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TRANSPORTATION PROBLEM WITH FUZZY COST BASED ON LEVEL 1 FUZZY NUMBERS

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Abstract. *The objective function of the crisp transportation is* $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$. *The constraints are* $\sum_{j=1}^{n} x_{ij} = a_i$, $i = 1, 2, \cdots, m$, $\sum_{i=1}^{m} x_{ij} = b_j$, $j = 1, 2, \cdots, n$, $x_{ij} \geq 0$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$ *and* $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. *We fuzzify* c_{ij} *to level 1 fuzzy numbers. Then we get fuzzy total cost and its centroid to represent the estimate of total cost in the fuzzy sense. From the numerical implementation, it shows that the proposed method is more practical and flexible.*

Keywords: Fuzzy numbers, Level 1 fuzzy numbers, Centroid method, Transportation problem

1. **Introduction.** In [3-6,9,15], they considered fuzzy transportation problems. They did not use level (λ, ρ) interval-valued fuzzy number, $0 < \lambda \leq \rho \leq 1$, to fuzzify. In [6], Chanas and Kuchta also considered fuzzy transportation problem, they used fuzzy number of the type L-L, $\tilde{c}_{ij}$, to fuzzify the transportation cost c_{ij} in the objective function, and they used λ-cut of $\tilde{c}_{ij}$ to derive interval c_{ij}^{λ}. With interval c_{ij}^{λ} as coefficient in the objective function, they chose two points in c_{ij}^{λ} to derive two objective functions. In this way, they got transportation problem in the fuzzy sense. Based on [7,10-13,18], we use level 1 fuzzy numbers to fuzzify objective function to obtain transportation problem in the fuzzy sense in this paper. We think that to use level 1 fuzzy number is closer to the real situations. Since the cost always fluctuates from certain value (crisp), a level 1 fuzzy number will be a good way to describe this uncertainty. In Section 2, we give some preliminary on crisp and fuzzy transportation problems and use centroid method to defuzzify. We give example and compare the results under crisp case and fuzzy case in Section 3. In Section 4, we make the concluding remarks.

2. **Fuzzy Objective Function.** For the crisp transportation problem [17], we use i and j to denote the ith origin and the jth destination, $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$. The production quantity at the ith origin is a_i and the demand at the jth destination is b_j. Let c_{ij} be the transportation cost per unit commodity from the ith origin to the jth destination.

Then we get the crisp transportation problem.

$$\min \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{1}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} = a_i, \quad a_i > 0, \quad i = 1, 2, \cdots, m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad b_j > 0, \quad j = 1, 2, \cdots, n \tag{3}$$

$$x_{ij} \geq 0, \quad i = 1, 2, \cdots, m, \quad j = 1, 2, \cdots, n \tag{4}$$

For consistency, we have

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{5}$$

$a_i, b_j, c_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$ are known positive numbers. $c_{ij}, i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$ can be determined from monopolist. Using simplex method [1,15], we can obtain the optimal solution.

Let the optimal solution be $x_{ij} = x_{ij}^{(0)}$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$. The transportation cost per unit commodity from the ith origin to the jth destination c_{ij} in crisp transportation problem (1)-(5) is a determined number. From (1)-(5), we have the optimal transportation quantity x_{ij}, $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$. There are two problems in this transportation plan.

(a) If the transportation plan executes only once and each c_{ij}, $i = 1, 2, \cdots, m$; $j = 1, 2, \cdots, n$ is fixed, then we do not need to consider the fuzzy case. If there are some difficulties in determining c_{ij} or the transportation cost may vary, the transportation cost per unit from the ith origin to the jth destination may lie in the interval $[c_{ij} - \Delta_{1ij}, c_{ij} + \Delta_{2ij}]$, where Δ_{1ij} and Δ_{2ij} will be determined by decision maker and $0 < \Delta_{1ij} < c_{ij}$, $0 < \Delta_{2ij}$. The decision maker will find an estimate in the interval $[c_{ij} - \Delta_{1ij}, c_{ij} + \Delta_{2ij}]$. How to find this estimate will be stated later.

(b) If the same transportation plan (i.e., a_i, b_j, $\forall i, j$ are fixed) excutes many times in the plan period T and in a perfect competitive market, c_{ij} is not the same in the plan period T, and the transportation cost per unit from the ith origin to the jth destination may lie in the interval $[c_{ij} - \Delta_{1ij}, c_{ij} + \Delta_{2ij}]$ as in (a), where Δ_{1ij} and Δ_{2ij} will be determined by decision maker and $0 < \Delta_{1ij} < c_{ij}$, $0 < \Delta_{2ij}$.

