ICIC Express Letters
Volume **10**, Number **10**, October **2016**

ICIC International ©2016 ISSN 1881-803X
pp. **2371–2376**

SPIN PREDICTION OF AIRCRAFT'S POST-STALL MANEUVERING BASED ON BIFURCATION ANALYSIS

Xiaogang Li, Yuhui Wang and Qingxian Wu

College of Automation Engineering
Nanjing University of Aeronautics and Astronautics
No. 29, Jiangjun Street, Jiangning District, Nanjing 211106, P. R. China
{ wangyh; xgli; wuqingxian }@nuaa.edu.cn

Received January 2016; accepted April 2016

Abstract. *Dangerous phenomena for aircraft's post-stall maneuvering are studied based on bifurcation theory. Firstly, equilibrium and bifurcation surfaces of an aircraft are identified in the state-control space based on continuation algorithm. Additionally, dangerous phenomena, such as spin, are analyzed and predicted by using bifurcation theory. Finally, the motion trajectories of aircraft are used to demonstrate the effectiveness of the predictions via the analysis of bifurcation phenomena.*

Keywords: Bifurcation analysis, Post-stall maneuvering, Spin, Continuation algorithm

1. **Introduction.** Bifurcation is an important physical phenomenon for a nonlinear dynamic system. Emergence of bifurcation often accompanies with some special physical phenomena [1-3]. For an aircraft, bifurcation usually appears when the aircraft flies in maneuvering flight regime which means the appearance of dangerous phenomena, especially in post-stall maneuvering.

Post-stall maneuverability is one of the important characters of modern advanced military aircraft [4,5]. The stability of flight at high angle of attack is a key problem which needs to be solved in post-stall maneuver, because the aircraft may undergo vortex flow, breakdown vortex flow and fully separated flow, meanwhile the control surfaces lose much of their efficiency due to low dynamic pressure. All of these may lead to some unexpected dangerous nonlinear bifurcation phenomena, such as wing rock, deep stall and spin [6,7]. In addition, with the existence of aerodynamics and inertial coupling, the stability of the aircraft will become more complex. Therefore, it is very dangerous for the safeties of pilot and aircraft in post-stall maneuvering. As a result, studying flight dynamics and developing predication techniques of dangerous regime at high angle of attack are necessary for safety, which also play an important role throughout the entire process of aircraft development [8,9].

Due to the fact that the aircraft is an inherent non-linear system, especially for the condition at high angle of attack, non-linear bifurcation phenomena are corresponding to several important aircraft maneuvers, including roll-coupling, stall and spin. Therefore, the linearized equations of aircraft motion cannot be used to accurately analyze these non-linear phenomena. The bifurcation theory was presented by Mehra for analyzing aircraft stability and non-linear bifurcation phenomenon of aircraft at high angle of attack [10]. With this approach, the aircraft global stability with respect to system states and control parameters which is called bifurcation diagram is presented. Based on this diagram, different dangerous phenomena including spin of aircraft post-stall maneuvering can be predicted. From then on, rich achievements [11-16] were achieved based on this approach.

As discussed above, the global stability and bifurcation phenomena of an aircraft's post-stall maneuvering process are analyzed in this paper. Analytic results indicate some special physical phenomena, such as spin, may occur at high angle of attack. Based on the analysis, a prediction method of the nonlinear dangerous phenomena is developed. In order to verify the accuracy of analysis, the time responses and the motion trajectories of the aircraft are given. The purpose of analysis is to provide theory basis for the aircraft recovering from these dangerous bifurcation phenomena.

The remainder of this paper is organized as follows. Section 2 presents the system and problem description. Section 3 describes the calculation of bifurcation diagram. Bifurcation analysis and spin prediction are presented in Section 4. Section 5 concludes the papers.

