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RADAR POWER ALLOCATION ALGORITHM FOR TARGET TRACKING BASED ON RADIO FREQUENCY STEALTH

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Abstract. *A novel power allocation strategy of radar networks based on grey relational grade (GRG) is presented in this paper. Firstly, the predicted tracking Cramér-Rao lower bounds (CRLB) of interacting multiple model particle filter (IMMPF) tracking method is computed. Secondly, the relational model between the radar's power and CRLB is built according to the measurement noise matrix. Finally, the algorithm controls the radiated power of radar according to GRG of predicted CRLB and desired CRLB. The tracking accuracy and low probability of intercept (LPI) performance are demonstrated in the Monte Carlo simulations. The results are validated through the comparison with other methods.*

Keywords: Power control, Radio frequency stealth, Target tracking, Radar network

1. **Introduction.** As we know, active sensors are irreplaceable on the airborne platforms as they have much more excellent performance than passive sensors. However, sensor platforms with active sensing equipment such as radars may betray their existence, by emitting energy that will be intercepted by enemy surveillance sensors thereby increasing the vulnerability of the whole combat system [1]. In order to achieve the important tactical requirement of low probability of intercept (LPI) [2], dynamically controlling the emission of the radar during the sensor management is very necessary. An overview of the theory, algorithms, and applications of sensor management is presented in [3], as it has developed over the past decades and as it stands today. A resource allocation strategy for the simultaneous multibeam working mode with the task of tracking is proposed in [4], and [5] describes an algorithm for scheduling electronically scanned array radar in order to perform the functions of searching and tracking. A joint scheme of antenna subset selection and optimal power allocation for localization is accomplished by solving a constrained optimization problem that is formulated to minimize the error in estimating target position, while conserving transmitter number and power budget [6]. [7] presents a new method for data association in multitarget tracking in the framework of evidence theory. Radar detection, multitarget tracking, and data fusion (DF) techniques are applied to experimental data collected during an HF-radar experiment [8], in order to solve the shortcomings such as poor range and azimuth resolution, and high nonlinearity of the sensor association. An optimal solution to the power allocation problem for object classification is presented in [9] for distributed active multiple-radar systems subject to different power constraints.

Almost all of those works concern the performance improvement of the radars; however, the radar network should not only have excellent detection and tracking performance, but also good LPI capability. On the basis of the studies mentioned above, a novel algorithm of power allocation in radar networks for LPI is proposed.

As the maneuvering target tracking could be formulated as a multiple model nonlinear filtering problem [10], a new method combing the interacting multiple model approach with a particle filter approach is presented in [11], which is able to deal with nonlinearities and non-Gaussian noise in a mode. So in this paper, the interacting multiple model particle filter (IMMPF) algorithm is employed for target tracking. The tracking performance in this paper is indicated by tracking error CRLB matrix. And grey relational grade (GRG) [12] is used to measure the similarity of the desired tracking error CRLB matrix and predicted error CRLB matrix which is predicted based on the limited tracking information of last tracking time. In our algorithm of radar power control, the higher value of GRG is taken into consideration as the stronger relational degree between the predicted tracking accuracy and desired tracking accuracy. Thus, the higher GRG indicates that the predicted error CRLB matrix is closer to the desired matrix which will be described in the next section.

The remainder of this paper is organized as follows. Section 2 presents the power allocation method in sensor network for LPI in details. Simulations of the proposed algorithms and comparison results with other methods are provided in Section 3. The conclusions are presented in Section 4.

2. **Power Allocation Algorithm Based on GRG.** Given the tracking system at time k:

$$\boldsymbol{X}(k) = \boldsymbol{F}\boldsymbol{X}(k-1) + \boldsymbol{U}(k-1) \tag{1}$$

$$\boldsymbol{Z}(k) = \boldsymbol{H}\boldsymbol{X}(k) + \boldsymbol{V}(k) \tag{2}$$

where, $\boldsymbol{X}(k) = [x(k), \dot{x}(k), y(k), \dot{y}(k)]$ is the dynamical state of the system, $(x(k), \dot{x}(k))$ and $(y(k), \dot{y}(k))$ are respectively the range, velocity of the direction of X and Y. $\boldsymbol{U}(k-1)$ and $\boldsymbol{V}(k)$ are the process noise vector and measurement noise vector with covariance matrices $\boldsymbol{Q}$ and $\boldsymbol{W}$ respectively. $\boldsymbol{F}$ is the dynamic matrix and $\boldsymbol{H}$ is the observation matrix of the system. $\boldsymbol{Z}(k)$ is the measurement vector. As we know, the signal to noise ratio of the measurement vector is decided by the radiated energy. There are M kinematics models and N particles for every model in the system. The estimated range and velocity of the target will be obtained by IMMPF algorithm.

