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A MODIFIED ORTHOGONAL MATCHING PURSUIT ALGORITHM FOR IDENTIFICATION OF A CLASS OF CLOSED-LOOP SYSTEMS

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Abstract. *In this paper, a modified orthogonal matching pursuit algorithm is proposed to indirectly identify both the control plant and the feedback controller of a class of closed-loop systems with unknown orders and large time-delays. The identification method contains two steps. The first step is to identify the orders, time-delays and the parameters of the plant using the proposed algorithm. The second step is to extract the parameters of the feedback controller. The proposed method can effectively identify the time-delays, orders and parameters of both the control plant and the feedback controller, and requires only a small number of sampling data. A simulation example is given to verify the effectiveness of the proposed algorithm.*

Keywords: Closed-loop system, Indirect identification, Orthogonal matching pursuit algorithm, Large time-delay

1. **Introduction.** Identification of a plant in closed-loop operation is of great significance in system identification, since feedback is very common in most dynamic systems. Closed-loop identification has received considerable attention in [1, 2, 3, 4]. The indirect closed-loop identification is commonly used when the input of the control plant is unmeasurable. The main idea is to firstly identify the model of the closed-loop system based on the measurable external input and the output data, and then to extract the parameter estimates of the control plant based on the knowledge of controller. In this literature, several least squares based methods have been developed, such as the bias-correction based least squares methods in [5, 6, 7] and the instrumental variable least squares method in [8, 9]. Generally, the indirect closed-loop system identification requires that the feedback controller is known. In this paper, we consider that the parameters of both the control plant and the controller are unknown. In most process industries, time-delays are unavoidable in their dynamical behavior [10, 11]. Therefore, it is important to identify the time-delays of a closed-loop system together with the parameters and orders. In this paper, we consider the case that both the control plant and the feedback controller have large time-delays, and the time-delays as well as the system orders are unknown. Therefore, the tasks of this paper seem complicated and are as follows:

- to estimate the parameters of the control plant and the feedback controller;
- to estimate the orders of the control plant and the feedback controller;
- to estimate the time-delays of the forward and backward channels.

Because the time-delays are unknown, the identification model of the closed-loop system is an over parameterized model with a high dimensional and sparse parameter vector. Inspired by the recovery theory of compressed sensing (CS) [12, 13], the orthogonal matching pursuit (OMP) algorithm is considered useful and can be improved for identifying the sparse parameter vector. The OMP algorithm is an iterative greedy algorithm, which can recover a high-dimensional sparse signal from a small set of measurements

[14, 15]. Compared with other sparse approximation algorithms, the major advantages of the OMP algorithm are its simplicity and fast speed. An improved OMP algorithm has been successfully applied to identifying the open-loop systems with multi-input finite impulse response systems and multi-input controlled autoregressive systems with unknown time-delays [16, 17]. Since the feedback is inherent in most dynamic systems, and in many cases, it is not possible to remove it during an identification experiment, it is an important work to extend the improved OMP algorithm to the closed-loop systems.

Briefly, the structure of this paper is as follows. Section 2 describes the identification problem of the closed-loop system. Section 3 presents a threshold orthogonal matching pursuit algorithm based on the compressed sensing theory. Section 4 provides a simulation example to show the effectiveness of the proposed algorithm. Finally, some concluding remarks are given in Section 5.

2. **System Description.** Consider a single input single output (SISO) closed-loop system with large time-delays depicted in Figure 1, where $u(t)$ and $y(t)$ are the system input and output, $v(t)$ is a stochastic noise process with zero mean and variance σ^2, z^{-1} is an unit backward shift operator: $[z^{-1}y(t) = y(t-1)]$, d_1 and d_2 are the time-delays of the plant and the feedback controller, respectively, and $A(z)$, $B(z)$ and $Q(z)$ are polynomials in the operator z^{-1} and are defined as

$$\begin{aligned} A(z) &:= 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{n_a} z^{-n_a}, \\ B(z) &:= b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{n_b} z^{-n_b}, \\ Q(z) &:= 1 + q_1 z^{-1} + q_2 z^{-2} + \cdots + q_{n_q} z^{-n_q}. \end{aligned}$$

Then the model of the closed-loop system can be described by

$$A(z)y(t) = z^{-d_1}B(z)u(t) + v(t), \tag{1}$$

$$u(t) = r(t) - z^{-d_2}Q(z)y(t). \tag{2}$$

Assume that the orders n_a, n_b, n_q, the parameters a_i, b_j, q_l and the time-delays d_1, d_2 are unknown.