2. **Description of the Problem.** According to the conditions mentioned in Remark 2.1 and Remark 2.2, the aircraft model can be described as the following equations:

$$\dot{\alpha} = q + \frac{1}{\cos\beta}\left[\left(-r\sin\beta - \frac{\bar{q}SC_x}{mV}\right)\sin\alpha + \left(-p\sin\beta + \frac{\bar{q}SC_z}{mV}\right)\cos\alpha\right] \tag{1}$$

$$\dot{\beta} = -\left[\left(\frac{\bar{q}SC_x}{mV}\right)\sin\beta + r\right]\cos\alpha + \left(\frac{\bar{q}SC_y}{mV}\right)\cos\beta - \left[\left(\frac{\bar{q}SC_z}{mV}\right)\sin\beta - p\right]\sin\alpha \tag{2}$$

$$\begin{aligned}\dot{p} = \frac{1}{I_xI_z - (I_{xz})^2}\big[&I_x\bar{q}SbC_l + I_{xz}\bar{q}SbC_n + I_{xz}(I_z + I_x - I_y)pq \\ &+ \left(I_yI_z - (I_z)^2 - (I_{xz})^2\right)qr\big]\end{aligned} \tag{3}$$

$$\dot{q} = \frac{1}{I_y}\left[\bar{q}ScC_m + (I_z - I_x)pr + I_{xz}\left(r^2 - p^2\right)\right] \tag{4}$$

$$\begin{aligned}\dot{r} = \frac{1}{I_xI_z - (I_{xz})^2}\big[&I_{xz}\bar{q}SbC_l + I_x\bar{q}SbC_n + \left((I_x)^2 - I_xI_y + (I_{xz})^2\right)pq \\ &+ I_{xz}(I_y - I_z - I_x)qr\big]\end{aligned} \tag{5}$$

where V is the aircraft speed, α is the angle of attack, β is the angle of sideslip, m is the aircraft mass, $\bar{q}$ is the dynamic pressure, ρ is the air density, S is the wing reference area, b is the wing span, g is the gravitational acceleration, c is the mean aerodynamic chord of the wing, p, q and r are the body-axis angular rates, I_x, I_y, I_z and I_{xz} are the moments of inertia, $C_x(\alpha,\beta,\delta_e,q)$, $C_y(\alpha,\beta,\delta_e,\delta_a,\delta_r,p,r)$ and $C_z(\alpha,\beta,\delta_e,\delta_a,\delta_r,p,r)$ are the aerodynamic force coefficients, and $C_l(\alpha,\beta,\delta_e,\delta_a,\delta_r,p,r)$, $C_m(\alpha,\beta,\delta_e,q)$ and $C_n(\alpha,\beta,\delta_e,\delta_a,\delta_r,p,r)$ are the aerodynamic moment coefficients. δ_e is elevator deflection, δ_a is aileron deflection and δ_r is rudder deflection.

Remark 2.1. *The thrust force T can be neglected due to the fact that the velocity V is invariant when the aircraft flies at high angle of attack α.*

Remark 2.2. *Similarly, gravity force mg has small influence on bifurcation results; thus, gravity force mg can be also neglected.*

3. **Calculation of Equilibrium Points.** Without loss of generality, the aircraft dynamic system can be rewritten as the following nonlinear differential equation:

$$\dot{X} = F(X,U) \tag{6}$$

where $X = [\alpha,\beta,p,q,r]^T$ is the state vector, and $U = [\delta_e,\delta_a,\delta_r]^T$ is the control vector.

When

$$\dot{X} = F(X,U)|_{(X_e,U_e)} = 0 \tag{7}$$

then (X_e,U_e) is called equilibrium point of system (6).

According to the bifurcation theory [17] and Remark 3.1, the system stability will change with the variation of control parameters of system (6), and even result in bifurcation phenomena, which indicate the sudden-changes of dynamical behavior. Especially for system (6), some special bifurcation phenomena, such as spin, deep stall and wing rock, may occur when the aircraft flies at high angle of attack α.

In order to study the bifurcation phenomena of system (6), global stability is studied firstly based on continuation algorithm and bifurcation theory, by which, steady states, and different types of bifurcation points can be calculated. With the calculation results, diagram of bifurcation and global steady-state distribution are presented, which is also called equilibrium and bifurcation surfaces.

