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LOW-COMPLEXITY VBLAST MIMO DETECTION THROUGH TPE-BASED DIFFERENTIAL DETECTION ALGORITHM

Jun Huang[1], Shengneng Huang[1], Song Xing[2], Zhengquan Li[1,3] and Tiecheng Song[3]

[1]College of Information Engineering
China Jiliang University
No. 258, Xueyuan Street, Hangzhou 310018, P. R. China
hjun@cjlu.edu.cn; lzq722@sina.com

[2]Department of Information Systems
California State University
Los Angeles 90032, USA
sxing@exchange.calstatela.edu

[3]National Mobile Communications Research Laboratory
Southeast University
No. 2, Sipailou, Nanjing 210096, P. R. China
{ lzq722; songtc }@seu.edu.cn

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Abstract. *In this paper, we propose a low-complexity differential detection algorithm for a constructed VBLAST (vertical bell laboratories layered space-time) code in massive multiple-input multiple-output (MIMO) systems when neither the transmitter nor the receiver has knowledge of the channel state information (CSI). To reduce the computational complexity of the proposed detection scheme, we utilize a truncated polynomial expansion (TPE) approximating the matrix inversion. Analysis and numerical results show that the proposed detection scheme can achieve a significant reduction in computational complexity in comparison with the ordinary VBLAST MIMO detector, while maintaining a good bit error rate (BER) performance.*
Keywords: Massive MIMO, VBLAST, TPE, Differential detection

1. **Introduction.** The massive multiple-input multiple-output (MIMO) system with large number of transmit and/or receiver antennas is one of the key technologies for next-generation wireless communications, which has the potential to bring tremendous improvement in data rates and energy efficiency [1]. In order to detect the received signal correctly, the receivers need to have an accurate estimate of the channel state information (CSI). However, due to a large number of antennas, an accurate estimation of CSI is very difficult to obtain and the system overhead rises significantly in massive MIMO systems. If neither the transmitter nor the receiver has knowledge of CSI, a differential detection algorithm can be applied [3,4]. In [4], for example, a linear differential detector close to zero forcing (ZF) performance is proposed for MIMO detection. Unfortunately, the detector in [4] incurs a large computational burden due to matrix inversion computing. In massive MIMO systems, the computational complexity of traditional differential detection algorithms is even higher when the large-dimensional matrix inversion needs to be performed. In order to reduce computational burden in detection, the large-dimensional matrix inversion can be replaced by a truncated polynomial expansion (TPE) with limited number of terms [2]. In this paper, we propose a linear-complexity differential detection algorithm based on a TPE [2,6-8] for a constructed VBLAST (vertical bell laboratories layered space-time) code [5] in the case that the receiver has no prior knowledge of CSI in the massive MIMO system. Furthermore, we compare the computational complexity and

the bit error rate (BER) performance of the proposed detection scheme with the ordinary VBLAST MIMO detector.

2. **System Model.** Consider an uplink massive MIMO wireless communication system consisting of a base station with N_r receiving antennas and the user mobile terminals, each of which has N_t transmitting antennas, where $N_t \ll N_r$. We assume that the transmitted signals are processed by VBLAST approach over the wireless channel which is assumed to be quasi-static and flat block-fading with the block size of T. In the kth block, if the transmitted and received signal matrices are denoted by $\mathbf{S}^k$ with the size of $N_t \times N_t$ and $\mathbf{Y}^k$ with the size of $N_t \times N_r$, respectively, the received signal matrix at time k will be given by

$$\mathbf{Y}^k = \mathbf{S}^k\mathbf{H} + \mathbf{N}^k, \tag{1}$$

where $\mathbf{H}$ is an $N_t \times N_r$ channel fading matrix whose element $h_{n_t n_r}$ is the channel fading coefficient from the transmitting antenna n_t to the receiving antenna n_r. Further the entries of $\mathbf{H}$ are modeled as independent and identically distributed (*i.i.d.*) complex Gaussian random variables with zero mean and unit variance. In (1), $\mathbf{N}^k$ is an $N_t \times N_r$ additive noise matrix, whose elements $n^k_{n_t n_r}$ are *i.i.d.* complex Gaussian random variables with zero mean and the variance 1/(2SNR) per real dimension.