Substituting Equation (2) into Equation (1), we have

$$A(z)y(t) = z^{-d_1}B(z)\left[r(t) - z^{-d_2}Q(z)y(t)\right] + v(t),$$

which follows that

$$\left[A(z) + z^{-d_1-d_2}B(z)Q(z)\right]y(t) = z^{-d_1}B(z)r(t) + v(t).$$

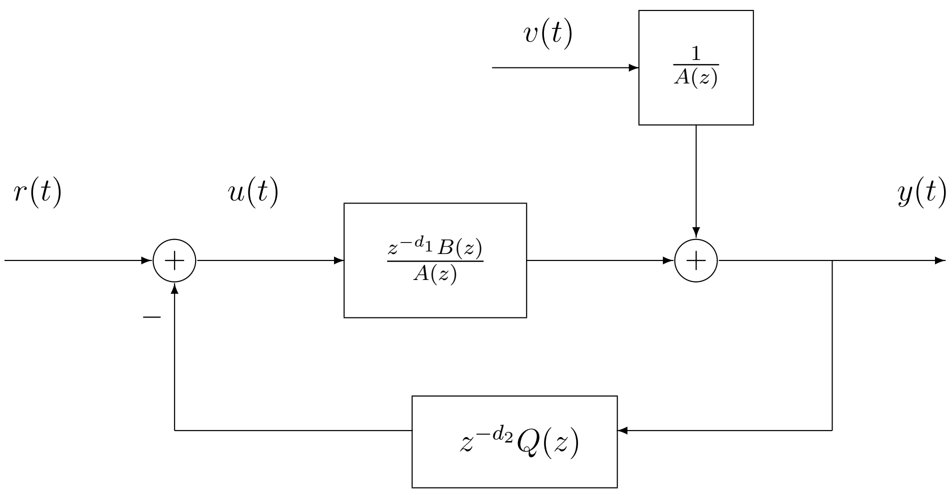

FIGURE 1. An SISO closed-loop system

Define

$$\begin{aligned}\gamma(z) &:= B(z)Q(z) = \gamma_1 z^{-1} + \cdots + \gamma_{n_\gamma} z^{-n_\gamma},\\ \alpha(z) &:= A(z) + z^{-(d_1+d_2)} B(z)Q(z)\\ &= 1 + a_1 z^{-1} + \cdots + a_{n_a} z^{-n_a} + z^{-(d_1+d_2)}\left(\gamma_1 z^{-1} + \cdots + \gamma_{n_\gamma} z^{-n_\gamma}\right),\end{aligned}$$

where $n_\gamma := n_b + n_q$. Then the model in (1) can be simplified as

$$\alpha(z)y(t) = z^{-d_1} B(z)r(t) + v(t). \tag{3}$$

In general, the following assumptions are made to study the identification problem of the closed-loop system [18]:

1. The external input $r(t)$ is stationary and persistently exciting of a sufficient order.
2. The noise $v(t)$ is stationary, and independent of the external input.
3. The polynomials $\alpha(z)$ and $B(z)$ are coprime.

The objective of this paper is to identify the coefficients of $\alpha(z)$ and $B(z)$ from measurable data $\{r(t), y(t), t = 1, 2, \cdots\}$, then to extract the parameter estimates of $Q(z)$ from the estimated parameters, and also to estimate the system orders and time-delays.

