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COLLABORATIVE SHIP PLANNING WITH SLOT EXCHANGE IN LINER SHIPPING

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Abstract. *Strategic alliance in liner shipping industry has been deployed with several types of collaborations such as joint fleet, slot chartering, slot exchange, slot purchase and share of port usage to avoid over-investment and excessive competition. Slot exchange, an advanced collaboration model of slot chartering, is a recommended presentation for two competing liner shipping companies to achieve win-win advantages. A mathematical model described in this study for slot exchange is based on a mixed integer linear programming model with bi-objective functions. A numerical example having bi-objective functions is described and a solution to this example problem is resolved by using Minsum method. Also a simple and reasonable method to reallocate the benefit obtained from slot exchange is proposed below.*

Keywords: Liner shipping, Strategic alliance, Slot exchange, Minsum method

1. **Introduction.** Strategic alliance among liner shipping companies is a significant way for avoiding severe competition as well as cost saving. There are various types of strategic alliances for liner shipping companies: joint fleet, slot exchange, slot chartering, slot purchase, and share of port usage [7]. Under the application of slot chartering model, only one company can rent a space of other company's ship and the vice versa is not approved. On the other hand, slot exchange model allows liner shipping companies to rent and lend some shipping space from/to other company each other. In this regard, slot exchange model can be considered as a more progressive model than the slot chartering. This study defines slot exchange problem for two companies, under the slot exchange agreement formed within each other and develops a mathematical formulation that minimizes the net cost required to transport all the container shipping demand. In this context, the net cost is calculated by subtracting the revenue from the total cost consisting of transportation cost and fixed and variable costs for slot chartering. The variable cost is proportional to the number of containers loaded to other company's ship and is paid to other company, but the fixed cost, which is also called as a setup cost, is not paid to other company. Chen and Zhen [3] developed a mathematical model for slot exchange alliance problem, but its feature of being a non-linear programming model became the limitation in applying it to the real world. Contrary to the Chen and Zhen's model, as the presented model in this paper is a linear model, it is easier to apply to the reality. Until now, several researches are in progress related to collaboration such as container slot chartering and exchange. Ting and Tzeng [9] proposed a conceptual model for liner shipping revenue management and a mathematical formulation for slot allocation. Lei et al. [6] suggested two strategies to build collaboration between two liner shipping companies and compared them to non-collaboration cases. Chen and Zhen [4] proposed a container slot exchange model, which was defined as an advanced standard than slot chartering, but it is relevant

to non-linear model. Lu et al. [7] continued on their investigation using Delphi method and concluded that the strategic alliance can be an essential tool for carriers to extend their service ranges in the global market. A mutual trust created between partners is a corner stone to ensure the success of alliances. In Shi et al.'s study [8], the liner carriers, whom were involved in slot chartering agreements, were regarded as the players and the pay-off of the games should be win-win games rather than zero-sum games. The principal idea of this research is to explain negotiation stages and to design an efficient mechanism to balance the slot requirements and the equilibrium prices under different circumstances set up in slot chartering agreements. An integrated model, a mixed-integer linear program, presented by Agarwal and Ergun [1] was a suggestion as a scheme of service network in liner shipping and this is a practicable model to solve both ship scheduling and cargo routing problems simultaneously. A greedy heuristic, a column generation-based algorithm, and a two phase Benders decomposition-based algorithm were also developed. In addition, this study proposes transportation networks that operate as an alliance among different carriers, especially in formation of alliance among carriers taking parts in liner shipping. Some tactical problems are addressed, such as the design of large scale networks and operational problems which can relate to the allocation of limited capacity on a transportation network among the carriers in the alliance [2]. Chung and Ko [5] presented an linear model for slot chartering alliance problem. In this study, determination of a certain level among several chartering space levels was suggested as a method to enhance the applicability of their model. This paper presents a mixed integer linear programming model for slot exchange and solves a numerical example using Excel Solver add-in program. While Chen and Zhen [3,4] proposed a nonlinear model for slot chartering and exchange with no excessive slot, this study proposes a linear model with excessive slot.

2. Problem Definition and Model Design.

This paper deals with container slot exchanging model with two liner shipping companies. Several assumptions to define the problem are mentioned below.

(1) There are two liner shipping companies which had agreed to exchange container slots each other. These two companies voyage the same route.

(2) The demands of container transportation of two companies are given for each route.