Remark 3.1. *For the bifurcation analysis and continuation, system (6) is assumed to be smooth.*

Remark 3.2. *From Equation (6), the system has three control surfaces, δ_e, δ_a and δ_r. However, the calculation step based on continuation algorithm can only take one surface as control variable and fix the other control surfaces as constants. So here, δ_a is taken as control variable and δ_e and δ_r are fixed as constants.*

Remark 3.3. *To calculate the equilibrium surfaces, the algorithm needs to start from an equilibrium state (X_0, U_0).*

The steps of calculating equilibrium surfaces are shown as follows.

Step 1: Initialization

According to implicit function theorem and Remark 3.1, the steady states X of system (6) are continuous function of the control variable δ_a, and $\delta_a \in [\delta_{a0}, \delta_{an}]$, which has $n+1$ discretization points $[\delta_{a0}, \delta_{a1}, \ldots, \delta_{an}]$. Thus, the corresponding solution of system (6) is $[X(\delta_{a0}), X(\delta_{a1}), \ldots, X(\delta_{an})]$. For convenience, take $[X_0, X_1, \ldots, X_n] = [X(\delta_{a0}), X(\delta_{a1}), \ldots, X(\delta_{an})]$. (X_0, U_0) is considered as initial equilibrium value, which satisfies the following equation:

$$\dot{X} = F(X, U)|_{(X_0, U_0)} = 0 \tag{8}$$

Step 2: Calculating new equilibrium point

Define:

$$X_{1(0)} = X_0 \tag{9}$$

According to the method of Newton-Raphsond, there is:

$$\wp F\left(X_{1(k)}\right)\left(X_{1(k+1)} - X_{1(k)}\right) + F\left(X_{1(k)}\right) = 0 \tag{10}$$

where

$$\wp F\left(X_{1(k)}\right) = \frac{\partial F}{\partial X}\Big|_{X = X_{1(k)}} \tag{11}$$

and then it yields that:

$$X_{1(k+1)} = X_{1(k)} - \left(\wp F\left(X_{1(k)}\right)^{-1} F\left(X_{1(k)}\right)\right) \tag{12}$$

where $k = [0, 1, \ldots, m]$ is the number of iterations.

$$\frac{\partial F}{\partial X} = \begin{bmatrix} \frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial \beta} & \frac{\partial F_1}{\partial p} & \frac{\partial F_1}{\partial q} & \frac{\partial F_1}{\partial r} \\ \frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial \beta} & \frac{\partial F_2}{\partial p} & \frac{\partial F_2}{\partial q} & \frac{\partial F_2}{\partial r} \\ \frac{\partial F_3}{\partial \alpha} & \frac{\partial F_3}{\partial \beta} & \frac{\partial F_3}{\partial p} & \frac{\partial F_3}{\partial q} & \frac{\partial F_3}{\partial r} \\ \frac{\partial F_4}{\partial \alpha} & \frac{\partial F_4}{\partial \beta} & \frac{\partial F_4}{\partial p} & \frac{\partial F_4}{\partial q} & \frac{\partial F_4}{\partial r} \\ \frac{\partial F_5}{\partial \alpha} & \frac{\partial F_5}{\partial \beta} & \frac{\partial F_5}{\partial p} & \frac{\partial F_5}{\partial q} & \frac{\partial F_5}{\partial r} \end{bmatrix}_{5 \times 5} \tag{13}$$

where $[F_1, F_2, F_3, F_4, F_5]^T = \left[\dot{\alpha}, \dot{\beta}, \dot{p}, \dot{q}, \dot{r}\right]^T$. When the iteration ends, it can be obtained that:

$$X_1 = X_{1(m)} \tag{14}$$

and then X_1 is the new equilibrium point.

