ICIC Express Letters ICIC International ©2016 ISSN 1881-803X
Volume **10**, Number **11**, November **2016** pp. **2555–2561**

HEURISTIC PERFORMANCE IMPROVEMENT METHOD FOR BACKWARD PATH-TRACKING CONTROL OF A TRACTOR-TRAILER MOBILE ROBOT

Nozomu Araki[1], Takahiro Inoue[2], Shintaro Nakatani[3]
Takao Sato[1] and Yasuo Konishi[1]

[1]Graduate School of Engineering
University of Hyogo
2167 Shosha, Himeji-shi, Hyogo 671-2201, Japan
araki@eng.u-hyogo.ac.jp

[2]Takara Standard Co., Ltd.
1-2-1 Shigino-higashi, Joto-ku, Osaka 536-8536, Japan

[3]Graduate School of Engineering
Tottori University
4-101 Koyama-cho Minami, Tottori 680-8550, Japan

Received April 2016; accepted July 2016

Abstract. *In this study, we considered the backward path-tracking control of articulated vehicles such as a tractor-trailer. An effective method using the exact feedback linearization was previously proposed in this research area. However, this method has a singular condition that the control law cannot be defined. Because of this, the controller has a strong dependence on the initial condition of the vehicle's position and behavior with respect to the desired path. Moreover, the range of application is too narrow for practical use. To solve this problem, we propose a performance improvement method for the exact feedback linearization-based controller. Our proposed method is obtained by adding a singular condition avoidance input, which is obtained heuristically, to the previous controller. The simulation results indicated that our proposed method is useful for controlling a tractor-trailer mobile robot.*
Keywords: Tractor-trailer mobile robot (TTMR), Backward path-tracking, Exact feedback linearization, Heuristic input

1. **Introduction.** Recently, technologies for autonomous cars have been actively researched, and some have already been applied to practical use. On the other hand, although there is a high demand for articulated vehicles such as tractor-trailers because they are widely used in industrial transportation, their automatic operation is still a problem that is hard to solve [1]. In particular, the backward movement of an articulated vehicle could fall into an uncontrollable condition called "jackknifing" as shown in Figure 1. Because of this, the backward operation of such a vehicle is difficult even for skilled drivers. In addition, from a theoretical point of view, the equation of motion of the backward movement of an articulated vehicle is highly nonlinear and unstable; therefore, the control design for the backward path-tracking is a very challenging problem and is actively being researched. For example, Sampei et al. proposed a method using the exact feedback linearization technique [2], and Martinez et al. proposed a pure geometric technique [3]. Moreover, methods using the fuzzy controller [4, 5] or neural networks are proposed as well [6, 7].

In this paper, we focused on the exact feedback linearization method proposed by Sampei et al., which provides a systematic control design for the backward path-tracking and has led to significant advances in this research area [1]. However, this method has a singular condition that the control law cannot be defined. Because of this, the controller