The proposed power allocation algorithm is designed based on the GRG between the predicted error CRLB and desired error CRLB. So in the algorithm, the computation method of tracking error CRLB should be presented firstly. Then measurement noise covariance matrix $\boldsymbol{W}$ which will have an impact on the predicted CRLB, is built by the emitted power. Finally, the radar and its power will be selected according to the GRG results.

2.1. **Computation method of tracking CRLB.** The mean squared error of any estimator cannot go below a bound $\boldsymbol{P}_{CRLB}$, which is given by

$$E\left\{(\hat{\boldsymbol{x}}_k - \boldsymbol{x}_k)(\hat{\boldsymbol{x}}_k - \boldsymbol{x}_k)^T\right\} \geq \boldsymbol{P}_{CRLB} \tag{3}$$

$$\boldsymbol{P}_{CRLB}^{-1} = E\left\{-\Delta_{\boldsymbol{x}_k}^{\boldsymbol{x}_k} \log p(\boldsymbol{x}_k, \boldsymbol{z}_k)\right\} \tag{4}$$

where $\boldsymbol{J}_k = \boldsymbol{P}_{CRLB}^{-1}$ is the Fisher information matrix. And the expectation is taken with respect to $p(\boldsymbol{x}, \boldsymbol{z})$, which is the joint PDF of the pair $(\boldsymbol{x}, \boldsymbol{z})$. Δ denotes the second-order derivative operator, namely

$$\Delta_x^y = \nabla_x \nabla_y^T \tag{5}$$

in which ∇ denotes the gradient operator. $\boldsymbol{J}_k$ can be calculated without manipulating the large matrices at each time k.

$$\boldsymbol{J}_{k+1} = \boldsymbol{D}_k^{22} - \boldsymbol{D}_k^{21}\left(\boldsymbol{J}_k + \boldsymbol{D}_k^{11}\right)^{-1}\boldsymbol{D}_k^{12} \tag{6}$$

where

$$\boldsymbol{D}_k^{11} = E\left\{-\Delta_{\boldsymbol{x}_k}^{\boldsymbol{x}_k} \log p\left(\boldsymbol{x}_{k+1}|\boldsymbol{x}_k\right)\right\} = \boldsymbol{F}^T\boldsymbol{Q}^{-1}\boldsymbol{F} \tag{7}$$

$$\boldsymbol{D}_k^{12} = E\left\{-\Delta_{\boldsymbol{x}_k}^{\boldsymbol{x}_{k+1}} \log p\left(\boldsymbol{x}_{k+1}|\boldsymbol{x}_k\right)\right\} = \left(\boldsymbol{D}_k^{21}\right)^T = -\boldsymbol{A}^T\boldsymbol{Q}^{-1} \tag{8}$$

$$\boldsymbol{D}_k^{22} = E\left\{-\Delta_{\boldsymbol{x}_{k+1}}^{\boldsymbol{x}_{k+1}} \left[\log p\left(\boldsymbol{x}_{k+1}|\boldsymbol{x}_k\right) + \log p\left(\boldsymbol{z}_{k+1}|\boldsymbol{x}_{k+1}\right)\right]\right\} = \boldsymbol{Q}^{-1} + \boldsymbol{H}^T\boldsymbol{W}^{-1}\boldsymbol{H} \tag{9}$$

2.2. **Computation of matrix $\boldsymbol{W}$.** In (7)-(9), $\boldsymbol{A}$, $\boldsymbol{F}$, $\boldsymbol{Q}$ and $\boldsymbol{H}$ are constant matrices, and measurement noise covariance matrix $\boldsymbol{W}$ is controlled by the emitted power. As we know, radar equation at time k is as follows:

$$R_k^4 = t_B^k \frac{P_{av}^k G_T G_R \lambda^2 \sigma_k}{(4\pi)^3 K T_R S_{NR}^k L} \tag{10}$$

where t_B^k is the single dwelling time of the beam from the normal direction at time k, P_{av}^k is the average radiated power, G_R is the receiver gain, σ_k is the radar cross section (RCS) of the target, K is Boltzmann constant, T_R and L are respectively effective noise temperature and radar system loss, R_k is the detection range, G_T is the transmit gain, and S_{NR}^k represents the signal to noise ratio of the system at time k. Suppose the target' range is R_0, the radar has to emit the power P_{av0}, and the radar equation becomes

$$R_0^4 = t_B \frac{P_{av0} G_T G_R \lambda^2 \sigma_0}{(4\pi)^3 K T_R S_{NR0} L} \tag{11}$$