Step 3: Calculating the eigenvalues of Jacobian matrix

According to Step 2, Jacobian matrix J at the new equilibrium point X_1 is given as follows:

$$J = \frac{\partial F}{\partial X}\Big|_{X=X_1} \tag{15}$$

Denote $\lambda(J) = [\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5]$ as the eigenvalues of J at new equilibrium point X_1.

Step 4: Calculating all equilibrium points

Denoting X_1 as new initial value for computing next equilibrium point X_2, and repeating the above steps until X_n, all equilibrium points can be obtained. Then the system's equilibrium and bifurcation surfaces are achieved. However, during the process of calculation, bifurcation points may lead to the end of the calculation. How to determine the locations and types of the bifurcation points can refer to [17,18].

Remark 3.4. *The above steps only chose* δ_a *as control variable, if taking* δ_e *and* δ_r *as variables, it has similar steps.*

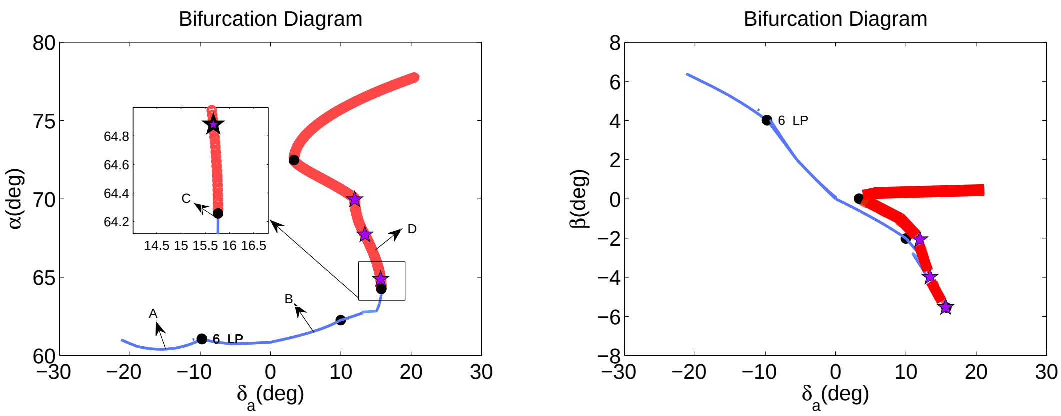
4. Analysis of Bifurcation and System Stability.

4.1. **Bifurcation analysis.** The aerodynamics model of aircraft in this paper is based on the wind-tunnel data from NASA [19]. Because the aerodynamic coefficients and the thrust force are usually defined as tabular functions of the motion parameters and control inputs, smooth curve fitting for tabular functions is necessary. Generally, cubic spline function is used to interpolate values for tabulate data that are obtained from wind tunnel tests approximately.

By analysis, it can be found that the equilibrium and bifurcation surfaces are mainly related to δ_a. So here, only bifurcation diagrams about δ_a are considered. The system parameters are given as $V = 60$m/s, $H = 3000$m.

Taking δ_a as control variable, the calculation result of equilibrium and bifurcation surfaces with δ_a is shown in Figure 1.

Figure 1 shows the equilibrium surfaces and bifurcation points of state variables α and β versus δ_a. In Figure 1, the blue thin lines represent stable equilibrium branches, which indicate the eigenvalues of equilibrium point in these branches are in the left-half complex plane. The red thick lines represent unstable equilibrium branches, which means that at least one of the eigenvalues in these branches crosses the imaginary axis. '•' represents limit point (LP) or saddle-node bifurcation point, which means a real eigenvalue crosses the left-half plane to the right-half plane at a specific parameter value of δ_a and indicates

