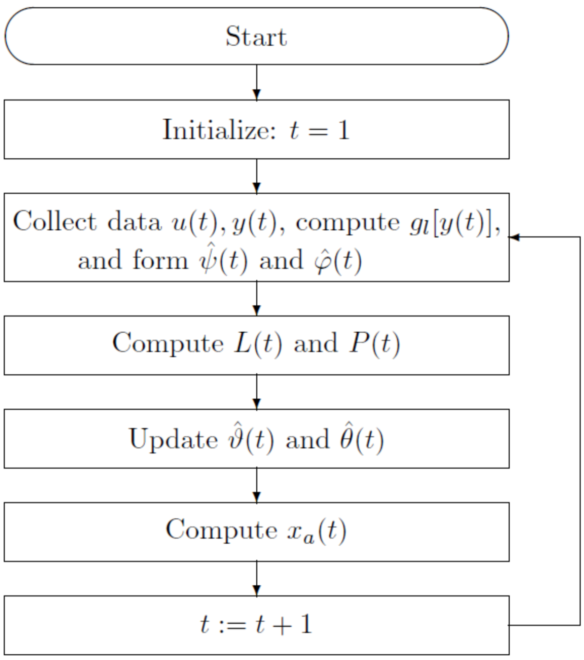

FIGURE 2. The flowchart of computing the W-RLS estimate $\hat{\vartheta}(t)$

4. **Example.** Consider the following Wiener system with the linear block,

$$x(t) = \frac{B(z)}{A(z)}u(t) + v(t),$$
$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} = 1 + 0.20z^{-1} + 0.35z^{-2},$$
$$B(z) = b_1 z^{-1} + b_2 z^{-2} = 1 + 0.15z^{-1} + 0.45z^{-2},$$

and the inversion of the nonlinear block,

$$r(t) = y(t) - 0.18y^2(t).$$

The input $\{u(t)\}$ is taken as an uncorrelated persistent excitation signal sequence with zero mean and unit variance, and $\{v(t)\}$ as a white noise sequence with zero mean and variance $\sigma^2 = 0.05^2$ and $\sigma^2 = 0.10^2$, respectively. Applying the W-RLS algorithm to estimate the parameters of this system, the parameter estimates and their errors are shown in Table 1, and the estimation errors $\delta := \|\hat{\vartheta}(t) - \vartheta\|/\|\vartheta\|$ are shown in Figure 3.

Table 1. The parameter estimates and errors with different σ^2

σ^2	t	d_2	a_1	a_2	b_1	b_2	δ (%)
0.05^2	100	−0.09481	0.1866	0.2744	0.1248	0.4100	19.1468
	200	−0.1496	0.1939	0.3219	0.1425	0.4349	6.9545
	500	−0.1720	0.2062	0.3429	0.1454	0.4420	2.3776
	1000	−0.1756	0.2056	0.3536	0.1485	0.4466	1.3662
	1500	−0.1796	0.2021	0.3519	0.1496	0.4462	0.7340
	2000	−0.1809	0.2034	0.3507	0.1494	0.4468	0.7484
0.10^2	100	−0.1123	0.1842	0.2665	0.1217	0.4088	18.4628
	200	−0.1700	0.1892	0.3072	0.1424	0.4378	7.3377
	500	−0.1956	0.2103	0.3349	0.1427	0.4404	4.1589
	1000	−0.1915	0.2094	0.3550	0.1474	0.4459	2.5257
	1500	−0.1956	0.2030	0.3511	0.1494	0.4439	2.6269
	2000	−0.1963	0.2058	0.3489	0.1488	0.4444	2.8188
True values		−0.1800	0.2000	0.3500	0.1500	0.4500	

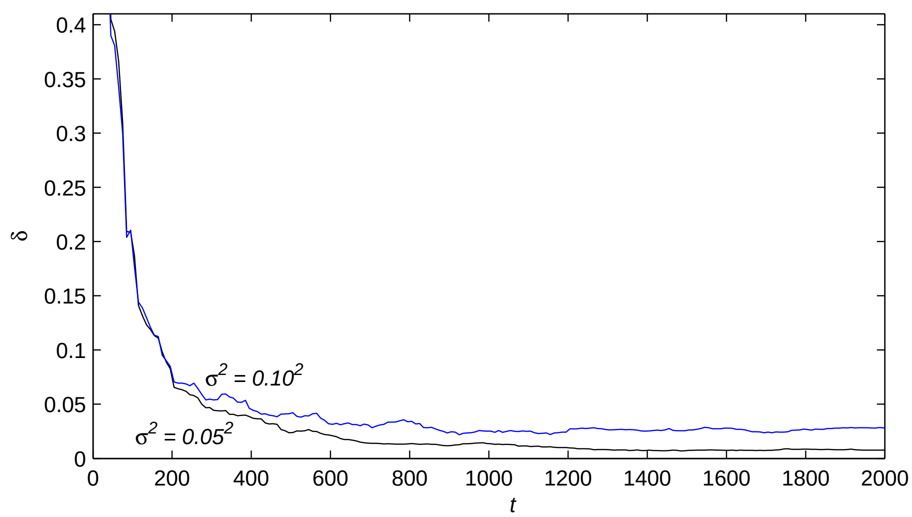

Figure 3. The estimation errors δ versus t with different σ^2

From Table 1 and Figure 3, we can draw the following conclusions.

- The parameter estimates given by the W-RLS algorithm converge to their true values as the noise variance becomes small.
- The parameter estimation errors given by the W-RLS algorithm become generally smaller and go to zero with the data length t increasing. This shows that the proposed algorithm is effective.

5. **Conclusion.** Due to the Wiener structure being a static nonlinear block following a linear dynamic block, it is difficult to get a simple output-input representation for the Wiener system with an output error linear element. In this paper, a Wiener output error system is parameterized as a linear-in-parameter form by using the auxiliary model idea and the invertible expression of the nonlinear part, and a recursive least squares algorithm is presented for the Wiener system. The simulation results show that the proposed algorithm is effective. For further research, we would study the simplified output-input representation and the simplified identification method for multivariable Wiener systems by using the auxiliary model idea.

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