3. **Algorithm Description.** The orthogonal matching pursuit (OMP), as a main CS recovery algorithm, can be used to recover any K-sparse signal for the merits of its ease of implementation and fast speed. In this paper, in order to reduce the estimation error caused by the noise, a modified OMP algorithm – threshold orthogonal matching pursuit (TH-OMP) algorithm is applied to estimate orders, time-delays and the parameters of the control plant with a small number of observations, and the parameters of the feedback controller will then be estimated by using the model equivalence principle.

Define the information vector $\boldsymbol{\varphi}(t)$ and the parameter vectors $\boldsymbol{\theta}$ as

$$\boldsymbol{\varphi}(t) := [-y(t-1), \cdots, -y(t-n_a), \cdots, -y(t-l_1), r(t-1), \cdots, r(t-l_2)]^{\mathrm{T}} \in \mathbb{R}^{l_1+l_2}, \tag{4}$$

$$\boldsymbol{\theta} := [a_1, \cdots, a_{n_a}, \underbrace{0, 0, \cdots, 0}_{d_1+d_2-n_a}, \gamma_1, \cdots, \gamma_{n_\gamma}, \underbrace{0, 0, \cdots, 0}_{l_1-d_2-n_\gamma}, b_1, \cdots, b_{n_b}, \underbrace{0, 0, \cdots, 0}_{l_2-n_b-d_1}]^{\mathrm{T}} \in \mathbb{R}^{l_1+l_2}, \tag{5}$$

where l_1 and l_2 are the maximum regression lengths of the input and output, satisfying $d_1+d_2+n_\gamma < l_1$ and $d_1+n_b < l_2$; $\boldsymbol{\varphi}(t)$ is the information vector consisting of the regression data; $\boldsymbol{\theta}$ is the parameter vector to be identified. Then Equation (3) can be written as

$$y(t) = \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta} + v(t). \tag{6}$$

Taking m ($m < l_1 + l_2$) consecutive observations and defining the stacked vectors

$$\begin{aligned}\boldsymbol{y}_m &:= [y(1), y(2), \cdots, y(m)]^{\mathrm{T}} \in \mathbb{R}^m,\\ \boldsymbol{\Phi}_m &:= [\boldsymbol{\varphi}(1), \boldsymbol{\varphi}(2), \cdots, \boldsymbol{\varphi}(m)]^{\mathrm{T}} \in \mathbb{R}^{m\times(l_1+l_2)},\\ \boldsymbol{v}_m &:= [v(1), v(2), \cdots, v(m)]^{\mathrm{T}} \in \mathbb{R}^m,\end{aligned}$$

we have

$$\boldsymbol{y}_m = \boldsymbol{\Phi}_m\boldsymbol{\theta} + \boldsymbol{v}_m, \tag{7}$$

where $\boldsymbol{\Phi}_m \in \mathbb{R}^{m\times(l_1+l_2)}$ is a known matrix whose columns are called the atoms in CS [14, 15].

From (5), we can see that most of the entries in the parameter vector $\boldsymbol{\theta}$ are zero, and only $n_a + n_\gamma + n_b$ of them are non-zero. It means that the parameter vector $\boldsymbol{\theta}$ is sparse, and the sparsity is $K := n_a + n_\gamma + n_b$. If there are enough measurements, i.e., $m \gg l_1 + l_2$, according to the least squares principle, the least squares estimate of $\boldsymbol{\theta}$ is $\hat{\boldsymbol{\theta}}_{\mathrm{LS}} = \left(\boldsymbol{\Phi}_m^{\mathrm{T}}\boldsymbol{\Phi}_m\right)^{-1}\boldsymbol{\Phi}_m^{\mathrm{T}}\boldsymbol{y}_m$. However, the dimension of $\boldsymbol{\theta}$ is $l_1 + l_2$, which is a large number. Therefore, it will take a lot of time and effort to get enough measurements. In order to improve the identification efficiency, this paper aims to identify the parameter vector $\boldsymbol{\theta}$ using a small number of observations ($K < m < l_1 + l_2$) based on the CS recovery theory.