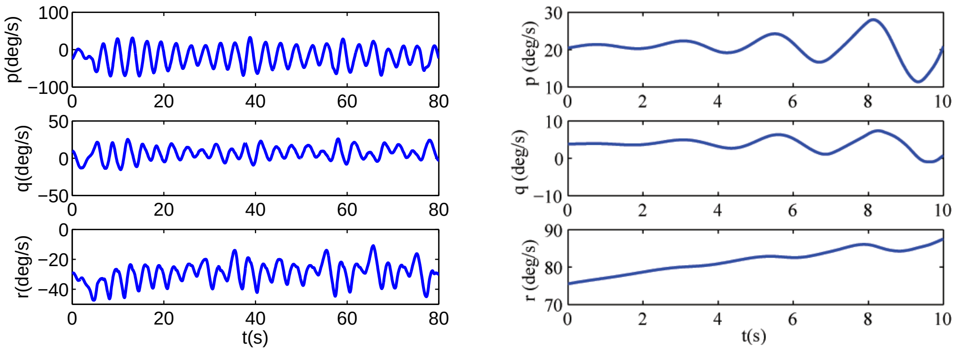


(A) $\alpha - \delta_a$ (B) $\beta - \delta_a$

FIGURE 1. Equilibrium bifurcation diagram versus δ_a

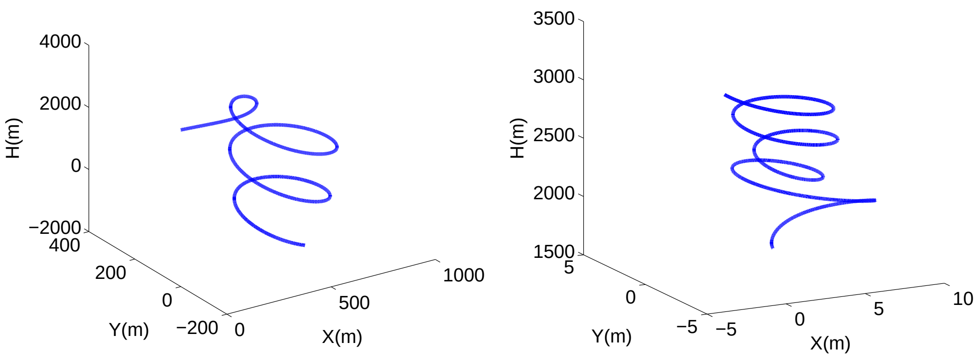
a sudden change of dynamical behavior. '★' represents hopf bifurcation (HB), which indicates a pair of complex conjugate eigenvalues cross the imaginary axis and may lead to very complicated dynamical behavior.

4.2. **Prediction of bifurcation phenomena.** Based on Section 4.1, it can be known that system (6) may be stable or unstable due to the existence of bifurcation points, which can lead to a curious dangerous phenomenon. Without loss of generality, only dangerous bifurcation phenomena spin is analyzed. Spin is a special bifurcation phenomenon of aircraft at high angle of attack, which causes the aircraft's motion trajectory like a downward spiral and the aircraft rotates around three body axes. Based on the analysis of Figure 1, the time responses of longitudinal-directional variable q and lateral-directional variables p, r indicate the aircraft's rotation around body axes in Figure 2, where the curves of state response in Figure 2(A) are finite amplitude oscillations, and those in Figure 2(B) are divergent oscillations. We can predict the blue branch in Figure 1 is a stable spin branch and the red is an unstable spin branch possibly. In order to verify the prediction, the motion trajectory of the aircraft is shown in Figures 3(A) and 3(B). Figure 3(A) indicates the motion trajectory is a regular spiral which represents stable spin and Figure 3(B) is irregular which represents unstable spin.



(A) Stable spin (B) Unstable spin

FIGURE 2. State time responses of two different types of spin



(A) Stable spin (B) Unstable spin

FIGURE 3. The motion trajectory of two different types of spin

5. **Conclusion.** The main aim of this work is to analyze the dangerous phenomena in post-stall maneuvering. Based on bifurcation and continuation algorithm, the non-linear flight dynamical phenomena, such as spin, are studied and a spin prediction method is developed. According to the prediction results, the bifurcation phenomena may occur when the aircraft flies at high angle of attack, which leads to dangerous phenomena, such as spin. The research results provide important theoretic significance to ensure the safety of aircraft when it flies at high angle of attack. Future work will look into designing aircraft maneuvers and control algorithm at high angle of attack based on this approach.