Combining (9) with (10), the emitting energy at time k can be written as

$$S_{NR}^k = \frac{P_{av}^k S_{NR0}}{P_{av0}} \frac{R_0^4}{R_k^4} \tag{12}$$

The single pulse signal is radiated by the radar, and the covariance of the measurement noise can be denoted as:

$$\boldsymbol{W} = \begin{bmatrix} \dfrac{c^2 T_p}{8 S_{NR}^k} & 0 \\ 0 & \dfrac{3c^2}{w_c^2 T_p^2 S_{NR}^k} \end{bmatrix} \tag{13}$$

where T_p is the pulse width, c is the wave velocity, and w_c is the carrier frequency. After the error covariance matrix $\boldsymbol{W}$ is computed, we can obtain the predicted error CRLB matrix.

2.3. **Power allocation method.** In order to select the optimal radiated power, the desired CRLB $\boldsymbol{P}_{CRLB0}$ should be set for the C radars in the network firstly. The power set which the radar c can radiate is denoted as $\boldsymbol{Pow}^c$:

$$\boldsymbol{Pow}^c = \{P_{av1}^c, P_{av2}^c, \ldots, P_{avn}^c\} \tag{14}$$

where $P_{av1}^c < P_{av2}^c, \ldots, < P_{avn}^c$.

Different radiated power from the set $\boldsymbol{Pow}^c$ can lead to different covariance matrix of measurement noise, which will have an impact on the result of predicted CRLB. However, during the tracking process, target range R_k is unknown before radar detection. So R_k in (12) is replaced by R_k^{pre} which is predicted by the tracking result at $k-1$. R_k^{pre} is presented as

$$R_k^{pre} = R_{k-1} + v_{k-1}T \tag{15}$$

R_{k-1} and v_{k-1} are the target's range and velocity which are estimated by the IMMPF tracking algorithm at time $k-1$, and T is the tracking interval. The predicted CRLB $\boldsymbol{PRE}_{CRLB}$ can be obtained from (7) to (9):

$$\boldsymbol{PRE}_{CRLB}^c = \{Pre_{CRLB1}^c, Pre_{CRLB2}^c, \ldots, Pre_{CRLBn}^c\} \tag{16}$$

Using the grey relational grade theory, we have the GRG results between the predicted CRLB and the desired CRLB:

$$\boldsymbol{GRG}^c_{CRLB} = \{GRG^c_{CRLB1}, GRG^c_{CRLB2}, \ldots, GRG^c_{CRLBn}\} \tag{17}$$

The power P^c_{avi} which leads to the maximum $\boldsymbol{GRG}^c_{CRLB}$ will be selected for the radars in the network. Then a power set can be given:

$$\boldsymbol{Pow} = \left\{P^1_{avi}, P^2_{avj}, \ldots, P^c_{avk}\right\} \tag{18}$$

The algorithm will choose the radar which will radiate the minimum power in the network for tracking at time k.

3. **Simulation Results.** In this section, Monte Carlo simulations are performed to analyze the performance of the proposed energy control method. Firstly, the tracking scene including the target trajectory and radar positions is designed according to approximately actual tracking scene, and then the tracking and LPI performance are illustrated to compare the proposed method with other methods.

3.1. **Trajectory design.** The IMM filter here is comprised of Constant Velocity model (CV) and Coordinated Turn rate model (CT). Figure 1 shows the target trajectory in 120s. There are 3 radars which are labeled A, B and C, in the sensor network. The positions of the radars are $(200, 100)$, $(300, 300)$ and $(400, 400)$ respectively.