For problems (a) and (b), it seems more practical that the decision maker changes c_{ij} (a fixed value) to an interval $[c_{ij} - \Delta_{1ij}, c_{ij} + \Delta_{2ij}]$. If $\Delta_{1ij} = \Delta_{2ij} = 0$, the interval reduces to a value c_{ij}. The decision maker wants to choose a value in this interval $[c_{ij} - \Delta_{1ij}, c_{ij} + \Delta_{2ij}]$ as the transportation cost per unit from the ith origin to the jth destination. If he chooses c_{ij}, then it makes no difference with the crisp cost c_{ij}. There will be no error, i.e., the error is 0. If the value is different from c_{ij}, then the error is larger as it deviates from c_{ij} farther. The error will be the largest at the two endpoints $c_{ij} - \Delta_{1ij}$ and $c_{ij} + \Delta_{2ij}$. From the fuzzy point of view, we can transform the error to confidence level. The confidence level will be the largest when the error is 0. We set it to be 1. In other words, the confidence level is 1 when we choose the crisp value c_{ij}. The confidence level is getting smaller as the value we choose deviates from c_{ij} farther. The confidence level will be the smallest at the two endpoints $c_{ij} - \Delta_{1ij}$ and $c_{ij} + \Delta_{2ij}$. In this case, we set the confidence level to be 0. From the reasons above and below, corresponding to the interval $[c_{ij} - \Delta_{1ij}, c_{ij} + \Delta_{2ij}]$, we set the level 1 fuzzy number in (6) as shown in Figure 1.

$$\tilde{c}_{ij} = (c_{ij} - \Delta_{1ij}, c_{ij}, c_{ij} + \Delta_{2ij}; 1), \quad 0 < \Delta_{1ij} < c_{ij}, \quad 0 < \Delta_{2ij} \tag{6}$$

$i = 1, 2, \cdots, m$, and $j = 1, 2, \cdots, n$.

In Figure 1, The membership grade of $\tilde{c}_{ij}$ at c_{ij} is 1. In interval $[c_{ij} - \Delta_{1ij}, c_{ij} + \Delta_{2ij}]$, the membership grade will be getting smaller when the number deviates from c_{ij} farther. The membership grade will be zero at the two endpoints $c_{ij} - \Delta_{1ij}$ and $c_{ij} + \Delta_{2ij}$. The membership grade shares the same property as confidence level. Therefore, we can

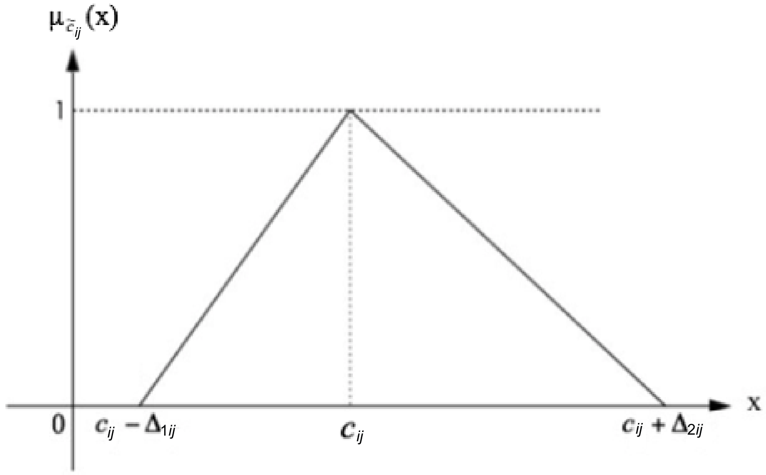

FIGURE 1. Level 1 fuzzy number $\tilde{c}_{ij}$

correspond membership grade to confidence level. The fact that we set level 1 fuzzy number (6) to correspond to the interval $[c_{ij}-\Delta_{1ij}, c_{ij}+\Delta_{2ij}]$ is reasonable.

Defuzzify $\tilde{c}_{ij}$ by the centroid method, and we have

$$c^*_{ij} \equiv c_{ij} + \frac{1}{3}(\Delta_{2ij}-\Delta_{1ij}) = \frac{2}{3}c_{ij} + \frac{1}{3}\Delta_{2ij} + \frac{1}{3}(c_{ij}-\Delta_{1ij}) > 0$$

and $c^*_{ij} \in [c_{ij}-\Delta_{1ij}, c_{ij}+\Delta_{2ij}]$. c^*_{ij} can be used as an estimate for the transportation cost per unit from the ith origin to the jth destination. When $\Delta_{1ij} = \Delta_{2ij}$, $c^*_{ij} = c_{ij}$. Then the estimate of the transportation cost c^*_{ij} in the fuzzy sense is the same as the crisp transportation cost c_{ij}. That means fuzzy case will contain the crisp case.