Acknowledgment. This paper is supported by National Natural Science Foundation of China (Grant No. 61374212 and No. 61304099).

REFERENCES

[1] L. Zhou, Y. Chen and F. Chen, Bifurcation and chaos of a new 3D quadratic system, *ICIC Express Letters*, vol.4, no.6(B), pp.2481-2488, 2010.
[2] E. E. Meyer, Continuation and bifurcation in linear flutter equations, *AIAA Journal*, vol.53, no.4, pp.1-3, 2015.
[3] W. Xiang, Equilibrium points and bifurcation control for Lorenz-Stenflo system, *ICIC Express Letters*, vol.3, no.1, pp.61-66, 2009.
[4] W. Y. Zhang and M. B. Tong, Status and trends of the post stall maneuvers, *Aeronautical Science and Technology*, vol.18, no.6, pp.18-21, 2006.
[5] S. Y. Liu and Y. B. Dong, The post stall maneuver's effect on air combat performance, *Journal of Jilin Teachers Institute of Engineering and Technology*, vol.28, no.8, pp.70-72, 2012.
[6] Y. L. Chen, *Aircraft's Analysis and Control of Nonlinear Dynamic Characteristics at High Angle of Attack*, Ph.D. Thesis, College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, 2007.
[7] Y. F. Liu, The prediction of spin through all process of the aircraft development, *Experiments and Measurements in Fluid Mechanics*, vol.13, no.4, pp.32-35, 1999.
[8] R. Weissman, Status of design criteria for predicting departure characteristics and spin susceptibility, *Journal of Aircraft*, vol.12, no.12, pp.989-993, 1975.
[9] W. J. Bihrle and B. Barnhart, Spin prediction techniques, *Journal of Aircraft*, vol.20, no.2, pp.97-101, 1983.
[10] R. K. Mehra, *Global Stability and Control Analysis of Aircraft at High Angles of Attack*, AD-A051850, 1978.
[11] D. Rezgui, M. H. Lowenberg and M. Jones, Continuation and bifurcation analysis in helicopter aeroelastic stability problem, *Journal of Guidance Control and Dynamics*, vol.37, no.3, pp.889-897, 2014.
[12] A. K. Khatri, J. Singh and N. K. Sinha, Aircraft maneuver design using bifurcation analysis and sliding mode control techniques, *Journal of Guidance Control and Dynamics*, vol.35, no.5, pp.1435-1449, 2012.
[13] Q. Xin and Z. Shi, Bifurcation analysis and stability design for aircraft longitudinal motion with high angle of attack, *Chinese Journal of Aeronautics*, vol.28, no.1, pp.250-259, 2015.
[14] J. Cai, H. Nie, M. Zhang et al., Effect of dual co-rotation wheels configuration on aircraft shimmy, *Journal of Vibroengineering*, vol.17, no.8, pp.4421-4431, 2015.
[15] S. J. Gill, M. H. Lowenberg, S. A. Neild et al., Impact of controller delays on the nonlinear of remotely piloted aircraft, *Journal of Guidance Control and Dynamics*, pp.1-9, 2015.
[16] A. A. Paranjape and N. Ananthkrishnan, Criterion for aircraft spin susceptibility, *Journal of Aircraft*, vol.47, no.5, pp.1804-1807, 2010.
[17] H. Y. Hu, *Applied Nonliear Dynamics*, Aviation Industry Press, 2002.
[18] A. Dhooge, W. Govaerts and Y. A. Kuznetsov, Matcon: A Matlab package for numerical bifurcation analysis of ODEs, *ACM Trans. Mathematical Software*, vol.29, no.2, pp.141-164, 2003.
[19] L. T. Nguyen et al., *Simulator Study of Stall/Post-stall Characteristics of a Fighter Airplane with Relaxed Longitudinal Static Stability*, NASA TP-1538, 1979.