According to the CS recovery theory, the identification problem of the model in (7) can be described as

$$\hat{\boldsymbol{\theta}} = \arg\min \|\boldsymbol{\theta}\|_0, \quad \text{s.t.} \quad \|\boldsymbol{y}_m - \boldsymbol{\Phi}_m \boldsymbol{\theta}\| < \varepsilon,$$

where $\varepsilon > 0$ is the error tolerance which is a chosen priori, and $\|\cdot\|_0$ counts the number of non-zero entries in $\boldsymbol{\theta}$.

Let θ_i be the ith parameter in $\boldsymbol{\theta}$ and $\boldsymbol{\phi}_i$ be the ith column of $\boldsymbol{\Phi}_m$. $k = 1, 2, \cdots$ denotes the iterative number; $\boldsymbol{r}_k$ denotes the residual of the kth iteration; λ_k denotes the number of the selected column of $\boldsymbol{\Phi}_m$ in the kth iteration, let $\Lambda_k = \{\lambda_1, \lambda_2, \cdots, \lambda_k\}$ be the index set and $\boldsymbol{\Phi}_{\Lambda_k}$ be the sub-information matrix which contains k columns of $\boldsymbol{\Phi}_m$ indexed by Λ_k.

The procedure of the TH-OMP algorithm is given as the following steps [17].

1. Initialize: Let $k = 0$, $\boldsymbol{r}_0 = \boldsymbol{y}_m$ and $\Lambda_k = \emptyset$; Given the error tolerance ε.
2. Compute the index λ_k by
$$\lambda_k = \arg \max_{k=1,2,\cdots} \left| \left\langle \boldsymbol{r}_{k-1}, \frac{\boldsymbol{\phi}_i}{\|\boldsymbol{\phi}_i\|} \right\rangle \right|.$$
3. Update the index set Λ_k and the sub-information matrix $\boldsymbol{\Phi}_{\Lambda_k}$ by
$$\Lambda_k = \Lambda_{k-1} \cup \{\lambda_k\}, \quad \boldsymbol{\Phi}_{\Lambda_k} = \boldsymbol{\Phi}_{\Lambda_{k-1}} \cup \boldsymbol{\phi}_i.$$
4. Compute the kth parameter estimate $\hat{\boldsymbol{\theta}}_{\Lambda_k}$ using
$$\hat{\boldsymbol{\theta}}_{\Lambda_k} = \left(\boldsymbol{\Phi}_{\Lambda_k}^{\mathrm{T}} \boldsymbol{\Phi}_{\Lambda_k} \right)^{-1} \boldsymbol{\Phi}_{\Lambda_k}^{\mathrm{T}} \boldsymbol{y}_m.$$
5. Set an appropriate small threshold ξ to filter the parameter estimation $\hat{\boldsymbol{\theta}}_{\Lambda_k}$. If $\left| \hat{\boldsymbol{\theta}}_{\Lambda_\xi} \right| < \xi$, let $\hat{\theta}_{\Lambda_\xi} = 0$, where $\hat{\theta}_{\Lambda_\xi}$ is the ith element of $\hat{\boldsymbol{\theta}}_{\Lambda_k}$. Denote $\hat{\boldsymbol{\theta}}_{\Lambda_k}$ as $\hat{\boldsymbol{\theta}}_{\Lambda_{k_\xi}}$ after filtering.
6. Compute the residual $\boldsymbol{r}_k$ by
$$\boldsymbol{r}_k = \boldsymbol{y}_m - \boldsymbol{\Phi}_{\Lambda_k} \hat{\boldsymbol{\theta}}_{\Lambda_{k_\xi}}.$$
7. If the residual $\boldsymbol{r}_k < \varepsilon$, stop the iteration; otherwise return to Step 2.
8. Recover the parameter estimate $\hat{\boldsymbol{\theta}}$ from $\boldsymbol{\Phi}_{\Lambda_{k_\xi}}$.