Then we have the following transportation problem in the fuzzy sense.

$$\min \quad \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}x_{ij} \tag{7}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} = a_i, \quad a_i > 0, \quad i = 1, 2, \cdots, m \tag{8}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad b_j > 0, \quad j = 1, 2, \cdots, n \tag{9}$$

$$x_{ij} \geq 0, \quad i = 1, 2, \cdots, m, \quad j = 1, 2, \cdots, n \tag{10}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{11}$$

Remark 2.1. $\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}x_{ij}$ *means that*

$$((\tilde{x}_{11})_1 \odot \tilde{c}_{11}) \oplus ((\tilde{x}_{21})_1 \odot \tilde{c}_{21}) \oplus \cdots \oplus ((\tilde{x}_{m1})_1 \odot \tilde{c}_{m1}) \oplus ((\tilde{x}_{12})_1 \odot \tilde{c}_{12}) \oplus \cdots \oplus$$
$$((\tilde{x}_{m2})_1 \odot \tilde{c}_{m2}) \oplus \cdots \oplus ((\tilde{x}_{1n})_1 \odot \tilde{c}_{1n}) \oplus \cdots \oplus ((\tilde{x}_{mn})_1 \odot \tilde{c}_{mn})$$

Property 2.1. *When we fuzzify c_{ij} in crisp transportation problem (1)-(5), we have fuzzy transportation (7)-(11). Then we get the transportation problem in the fuzzy sense.*

$$\min \quad M_Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \frac{1}{3}\sum_{i=1}^{m}\sum_{j=1}^{n}(\Delta_{2ij}-\Delta_{1ij})x_{ij} \tag{12}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} = a_i, \quad a_i > 0, \quad i = 1, 2, \cdots, m \tag{13}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad b_j > 0, \quad j = 1, 2, \cdots, n \tag{14}$$

$$x_{ij} \geq 0, \quad i = 1, 2, \cdots, m, \quad j = 1, 2, \cdots, n \tag{15}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_i \tag{16}$$

Proof: Since $x_{ij} \geq 0$, $\forall i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$, then from (6), we have $\tilde{Z} = \left(\sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij} - \Delta_{1ij})x_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij} + \Delta_{2ij})x_{ij}; 1\right)$. Defuzzify $\tilde{Z}$ by the centroid method, and we have $M_Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \frac{1}{3}\sum_{i=1}^{m}\sum_{j=1}^{n}(\Delta_{2ij} - \Delta_{1ij})x_{ij}$. This is the objective function in the fuzzy sense, i.e., (12).

Remark 2.2. *The method for solving the optimal solution for the transportation problem in Property 2.1 is the same as the crisp transportation problem. When $\Delta_{2ij} = \Delta_{1ij}$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$, $\tilde{c}_{ij}$ is symmetric with respect to c_{ij}. Then Property 2.1 reduces to the crisp transportation problem (1)-(5).*

The meaning of Property 2.1 is that when we fuzzify c_{ij} to $\tilde{c}_{ij}$ (in (7)), the c_{ij} in objective function (7) should be replaced by the estimate $c_{ij}^* \equiv c_{ij} + \frac{1}{3}(\Delta_{2ij} - \Delta_{1ij})$ through defuzzification by centroid.

3. **Example.** In this section, we give an example to implement Property 2.1.

Case 0: Crisp transportation problem.

A company has two factories F_1 and F_2 and three retail warehouses W_1, W_2 and W_3. The production quantities per month for F_1 and F_2 are 10 tons and 8 tons respectively. The demands for W_1, W_2 and W_3 are 5 tons, 6 tons and 7 tons. The transportation costs per unit are $c_{11} = 16$, $c_{12} = 15$, $c_{13} = 25$, $c_{21} = 19$, $c_{22} = 24$ and $c_{23} = 12$. We obtain the following crisp transportation problem.

$$\begin{aligned}
\text{min} \quad & Z = 16x_{11} + 15x_{12} + 25x_{13} + 19x_{21} + 24x_{22} + 12x_{23} \\
\text{s.t.} \quad & x_{11} + x_{12} + x_{13} = 10 \\
& x_{21} + x_{22} + x_{23} = 8 \\
& x_{11} + x_{21} = 5 \\
& x_{12} + x_{22} = 6 \\
& x_{13} + x_{23} = 7 \\
& x_{ij} \geq 0, \quad i = 1, 2, \quad j = 1, 2, 3
\end{aligned}$$

We can rewrite the constraints as follows.

$$\begin{aligned}
x_{21} &= 8 - x_{22} - x_{23} \\
x_{11} &= 5 - x_{21} = -3 + x_{22} + x_{23} \\
x_{12} &= 6 - x_{22} \\
x_{13} &= 7 - x_{23},
\end{aligned}$$

where $x_{ij} \geq 0$, $i = 1, 2$; $j = 1, 2, 3$.