3. **TPE-Based Differential Detection.** We assume that neither the transmitter nor the receiver has the knowledge of CSI. Hence a differential modulation scheme is utilized [3,4]. Specifically, for the kth block, a VBLAST code can be constructed as

$$\mathbf{U}^k = \begin{pmatrix} s_1^k & s_{N_t}^k & s_{N_t-1}^k & \cdots & s_2^k \\ s_2^k & s_1^k & s_{N_t}^k & \cdots & s_3^k \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{N_t-1}^k & s_{N_t-2}^k & s_{N_t-3}^k & \cdots & s_{N_t}^k \\ s_{N_t}^k & s_{N_t-1}^k & s_{N_t-2}^k & \cdots & s_1^k \end{pmatrix}. \tag{2}$$

Consider two consecutive block transmissions, k and $k-1$, where the $(k-1)$th block is the reference for decoding the kth block. Then the relationship between the $(k-1)$th and the kth transmission matrices is satisfied by [3]

$$\mathbf{S}^k = \frac{\mathbf{U}^k\mathbf{S}^{k-1}}{d^{k-1}}, \tag{3}$$

where, $d^{k-1} = \sqrt{\left|s_1^{k-1}\right|^2 + \left|s_2^{k-1}\right|^2 + \cdots + \left|s_{N_t}^{k-1}\right|^2}$. Define $\mathbf{S}^0 = \mathbf{I}$ when $k = 0$.

Without loss of generality, we only study the signals received by the first receiving antenna at the base station. Hence the received signals can be explicitly written as, following from [3,4],

$$\mathbf{y}_1^k = \mathbf{S}^k\mathbf{h}_1 + \mathbf{n}_1^k = \frac{\mathbf{U}^k\mathbf{y}_1^{k-1}}{d^{k-1}} + \underbrace{\left(\mathbf{n}_1^k - \frac{\mathbf{U}^k\mathbf{n}_1^{k-1}}{d^{k-1}}\right)}_{\tilde{\mathbf{n}}_1^k}, \tag{4}$$

where $\mathbf{h}_1$ and $\mathbf{n}_1^k$ are the first column of the channel matrix $\mathbf{H}$ and the noise matrix $\mathbf{N}$, respectively. In (4), $\tilde{\mathbf{n}}_1^k$ is the additive white Gaussian noise vector in which elements are *i.i.d.* zero-mean complex Gaussian random variables with variance $\tilde{\sigma}^2 = \left[1 + (d^k)^2/(d^{k-1})^2\right](1/\gamma)$ real dimension, where γ is the input SNR.

For the sake of simplicity, we omit the subscript "1" in variable notations in (4). Then (4) becomes

$$\mathbf{y}^k = \frac{\mathbf{U}^k\mathbf{y}^{k-1}}{d^{k-1}} + \tilde{\mathbf{n}}^k. \tag{5}$$

Denote the received signal $\mathbf{y}^k$ of the kth block transmission by $\mathbf{y}^k = \left(y_1^k\ y_2^k\ \cdots\ y_{N_t}^k\right)^{\mathrm{T}}$. Then from (5), we have

$$\underbrace{d^{k-1}\begin{pmatrix} y_1^k \\ y_2^k \\ \vdots \\ y_{N_t}^k \end{pmatrix}}_{\hat{\mathbf{r}}^k} = \mathbf{R}^{k-1}\underbrace{\begin{pmatrix} s_1^k \\ s_2^k \\ \vdots \\ s_{N_t}^k \end{pmatrix}}_{\mathbf{s}^k} + \underbrace{d^{k-1}\begin{pmatrix} \tilde{\mathbf{n}}_1^k \\ \tilde{\mathbf{n}}_2^k \\ \vdots \\ \tilde{\mathbf{n}}_{N_t}^k \end{pmatrix}}_{\hat{\mathbf{n}}^k}, \tag{6}$$

where $\mathbf{R}^{k-1}$ is an $N_t \times N_t$ matrix, which can be expressed as

$$\mathbf{R}^{k-1} = \begin{pmatrix} y_1^{k-1} & y_{N_t}^{k-1} & y_{N_t-1}^{k-1} & \cdots & y_2^{k-1} \\ y_2^{k-1} & y_1^{k-1} & y_{N_t}^{k-1} & \cdots & y_3^{k-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{N_t-1}^{k-1} & y_{N_t-2}^{k-1} & y_{N_t-3}^{k-1} & \cdots & y_{N_t}^{k-1} \\ y_{N_t}^{k-1} & y_{N_t-1}^{k-1} & y_{N_t-2}^{k-1} & \cdots & y_1^{k-1} \end{pmatrix}. \tag{7}$$