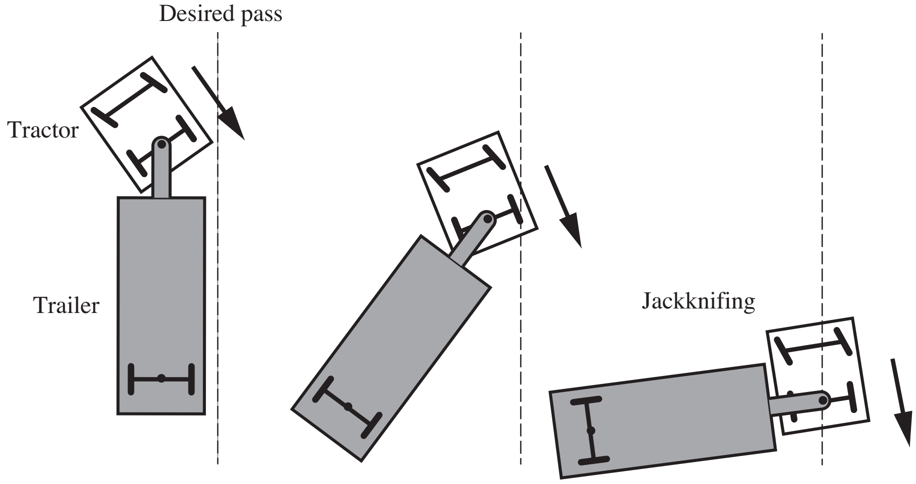

FIGURE 1. Jackknifing by an articulated vehicle

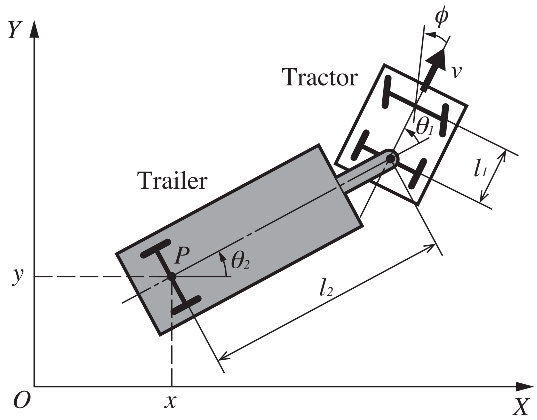

FIGURE 2. TTMR model

has a strong dependence on the initial condition of the vehicle's position and its behavior with respect to the desired path, and the range of application is too narrow for practical use. To solve this problem, we propose a performance improvement method for the exact feedback linearization-based controller. Our proposed method is obtained by adding a singular condition avoidance input, which is obtained heuristically, to the previous controller. We performed computer simulations to confirm the effectiveness of our proposed method.

2. **Mathematical Model of a Tractor-Trailer Mobile Robot.** In this section, we describe the mathematical model of a tractor-trailer mobile robot (TTMR), which was considered in this paper. Consider the backward linear path-tracking control of a TTMR that moves on the x-y coordinate plane, as shown in Figure 2.

The wheels of the TTMR are assumed to have no sideslip; i.e., this TTMR is a drift-free system. Then, the state equation of the TTMR is given as follows:

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} v\cos\theta_1\cos\theta_2 \\ v\cos\theta_1\sin\theta_2 \\ v\left(\dfrac{\tan\phi}{l_1} - \dfrac{\sin\theta_1}{l_2}\right) \\ v\dfrac{\sin\theta_1}{l_2} \end{bmatrix}, \tag{1}$$

where (x, y) are the coordinates of the center of the axle of the trailer, θ_1 is the angle of the direction of the tractor movement with reference to the trailer, and θ_2 is the angle of the direction of the trailer movement with reference to the x-axis. l_1 is the wheel base length of the tractor, and l_2 is the length from the tractor-trailer joint to the center of the trailer axle. In addition, v and ϕ are the velocity and steering angle, respectively, of the tractor. Because the purpose of the TTMR control is backward linear path-tracking, it is assumed that the desired path is along the x-axis and the tractor velocity v is a negative constant value. Moreover, to achieve the backward path-tracking, we consider to make the state (y, θ_1, θ_2) asymptotically stable, where the state x decreases monotonically. Then, the state vector is chosen to be $\boldsymbol{\xi} = [y \quad \theta_1 \quad \theta_2]^T$. By rewriting the state equation of (1) into the form of $\boldsymbol{\xi}$ and differentiating with respect to $x' = -x$ instead of the time t, we obtain:

$$\frac{d\boldsymbol{\xi}}{dx'} = -\frac{d\boldsymbol{\xi}}{dt}\frac{dt}{dx} = \boldsymbol{f}(\boldsymbol{\xi}) + \boldsymbol{g}(\boldsymbol{\xi})u, \tag{2}$$

where

$$\boldsymbol{f}(\boldsymbol{\xi}) = \begin{bmatrix} -\tan\theta_2 \\ \dfrac{\tan\theta_1}{l_2\cos\theta_2} \\ -\dfrac{\tan\theta_1}{l_2\cos\theta_2} \end{bmatrix}, \quad \boldsymbol{g}(\boldsymbol{\xi}) = \begin{bmatrix} 0 \\ -\dfrac{1}{l_1\cos\theta_1\cos\theta_2} \\ 0 \end{bmatrix}, \quad u = \tan\phi.$$

The details of this formulation are in [2]. For (2), if we design the control law that achieves $\boldsymbol{\xi} \to 0$ for $x \to \infty$, then the TTMR converges to the x-axis with backward motion.