(3) The allowed container slot size to load for the other company is predetermined. Therefore, a company can load containers to the other company's ship within the predetermined size.

(4) Fixed and variable costs take effect when a company loads their containers to the other company's ship and are defined as follows. Fixed cost is a setup cost associated with loading containers and is irrelevant to the number of containers being loaded. On the other hand, variable cost is equivalent to the number of containers loaded to the other company's ship. As the variable cost is paid to the other company, this becomes the revenue for the other company.

Some notations and decision variables are introduced as follows in order to formulate the mathematical model.

(Notations)

I_1: Set of ships of company 1

I_2: Set of ships of company 2

K: Set of routes that liner shipping companies voyage to transport containers during planning time period

D_k^1: Container transportation demand that company 1 should transport on the route k, $k \in K$

D_k^2: Container transportation demand that company 2 should transport on the route k, $k \in K$

c^1_{ik}: Transportation cost occurred when ship i of company 1 voyages the route k one time, $i \in I_1$, $k \in K$
c^2_{ik}: Transportation cost occurred when ship i of company 2 voyages the route k one time, $i \in I_2$, $k \in K$
d^1_k: Unit penalty cost for undelivered containers that company 1 does not transport on route k, $k \in K$
d^2_k: Unit penalty cost for undelivered containers that company 2 does not transport on route k, $k \in K$
e^1_{ik}: Variable cost that company 2 pays for slot chartering of one TEU from company 1 on route k, $i \in I_1$, $k \in K$
e^2_{ik}: Variable cost that company 1 pays for slot chartering of one TEU from company 2 on route k, $i \in I_2$, $k \in K$
f^1_{ik}: Fixed cost occurred when company 2 loads his containers to ship i of company 1 on the route k, $i \in I_1$, $k \in K$
f^2_{ik}: Fixed cost occurred when company 1 loads his containers to ship i of company 2 on the route k, $i \in I_2$, $k \in K$
m^1_{ik}: maximum number of times that ship i of company 1 can voyage the route k during planning time period, $i \in I_1$, $k \in K$
m^2_{ik}: maximum number of times that ship i of company 2 can voyage the route k during planning time period, $i \in I_2$, $k \in K$
Q^1_i: Container loading capacity of ship i of company 1, $i \in I_1$
Q^2_i: Container loading capacity of ship i of company 2, $i \in I_2$
U^1_{ik}: Maximum allowable amount company 2 can load to company 1's ship i voyaging route k, $i \in I_1$, $k \in K$
U^2_{ik}: Maximum allowable amount company 1 can load to company 2's ship i voyaging route k, $i \in I_2$, $k \in K$

(Decision variables)
x^1_{ik}: Number of voyages of company 1's ship i in route k during planning time period, $i \in I_1$, $k \in K$
x^2_{ik}: Number of voyages of company 2's ship i in route k during planning time period, $i \in I_2$, $k \in K$
y^1_{ik}: Number of times that company 2 charters ship i of company 1 in route k, $i \in I_1$, $k \in K$
y^2_{ik}: Number of times that company 1 charters ship i of company 2 in route k, $i \in I_2$, $k \in K$
w^1_{ik}: Number of containers that company 2 loads to ship i of company 1 in route k, $i \in I_1$, $k \in K$
w^2_{ik}: Number of containers that company 1 loads to ship i of company 2 in route k, $i \in I_2$, $k \in K$
z^1_k: Number of undelivered company 1's containers in route k, $k \in K$
z^2_k: Number of undelivered company 2's containers in route k, $k \in K$

The problem can be formulated as follows.

(P)

$$\text{Min } Z_1 = \sum_{i\in I_1}\sum_{k\in K} c^1_{ik}x^1_{ik} + \sum_{i\in I_2}\sum_{k\in K} e^2_{ik}w^2_{ik} + \sum_{i\in I_2}\sum_{k\in K} f^2_{ik}y^2_{ik} + \sum_{k\in K} d^1_k z^1_k - \sum_{i\in I_1}\sum_{k\in K} e^1_{ik}w^1_{ik} \quad (1)$$

$$\text{Min } Z_2 = \sum_{i\in I_2}\sum_{k\in K} c^2_{ik}x^2_{ik} + \sum_{i\in I_1}\sum_{k\in K} e^1_{ik}w^1_{ik} + \sum_{i\in I_1}\sum_{k\in K} f^1_{ik}y^1_{ik} + \sum_{k\in K} d^2_k z^2_k - \sum_{i\in I_2}\sum_{k\in K} e^2_{ik}w^2_{ik} \quad (2)$$