We can change this linear programming problem with 6 variables to an equivalent linear programming with two variables.

Substituting these into the objective function, we get $Z = 369 + 6x_{22} - 16x_{23}$.

Then we have

$$\begin{aligned}
\text{min} \quad & Z = 369 + 6x_{22} - 16x_{23} \\
\text{s.t.} \quad & 0 \leq x_{22} \leq 6,
\end{aligned}$$

$$0 \le x_{23} \le 7,$$
$$x_{22} + x_{23} \le 8,$$
$$x_{22} + x_{23} \ge 3$$

The optimal solution is that $x_{11} = 4$, $x_{12} = 6$, $x_{13} = 0$, $x_{21} = 1$, $x_{22} = 0$, $x_{23} = 7$ and minimal cost is $Z = 257$ ($\equiv Z_0$).

Note: We can also use linear programming solver (e.g., LINDO) to solve it.

Case 1: Property 2.1.

Let $\tilde{c}_{11} = (16-1, 16, 16+2; 1)$, $\tilde{c}_{12} = (15-1, 15, 15+0.5; 1)$, $\tilde{c}_{13} = (25-1, 25, 25+2; 1)$, $\tilde{c}_{21} = (19-2, 19, 19+3; 1)$, $\tilde{c}_{22} = (24-0.5, 24, 24+3.5; 1)$ and $\tilde{c}_{23} = (12-1, 12, 12+1.5; 1)$. From Property 2.1, (12)-(16) we get

$$\begin{aligned}
\min \quad & M_Z = 16.333x_{11} + 14.833x_{12} + 25.333x_{13} + 19.333x_{21} + 25x_{22} + 12.167x_{23} \\
\text{s.t.} \quad & x_{11} + x_{12} + x_{13} = 10 \\
& x_{21} + x_{22} + x_{23} = 8 \\
& x_{11} + x_{21} = 5 \\
& x_{12} + x_{22} = 6 \\
& x_{13} + x_{23} = 7 \\
& x_{ij} \ge 0, \quad i = 1, 2, \quad j = 1, 2, 3
\end{aligned}$$

Alternatively, we rewrite the constraints $x_{21} = 8 - x_{22} - x_{23}$, $x_{12} = 6 - x_{22}$, $x_{13} = 7 - x_{23}$ and $x_{11} = 5 - x_{21} = -3 + x_{22} + x_{23}$, where $x_{ij} \ge 0$, $i = 1, 2$, $j = 1, 2, 3$.

We can change the linear programming problem with six variables to an equivalent linear programming with two variables.

Substituting these constraints into the objective function, we get $M_Z = 371.994 + 7.167x_{22} - 16.166x_{23}$. Therefore, we have

$$\begin{aligned}
\min \quad & M_Z = 371.994 + 7.167x_{22} - 16.166x_{23} \\
\text{s.t.} \quad & 0 \le x_{22} \le 6, \\
& 0 \le x_{23} \le 7, \\
& x_{22} + x_{23} \le 8, \\
& x_{22} + x_{23} \ge 3
\end{aligned}$$

The optimal solution is that $x_{11} = 4$, $x_{12} = 6$, $x_{13} = 0$, $x_{21} = 1$, $x_{22} = 0$, $x_{23} = 7$ and minimal cost is $M_Z = 258.832$ ($\equiv Z_1$).

We can also use linear programming solver (e.g., LINDO) to solve it.

The comparison between case 0 and case 1 is $\dfrac{Z_1 - Z_0}{Z_0} \times 100\% = 0.71\%$.

4. **Conclusions.** We made some comments about this paper as follows.

(A) From the example implementation in Section 3, we have that the relative error between the crisp case and our proposed level 1 fuzzy number method is very small. We can show that the proposed method is more practical and flexible.

(B) If the transportation plan executes only once and without statistical data in the past, we use Property 2.1, i.e., the estimate of the crisp cost c_{ij} should be changed to $c_{ij} + \frac{1}{3}(\Delta_{2ij} - \Delta_{1ij})$ which is the estimate in the fuzzy sense.

(C) The future research can be extended to a more general case, i.e., the cost is a level $(\lambda, 1)$ interval-valued fuzzy numbers or apply signed distance method to defuzzifying and comparing the results.

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