3. **Controller Design Using Exact Feedback Linearization.** For the backward path-tracking control design in (2), Sampei et al. proposed an excellent approach that uses the exact feedback linearization method and state feedback for a linearized system [2]. In this section, we briefly discuss their control design method and practical issues.

First, to obtain the equivalent linear state equation of (2) without approximation, we apply the exact feedback linearization method, which transforms a nonlinear state equation to a linear form without any approximation, based on the theoretically defined nonlinear coordinate transformation and nonlinear feedback input [8]. For the nonlinear state equation of the TTMR in (2), we choose the new state vector $\boldsymbol{\zeta}$, which is obtained by the nonlinear coordinate transformation of $\boldsymbol{\xi}$ and the nonlinear feedback input u as follows:

$$\boldsymbol{\zeta} = \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix} = \begin{bmatrix} y \\ -\tan\theta_2 \\ \dfrac{\tan\theta_1}{l_2\cos^3\theta_2} \end{bmatrix}, \tag{3}$$

$$u = \frac{l_1\cos\theta_1\left\{3\sin^2\theta_1\tan\theta_2 - \tan\theta_1\right\}}{l_2} - l_1 l_2\cos^3\theta_1\cos^4\theta_2 w, \tag{4}$$

where w in (4) is a new input for the obtained linear state equation. Moreover, the new coordinate $\boldsymbol{\zeta}$ in (3) is $\boldsymbol{\zeta} \to 0$ when $\boldsymbol{\xi} \to 0$. By using (3) and (4), (2) is translated into a linear form as follows:

$$\frac{d\boldsymbol{\zeta}}{dx'} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\boldsymbol{\zeta} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w. \tag{5}$$

Because the linear system given by (5) is controllable, a stable controller for (5) can easily be designed by using the linear control theory. If we design the state feedback control law with the following form:

$$w = -\boldsymbol{K}\boldsymbol{\zeta} = -[k_1 \quad k_2 \quad k_3]\boldsymbol{\zeta}, \tag{6}$$

then, we can use (3), (4) and (6) to obtain the steering angle ϕ to achieve backward path-tracking of the TTMR as follows:

$$\phi = \tan^{-1} u, \tag{7}$$

where

$$u = \frac{l_1 \cos\theta_1 \left\{3\sin^2\theta_1 \tan\theta_2 - \tan\theta_1\right\}}{l_2} - l_1 l_2 \cos^3\theta_1 \cos^4\theta_2 \left\{-k_1 y + k_2 \tan\theta_2 - k_3 \frac{\tan\theta_1}{l_2\cos^3\theta_2}\right\}.$$

To confirm the performance of the backward tracking controller in (7), we carried out simulations under various initial conditions. In these simulations, l_1 and l_2 in Figure 2 were set to 0.3 m and 0.625 m, respectively. The velocity of the tractor v was set to -0.2 m/s. The state feedback controller in (6) was designed with the pole placement method, where all poles were set to -2. Then the gain $\boldsymbol{K}$ in (6) was obtained as $\boldsymbol{K} = [8 \quad 12 \quad 6]$. To avoid unrealistic situations, we restricted the steering angle ϕ to $-\pi/6 \leq \phi \leq \pi/6$ in our simulations. Figure 3 shows the TTMR control results for different initial values. The solid line is the control trajectory of the axle center of the trailer, and the behavior of the TTMR is also plotted. In the case of the initial condition $(x, y, \theta_1, \theta_2) = (0, 0.5, 0, 0)$, the control trajectory converged well along the x-axis. However, when the initial condition was $(x, y, \theta_1, \theta_2) = (0, 1.0, 0, 0)$, the TTMR jackknifed at $x = -0.4$ m; after that, it converged to the x-axis with an impossible condition where the tractor and trailer overlapped.