$$\text{s.t. } \sum_{k\in K}\frac{1}{m^1_{ik}}x^1_{ik} \le 1, \quad i \in I_1 \quad (3)$$

$$\sum_{k\in K}\frac{1}{m_{ik}^2}x_{ik}^2 \le 1, \quad i\in I_2 \tag{4}$$

$$\sum_{i\in I_1} Q_i^1 x_{ik}^1 - \sum_{i\in I_1} w_{ik}^1 + \sum_{i\in I_2} w_{ik}^2 + z_k^1 = D_k^1, \quad k\in K \tag{5}$$

$$\sum_{i\in I_2} Q_i^2 x_{ik}^2 - \sum_{i\in I_2} w_{ik}^2 + \sum_{i\in I_1} w_{ik}^1 + z_k^2 = D_k^2, \quad k\in K \tag{6}$$

$$0 \le w_{ik}^1 \le U_{ik}^1 y_{ik}^1, \quad i\in I_1,\ k\in K \tag{7}$$

$$0 \le w_{ik}^2 \le U_{ik}^2 y_{ik}^2, \quad i\in I_2,\ k\in K \tag{8}$$

$$y_{ik}^1 \le x_{ik}^1, \quad i\in I_1,\ k\in K \tag{9}$$

$$y_{ik}^2 \le x_{ik}^2, \quad i\in I_2,\ k\in K \tag{10}$$

$$x_{ik}^1, y_{ik}^1 \ge 0,\ x_{ik}^1, y_{ik}^1:\ \text{integer}, \quad i\in I_1,\ k\in K \tag{11}$$

$$x_{ik}^2, y_{ik}^2 \ge 0,\ x_{ik}^2, y_{ik}^2:\ \text{integer}, \quad i\in I_2,\ k\in K \tag{12}$$

$$z_k^1, z_k^2 \ge 0, \quad k\in K \tag{13}$$

Objective functions (1) and (2) represent that the total costs of companies 1 and 2 should be minimized respectively. The total cost of company 1 can be calculated as the sum of transportation cost, slot chartering cost paid to company 2, fixed cost for slot chartering, and penalty cost for undelivered containers, subtracting the revenue obtained from loading the containers of company 2 to their ships. The total cost of company 2 can be calculated in the same way. Constraints (3) and (4) mean that each ship cannot voyage more than the limited number of voyage allowance. Constraints (5) and (6) present that the container transportation demand for each route should be satisfied to both companies but if it is not, penalty cost is accrued from undelivered amount. Constraints (7) and (8) express that the amount of containers loaded to the other company's ship has an upper limit. Constraints (9) and (10) show that the frequency of using the other company's ship for each route is limited to the voyage number of the other company's ship. Constraints (11)-(13) define variable types.

3. Solution Procedure.

3.1. Numerical example.

Company 1 has 20 ships which consist of 5 6,000TEU ships, 8 4,000 TEU ships, and 7 2,000TEU ships, and company 2 has 15 ships which consist of 8 4,000 TEU ships, and 7 2,000TEU ships. Tables 1 and 2 show the container transportation

Table 1. Container transportation demands for routes (unit: TEU)

Company	Route			
	1	2	3	4
1	40,000	80,000	21,000	80,000
2	40,000	30,000	30,000	40,000

Table 2. Maximum number of voyages for each ship on the route

Company	TEU	Route			
		1	2	3	4
1	6,000	3	2	3	1
	4,000	4	3	3	2
	2,000	5	5	5	2
2	4,000	3	2	1	3
	2,000	3	4	2	3

Table 3. Transportation cost

Company	TEU	Route			
		1	2	3	4
1	6,000	1,050	1,100	1,200	1,400
	4,000	850	900	1,000	1,000
	2,000	600	800	800	900
2	4,000	900	1,000	1,100	1,200
	2,000	700	850	900	950

Table 4. Slot chartering cost

Company	Route			
	1	2	3	4
1	0.8	0.5	0.7	0.6
2	3.5	1.3	2.5	2.0

Table 5. Penalty cost

Company	Route			
	1	2	3	4
1	9	3.5	6	5
2	2.4	1.5	1.8	1.5

demands for the two companies in each route and the maximum number of voyages for each ship on the route.