From (5), we can see that there are 3 non-zero blocks and 3 zero blocks in $\boldsymbol{\theta}$. The numbers of entries of the non-zero blocks, orders, are the estimated orders $\hat{n}_a$, $\hat{n}_r$ and $\hat{n}_b$, respectively. And $\hat{n}_q$ can be estimated by $\hat{n}_q = \hat{n}_r - \hat{n}_b$. Denote the numbers of zeros of the zero blocks as z_1, z_2 and z_3. Based on the estimated orders $\hat{n}_a$, $\hat{n}_r$ and $\hat{n}_b$, the time-delays can be computed by

$$\hat{d}_2 = l_2 - z_3 - \hat{n}_b, \text{ or } \hat{d}_2 = l_1 - \hat{n}_\gamma - z_2, \tag{8}$$

$$\hat{d}_1 = z_1 + n_a - \hat{d}_2. \tag{9}$$

The parameter estimates $\hat{a}_i$, $i = 1, \cdots, n_a$, $\hat{b}_j$, $j = 1, \cdots, n_b$, and $\hat{\gamma}_p$, $p = 1, \cdots, n_\gamma$ can be directly read from the estimated parameter vector $\hat{\boldsymbol{\theta}}$.

The following is to get the estimates of $\hat{q}_l$, $l = 1, \cdots, n_q$. According to the model equivalence principle [19, 20], and from (3), we have

$$\hat{\gamma}(z) = \hat{B}(z)\hat{Q}(z).$$

Assuming that $n_b > n_q$, and comparing the coefficients on both sides, we have

$$\hat{\boldsymbol{S}} = \hat{\boldsymbol{\psi}} \hat{\boldsymbol{\theta}}_q, \tag{10}$$

where

$$\hat{\boldsymbol{S}} = \left[\hat{\gamma}_2 - \hat{b}_2, \hat{\gamma}_3 - \hat{b}_3, \cdots, \hat{\gamma}_{n_q+1} - \hat{b}_{n_q+1}, \hat{\gamma}_{n_q+2} - \hat{b}_{n_q+2}, \cdots, \hat{\gamma}_{n_b} - \hat{b}_{n_b}, \hat{\gamma}_{n_b+1}, \hat{\gamma}_{n_b+2}, \cdots, \hat{\gamma}_{n_\gamma}\right]^{\mathrm{T}},$$

$$\hat{\boldsymbol{\theta}}_q = \left[\hat{q}_1, \hat{q}_2, \cdots, \hat{q}_{n_q}\right]^{\mathrm{T}}, \quad \hat{\boldsymbol{\psi}} = \begin{bmatrix} \hat{b}_1 & 0 & \cdots & 0 \\ \hat{b}_2 & \hat{b}_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{b}_{n_q} & \hat{b}_{n_q-1} & \cdots & \hat{b}_1 \\ \hat{b}_{n_q+1} & \hat{b}_{n_q} & \cdots & \hat{b}_2 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{b}_{n_b-1} & \hat{b}_{n_b-2} & \cdots & \hat{b}_{n_b-n_q} \\ \hat{b}_{n_b-1} & \hat{b}_{n_b-2} & \cdots & \hat{b}_{n_b-n_q} \\ \hat{b}_{n_b} & \hat{b}_{n_b-1} & \cdots & \hat{b}_{n_b+1-n_q} \\ 0 & \hat{b}_{n_b} & \cdots & \hat{b}_{n_b+2-n_q} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{b}_{n_b-n_q} \end{bmatrix}.$$

Then $\hat{\boldsymbol{\theta}}_q$ can be computed by using the least squares principle,

$$\hat{\boldsymbol{\theta}}_q = \left(\hat{\boldsymbol{\psi}}^{\mathrm{T}}\hat{\boldsymbol{\psi}}\right)^{-1}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\hat{\boldsymbol{S}}. \tag{11}$$

4. **Numerical Example.** Consider an SISO closed-loop sysetm,

$$\left(1 - 0.50z^{-1} + 0.6z^{-2}\right)y(t) = z^{-64}(0.87z^{-1} - 0.95z^{-2})u(t) + 1 - 0.50z^{-1} + 0.60z^{-2}v(t),$$
$$u(t) = r(t) - z^{-55}\left(1 - 1.26z^{-1}\right)y(t).$$