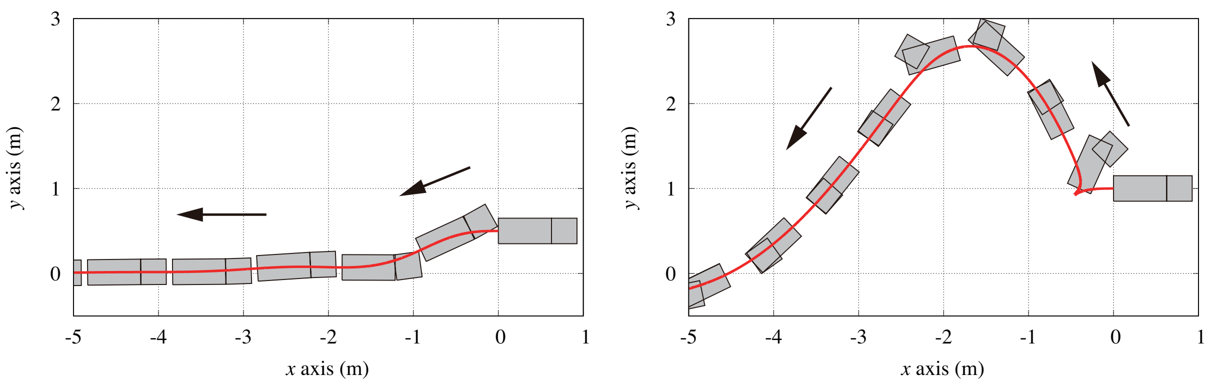

FIGURE 3. TTMR backward tracking control simulation results with the conventional controller given by (7): (left) initial condition $(x, y, \theta_1, \theta_2) = (0, 0.5, 0, 0)$, (right) initial condition $(x, y, \theta_1, \theta_2) = (0, 1.0, 0, 0)$

This control failure was caused by the singular condition of the nonlinear coordinate transformation in (3). Here, the translated state vector $\boldsymbol{\zeta}$ has the singular condition $\theta_1 = \pm\pi/2$ and $\theta_2 = \pm\pi/2$, where the coordinate transformation cannot be defined. With regard to the control failure, the TTMR jackknifed because the angle θ_1 increased to more than the singular condition $\theta_1 = \pi/2$. Unfortunately, the backward path-tracking controller in (7) has no function to handle such a singular condition; thus, this method has a strong dependence on the initial condition.

4. **Heuristic Performance Improvement Method for Backward Tracking Control of the TTMR.** Based on the discussion of the backward tracking control result for the TTMR using (7) in the previous section, the control performance can be improved if we can prevent θ_1 from approaching the singular condition $\theta_1 = \pm\pi/2$. For the backward operation of an articulated vehicle, skilled drivers know from experiments that the angle θ_1 does not increase if the steering angle ϕ has the same direction as θ_1. Based on this fact, $|\theta_1|$ decreases if the steering angle ϕ has the same direction as θ_1, and one such steering

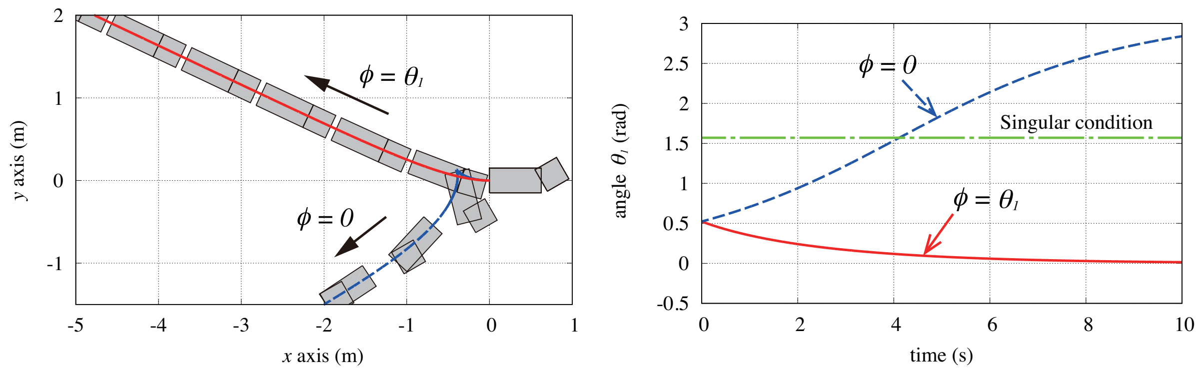

FIGURE 4. Simulation results of TTMR with steering angles $\phi = \theta_1$ and $\phi = 0$: (left) trajectory of the TTMR, (right) time series of the tractor-trailer angle θ_1