Transportation cost, slot chartering cost and penalty cost for undelivered containers are shown in Tables 3, 4 and 5. Slot chartering cost is assumed to be the same irrelevant to ship types.

It can be assumed that the number of undelivered containers is permitted at most 20% of container transportation demand in each route and slot chartering amount is allowable up to predetermined portion of the ship capacity.

3.2. **Results and analysis.** The numerical example problems are solved considering various sizes of slot chartering for the other company and Premium Solve Platform are used to solve these problems. The size of slot chartering is set up starting from 5% to 50% to ship's capacity at 5% intervals and under this arrangement, 10 problems are figured out. Minsum method is used to deal with two objective functions. Optimal solutions are obtained for 10 problems and are compared to the results drawn from no-alliance. The results are shown in Tables 6, 7 and 8.

Compared to no-alliance, slot exchanging alliance model definitely has larger benefit. As the permissible size of slot chartering increases, the cost reduction amount is also continuously increasing. However, it is interesting to find that cost reduction ratio decreases as the allowable size for slot chartering increases in view of each company. As shown in company 1, cost reduction ratio constantly increases until 30% of the slot chartering allowable size, but starts to decrease after 30%. Nevertheless, the total sum cost of companies 1 and 2 is steadily increasing as the chartering size increases. In this regard, an important problem arises as for how to allocate the benefit to each company.

There is a very simple and rational way to deal with this benefit allocation problem. Without alliance, the optimal costs of companies 1 and 2 are 126,400 and 68,600 and the sum is 195,000. The ratio of each cost to total sum is 65% and 35% for company 1 and company 2 respectively. According to this ratio, the total cost reduction amount is reallocated to each company as in Table 9.

Table 6. Optimal cost

	Company 1	Company 2	Sum
No alliance	126,400	68,600	195,000
5%	121,380	71,640	193,020
10%	121,890	55,320	177,210
15%	112,840	52,275	165,115
20%	99,860	48,995	148,855
25%	93,630	41,280	134,910
30%	85,930	34,290	120,220
35%	89,180	24,480	113,660
40%	92,140	17,190	109,330
45%	98,910	8,165	107,075
50%	108,050	−2,525	105,525

Table 7. Cost reduction

	Company 1	Company 2	Sum
5%	5,020	−3,040	1,980
10%	4,510	13,280	17,790
15%	13,560	16,325	29,885
20%	26,540	19,605	46,145
25%	32,770	27,320	60,090
30%	40,470	34,310	74,780
35%	37,220	44,120	81,340
40%	34,260	51,410	85,670
45%	27,490	60,435	87,925
50%	18,350	71,125	89,475

Table 8. Cost reduction ratios

	Company 1	Company 2	Sum
5%	4.0%	−4.4%	1.0%
10%	3.6%	19.4%	9.1%
15%	10.7%	23.8%	15.3%
20%	21.0%	28.6%	23.7%
25%	26.0%	39.8%	30.8%
30%	32.0%	50.0%	38.4%
35%	29.5%	64.3%	41.7%
40%	27.1%	74.9%	43.9%
45%	21.8%	88.1%	45.1%
50%	14.5%	103.7%	45.9%

4. **Conclusions.** Container slot exchange model gave a way to establish strategic alliance in liner shipping industry. The model has more progressive features than the slot chartering model. In this study, the mixed integer linear programming formulation is suggested and a numerical example is solved using Excel add-in Premium Solver Platform. 10 cases with different values from 5% to 50% of slot chartering size on the same example are also described. Regarding the optimal result drawn from using Minisum method, the total sum of cost reduction is consistently increasing. But the cost reduction ratio for company 1 only increases until a certain interval and decreases after this interval region.

Table 9. Cost reduction benefit reallocation

	Company 1	Company 2	Sum
5%	1,287	693	1,980
10%	11,564	6,226	17,790
15%	19,425	10,460	29,885
20%	29,994	16,151	46,145
25%	39,059	21,031	60,090
30%	48,607	26,173	74,780
35%	52,871	28,469	81,340
40%	55,686	29,984	85,670
45%	57,151	30,774	87,925
50%	58,159	31,316	89,475

In addition, a modest and rational way of reallocating the cost reduction benefit to each company is proposed.

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