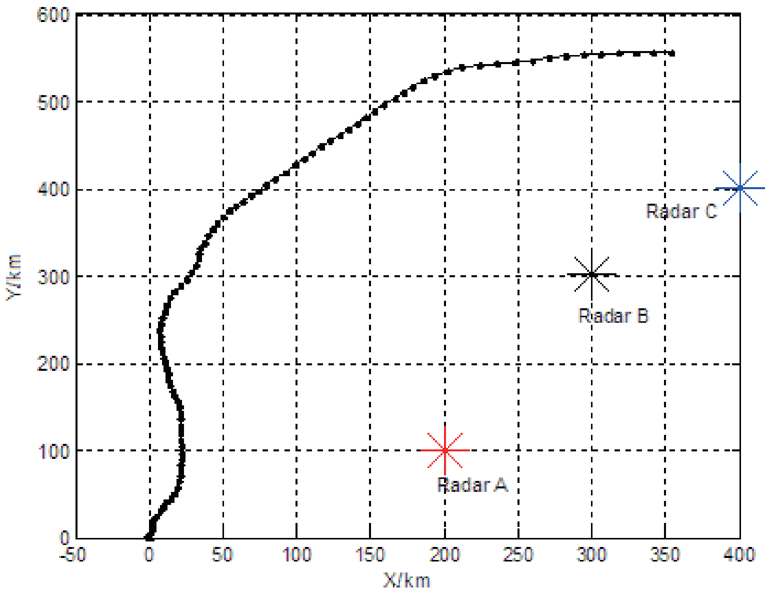
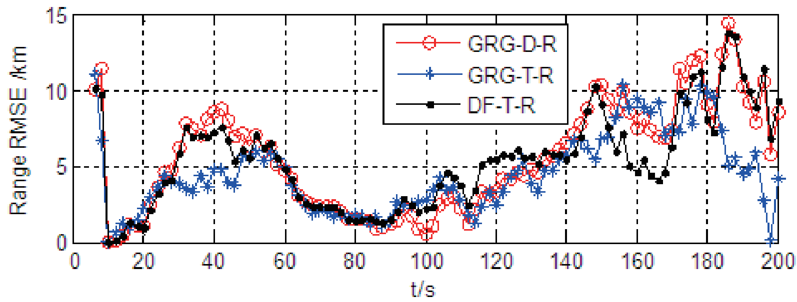
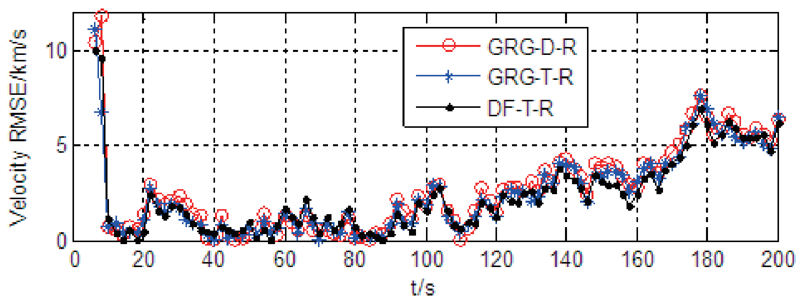


FIGURE 1. Trajectory of the target and the positions of the radars

3.2. **Comparison of tracking performance.** Based on the grey relational grade algorithm, the proposed radiated energy methods of double radars (GRG-D-R) and three radars (GRG-T-R) in the sensor network, are realized in the simulation. The performance is also compared with the data fusion methods of three radars (DF-T-R) [8]. Figure 2 and Figure 3 show the Root-mean-square error (RMSE) [1] of the three methods in X and Y direction respectively. Table 1 shows the average Root-mean-square error (ARMSE) [1] of the range and the velocity. Compared with DF-T-R, we can see that the proposed methods GRG-D-R and GRG-T-R present almost the same excellent tracking accuracy.

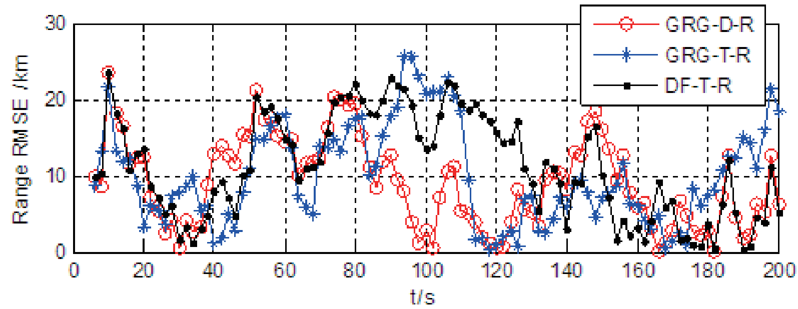


(a) RMSE of tracking range

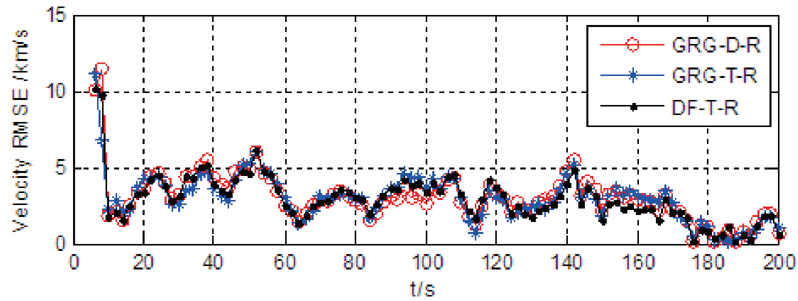


(b) RMSE of tracking velocity

FIGURE 2. Comparison of tracking performance (X direction)