angle is $\phi = \theta_1$, which we call the "heuristic input" in this study. To confirm the effect of the heuristic input, we carried out a simulation with the steering angle $\phi = \theta_1$ and the initial condition $(x, y, \theta_1, \theta_2) = (0, 0, \pi/6, 0)$. Figure 4 shows the simulation results for the TTMR trajectory and the time series of the tractor-trailer angle θ_1. For comparison, the simulation result using the steering angle $\phi = 0$ is also shown. While the TTMR jackknifed in the case of $\phi = 0$, the tractor-trailer angle θ_1 converged to zero when the steering angle $\phi = \theta_1$ was used.

For this heuristic input, we also carried out the stability analysis. First, we consider the positive definite function as follows:

$$V = \frac{1}{2}\,\theta_1^2 \geq 0. \tag{8}$$

From (1), the time derivative of V along the system trajectory is given by

$$\frac{d}{dt}V = \theta_1 \cdot \frac{d}{dt}\theta_1 = \theta_1 \cdot v\left(\frac{\tan\phi}{l_1} - \frac{\sin\theta_1}{l_2}\right). \tag{9}$$

Substituting the heuristic input $\phi = \theta_1$ into (9), we have

$$\frac{d}{dt}V = \theta_1 \cdot v\left(\frac{\tan\theta_1}{l_1} - \frac{\sin\theta_1}{l_2}\right) = \theta_1 \tan\theta_1 \cdot v\left(\frac{l_2 - l_1\cos\theta_1}{l_1 l_2}\right). \tag{10}$$

In the right term of above equation, $\theta_1 \tan\theta_1 \geq 0$ for $\theta_1 \in (-\pi/2, \pi/2)$ is satisfied and

$$\frac{l_2 - l_1\cos\theta_1}{l_1 l_2} \geq 0,$$

if $l_1 \leq l_2$. The condition $l_1 \leq l_2$ holds for the TTMR. Moreover, the tractor velocity v was chosen as a negative constant value. Therefore, (10) satisfies

$$\frac{d}{dt}V \leq 0. \tag{11}$$

From (8), (9) and the Lyapunov's stability theorem, the subsystem for θ_1 is asymptotically stable by using the heuristic input. These simulations and stability analysis result indicate that the steering angle $\phi = \theta_1$ prevents the tractor-trailer angle θ_1 from approaching the singular condition.

Based on this, we propose a method for improving the backward tracking control of the TTMR. Our proposed method is obtained by adding the heuristic input into Sampei el al.'s controller in (7) as follows:

$$\phi = \tan^{-1} u + \theta_1 \tag{12}$$

where

$$u = \frac{l_1 \cos\theta_1 \left\{3\sin^2\theta_1 \tan\theta_2 - \tan\theta_1\right\}}{l_2} - l_1 l_2 \cos^3\theta_1 \cos^4\theta_2 \left\{-k_1 y + k_2 \tan\theta_2 - k_3 \frac{\tan\theta_1}{l_2\cos^3\theta_2}\right\}.$$

The second term in (12) is the singular condition avoidance input obtained from the skilled drivers' empirical knowledge, as mentioned above.

Figure 5 shows the simulation results of our proposed controller (12) under the same conditions for the conventional controller given by (7), as shown in Figure 3. The solid line is the control trajectory of our proposed controller in (12), while the dashed line is that of the conventional controller in (7) proposed by Sampei et al. In the case of the initial condition $(x, y, \theta_1, \theta_2) = (0, 0.5, 0, 0)$, although both our proposed controller and the conventional controller were able to control the TTMR, our proposed controller has a smoother trajectory. When the initial condition was $(x, y, \theta_1, \theta_2) = (0, 1.0, 0, 0)$, the conventional controller resulted in jackknifing; however, our proposed controller was able to achieve backward path-tracking.