Let $l_1 = 130$ and $l_2 = 80$, and then the parameter vector $\boldsymbol{\theta}$ is

$$\boldsymbol{\theta} = [-0.50, 0.60, \mathbf{0}_{117}, 0.8700, -2.0462, 1.197, \mathbf{0}_{73}, 0.87, -0.95, \mathbf{0}_{13}]^{\mathrm{T}} \in \mathbb{R}^{l_1+l_2}.$$

It is obvious that the dimension of $\boldsymbol{\theta}$ is 210, and the sparsity is $K = 7$. In simulation, the external input $\{r(t)\}$ is taken as a persistent excitation signal sequence with zero mean and unit variance, and $\{v(t)\}$ as a white noise sequence with zero mean and variance $\sigma^2 = 0.30^2$.

Let $m = 150$, $\xi = 0.06$ and by using the TH-OMP algorithm to estimate the parameter vector $\hat{\boldsymbol{\theta}}$, we can get

$$\hat{\boldsymbol{\theta}}_{\xi=0.06} = [-0.4968, 0.5972, \mathbf{0}_{117}, 0.8566, -2.0192, 1.1930, \mathbf{0}_{73}, 0.8533, -0.9743, \mathbf{0}_{13}]^{\mathrm{T}} \in \mathbb{R}^{l_1+l_2}. \tag{12}$$

If we set $\xi = 0.01$, then we can get

$$\hat{\boldsymbol{\theta}}_{\xi=0.01} = [-0.4949, 0.5973, \mathbf{0}_{35}, -0.0143, \mathbf{0}_{31}, 0.0224, \mathbf{0}_{49}, 0.8760, -2.0500, 1.2069,$$
$$\mathbf{0}_{56}, 0.0516, -0.0474, \mathbf{0}_{15}, 0.8569, -0.9924, \mathbf{0}_{13}]^{\mathrm{T}} \in \mathbb{R}^{l_1+l_2}.$$

From the above equation, we can see that there exist 4 undesirable parameter estimates $\hat{\theta}_{38}$, $\hat{\theta}_{70}$, $\hat{\theta}_{179}$ and $\hat{\theta}_{180}$, which are much smaller than other parameters. Based on the the structure of $\hat{\boldsymbol{\theta}}$, these 4 parameters are of little values and can be set to zeros. It implies that ξ should be chosen as an appropriate small number, and even a too small number is chosen, we can still obtain an effective estimate according to the structure of the sparse parameter vector.

From (12), we can get the estimated orders and the numbers of zeros as

$$\hat{n}_a = 2, \quad \hat{n}_\gamma = 3, \quad \hat{n}_b = 2, \quad z_1 = 117, \quad z_2 = 73, \quad z_3 = 13,$$

and $\hat{n}_q$ can be computed by

$$\hat{n}_q = \hat{n}_\gamma - \hat{n}_b = 1.$$

According to Equations (8) and (9), the time-delays can be estimated by

$$\hat{d}_1 = l_2 - z_3 - n_b = 64, \quad \hat{d}_2 = z_1 + n_a - \hat{d}_1 = 55.$$

According to Equation (8), we can obtain $\hat{q}_1 = -1.9999$, It is obvious that the estimated orders and time-delays are correct, and the parameter estimates are effective. Define

$$\boldsymbol{\vartheta} := [a_1, a_2, \cdots, a_{n_a}, b_1, b_2, \cdots, b_{n_b}, q_1, q_2, \cdots, q_{n_q}]^{\mathrm{T}},$$
$$\hat{\boldsymbol{\vartheta}} := \left[\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_{n_a}, \hat{b}_1, \hat{b}_2, \cdots, \hat{b}_{n_b}, \hat{q}_1, \hat{q}_2, \cdots, \hat{q}_{n_q}\right]^{\mathrm{T}},$$