(a) RMSE of tracking range



(b) RMSE of tracking velocity

FIGURE 3. Comparison of tracking performance (Y direction)

Table 1. Comparison of ARMSE

Method	**X-ARMSE (km)**	**X-ARMSE (km/s)**	**Y-ARMSE (km)**	**Y-ARMSE (km/s)**
GRG-D-R	5.4795	2.5900	9.3212	2.9917
GRG-T-R	4.4558	2.4017	10.2888	2.9515
DF-T-R	5.3204	2.3161	11.3708	2.9067

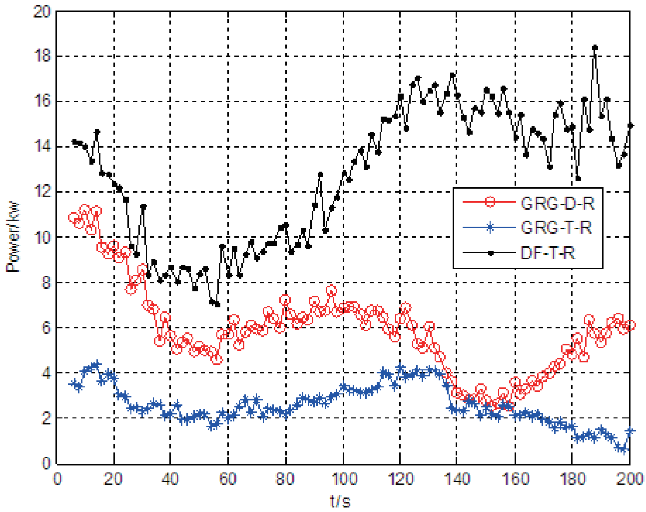


Figure 4. Comparison of radiated power

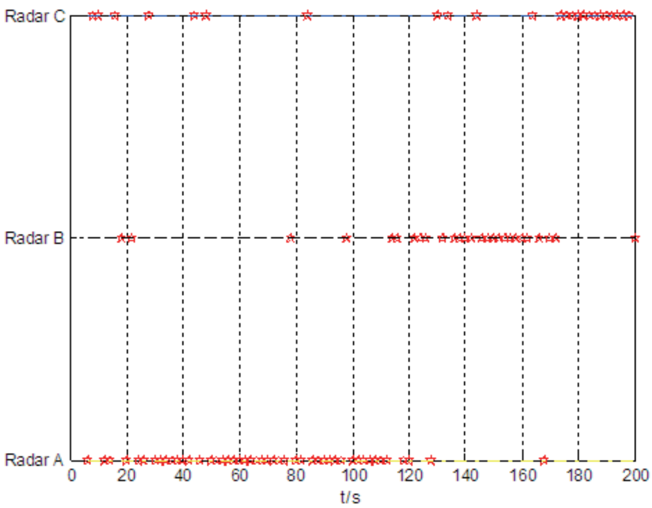


Figure 5. Radiation label of the radars

3.3. **Comparison of LPI performance.** Radiated power of DF-T-R, GRG-D-R and GRG-T-R are shown in Figure 4. Compared with DF-T-R, we can see that the proposed method not only presents excellent tracking accuracy, but also reduces more radiated energy. Figure 5 shows the radiation label of the radars, and we can see the radars work in turn in order to obtain excellent tracking and LPI performance. As a result, the GRG-T-R method has better LPI ability than GRG-D-R, as more radars can be selected to work according to their positions and predicted tracking performance. However, the DF-T-R method was proposed to improve the tracking performance instead of the LPI ability.

4. **Conclusions.** In this paper, we have presented a new strategy of power allocation for LPI taking advantage of GRG theory. Firstly, the computation method of tracking error CRLB matrix is presented, and the relational model between emitted power and predicted error CRLB is built. Then the GRG method is used to measure the similarity of desired error CRLB and predicted error CRLB. The larger GRG indicates the closer relation between the predicted CRLB and the desired CRLB. According to the GRG results between the two CRLBs, the radar with the minimum radiation power in the network will be selected to track the target. The simulation results show that the proposed method has excellent performance of tracking and LPI, with the comparison of other methods. In the future work, the power control method will be improved for multi-target tracking in clutter.

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