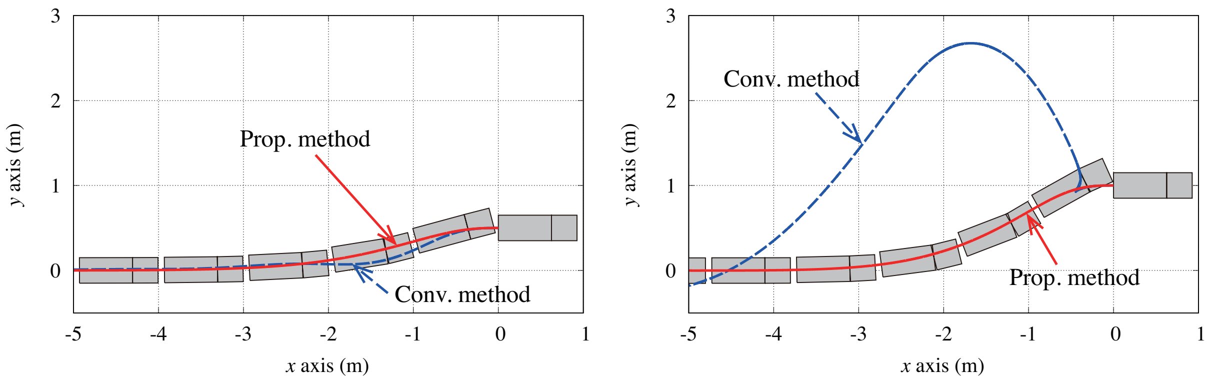

Figure 5. TTMR backward tracking control simulation results using the proposed controller given by (12): (left) initial condition $(x, y, \theta_1, \theta_2) = (0, 0.5, 0, 0)$, (right) initial condition $(x, y, \theta_1, \theta_2) = (0, 1.0, 0, 0)$

These simulation results indicate that the additional term in our proposed method works well to prevent the tractor-trailer angle θ_1 from approaching the singular condition and improves the control performance of the conventional controller (7). Therefore, our proposed controller is effective for the backward path-tracking of a TTMR in practical application.

5. **Conclusion.** On considereing the backward path-tracking control of a TTMR in this study, it was found that the conventional exact feedback linearization-based controller has a strong dependence on the initial condition, which is caused by the singular condition of the nonlinear coordinate transformation. To prevent the system trajectory from converging into the singular condition, we added a heuristic input to the conventional controller. The simulation results indicated that our proposed method is applicable to a wider range of initial conditions and is effective for practical use.

REFERENCES

[1] J. David and P. V. Manivannan, Control of truck-trailer mobile robots: A survey, *Intelligent Service Robotics*, no.7, pp.245-258, 2014.

[2] M. Sampei, T. Tamura, T. Itoh and M. Nakamichi, Path tracking control of trailer-like mobile robot, *IEEE/RSJ Int. Workshop on Intelligent Robots and System IROS'91*, pp.193-198, 1991.

[3] J. L. Martinez, M. Paz and A. Garcia-Cerezo, Path tracking for mobile robot with a trailer, *Proc. of the 15th IFAC World Congress*, vol.35, no.1, pp.329-334, 2002.
[4] K. Tanaka and K. Yoshioka, Fuzzy trajectory control and GA-based obstacle avoidance of a truck with five trailers, *Proc. of IEEE International Conference on Systems, Man and Cybernetics, Intelligent Systems for the 21st Century*, vol.5, pp.4378-4382, 1995.
[5] J. Cheng, Y. Zhang and Z. Wang, Backward tracking of mobile robot with one trailer via fuzzy line-of-sight method, *Proc. of the 6th International Conference on Fuzzy Systems and Knowledge Discovery*, vol.4, pp.66-70, 2009.
[6] D. Nguyen and B. Widrow, Truck backer-upper: An example of self-learning in neural networks, *Proc. of SPIE 1293, Applications of Artificial Intelligence*, pp.11-17, 1990.
[7] D. F. Hougen, M. Gini and J. Slagle, Rapid unsupervised connectionist learning for backing a robot with two trailers, *Proc. of IEEE International Conference on Robotics and Automation*, pp.2950-2955, 1997.
[8] H. K. Khalil, *Nonlinear Systems Second Edition*, Prentice-Hall, 1996.