and the relative parameter estimation error as $\delta := \left(\left\|\boldsymbol{\vartheta} - \hat{\boldsymbol{\vartheta}}\right\|^2 \Big/ \|\boldsymbol{\vartheta}\|^2\right)$. Applying the proposed TH-OMP algorithm and the OMP algorithm to estimate the closed-loop system. The parameter estimation errors δ versus k are shown in Figure 2. From Figure 2, we can see that the estimation accuracy of the TH-OMP algorithm is higher than that of the OMP algorithm. The estimation error δ versus k with different variances $\sigma^2 = 0.30^2$, $\sigma^2 = 0.50^2$ and $\sigma^2 = 0.70^2$ is shown in Figure 3. From Figure 3, we can see that the TH-OMP algorithm is sensitive to noise, and a lower noise level leads to a better estimation result.

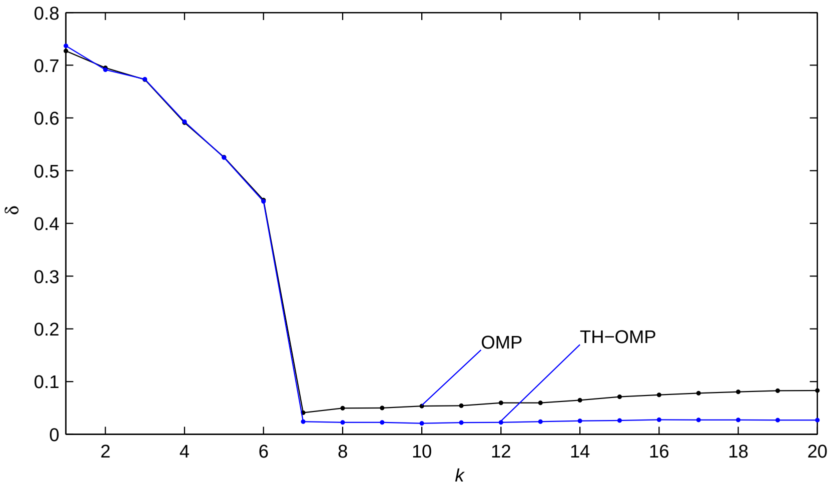

FIGURE 2. The estimation error δ versus iteration k

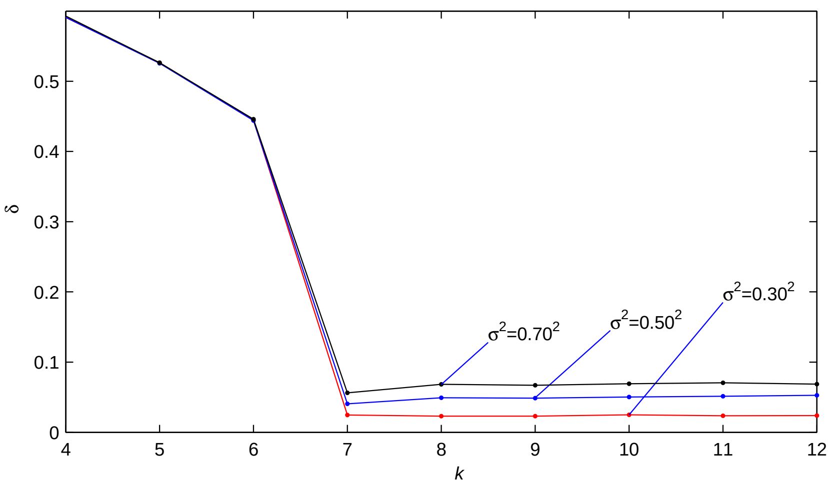

FIGURE 3. The estimation error δ versus k with variance

5. **Conclusions.** In this paper, we studied the identification problem of a class of closed-loop systems with large time-delays that the parameters and orders of both the control plant and feedback controller are unknown, and the time-delays of both the forward and backward channels are also unknown. The TH-OMP algorithm applied in the indirect identification framework can simply identify the parameters, the orders and the time-delays with only a few iterations. The simulation results show that the algorithm is effective. Moreover, for the over parameterized model, the proposed method requires much smaller observations compared to conventional identification methods. In the future, we aim to study more robust algorithms such as without setting a threshold ξ, and for more complex systems models.

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