We are now in position to derive an improved low-complexity linear differential detection algorithm based on the replacement of the matrix inversion with a truncated polynomial expansion. We start it by considering the ordinary MIMO detection for our constructed VBLAST code in (2) and defining $\mathbf{W}^{k-1}$ as

$$\mathbf{W}^{k-1} = \left(\mathbf{R}^{k-1\mathrm{H}}\mathbf{R}^{k-1}\right)^{-1}\mathbf{R}^{k-1\mathrm{H}}, \tag{8}$$

where $(\cdot)^{\mathrm{H}}$ denotes Hermitian operation. Obviously, the matrix $\mathbf{W}^{k-1}$ size of $N_t \times N_t$ increases with N_t. Left-multiplication of (6) by $\mathbf{W}^{k-1}$ will get the transmitted signal detected.

In massive MIMO systems, however, inversion of very large matrix in (8) incurs great computational complexity. To reduce the computational burden in detection, we use a truncated polynomial expansion to replace the matrix inversion. Specifically, it is known that if a parameter α is selected such that $0 < \alpha < 2/\max\lambda(\mathbf{X})$, where $\lambda(\mathbf{X})$ is the eigenvalue of the matrix $\mathbf{X}$, the inverse of any positive definite Hermitian matrix $\mathbf{X}$ can be expressed by a truncated polynomial expansion as $\mathbf{X}^{-1} = \alpha\big(\mathbf{I}-(\mathbf{I}-\alpha\mathbf{X})\big)^{-1} = \alpha\sum\limits_{n=0}^{\infty}(\mathbf{I}-\alpha\mathbf{X})^n$. Thus, to exploit the truncated polynomial expansion technique to approximate the inversion of $\mathbf{W}^{k-1}$ with a matrix polynomial, we first express

$$\left(\mathbf{R}^{k-1\mathrm{H}}\mathbf{R}^{k-1}\right)^{-1} = \alpha\left(\mathbf{I} - \left(\mathbf{I} - \alpha\mathbf{R}^{k-1\mathrm{H}}\mathbf{R}^{k-1}\right)\right)^{-1} = \alpha\sum_{n=0}^{\infty}\left(\mathbf{I} - \alpha\mathbf{R}^{k-1\mathrm{H}}\mathbf{R}^{k-1}\right)^n. \tag{9}$$

Then a finite series expansion $(a+b)^n = \sum\limits_{l=0}^{n}\begin{pmatrix} n \\ l \end{pmatrix} a^{n-l}b^l$ is employed in the right-hand side of (9) and the result is then plugged into (8). Considering a truncated polynomial expansion using only the first J terms in (8), we will approximate

$$\mathbf{W}^{k-1} \approx \sum_{n=0}^{J-1}\left[\alpha\sum_{l=0}^{n}\begin{pmatrix} n \\ l \end{pmatrix}(-\alpha)^l\left(\mathbf{R}^{k-1\mathrm{H}}\mathbf{R}^{k-1}\right)^l\right]\mathbf{R}^{k-1\mathrm{H}}. \tag{10}$$

Further, $\mathbf{W}^{k-1}$ in (10) can be expressed as

$$\begin{aligned}\mathbf{W}^{k-1} &= \sum_{n=0}^{J-1}\left[\alpha\sum_{l=n}^{J-1}\binom{l}{n}(-\alpha)^n\left(\mathbf{R}^{k-1\mathrm{H}}\mathbf{R}^{k-1}\right)^n\right]\mathbf{R}^{k-1\mathrm{H}} \\ &= \sum_{n=0}^{J-1}\underbrace{\left[\alpha\sum_{l=n}^{J-1}\binom{l}{n}(-\alpha)^n\right]}_{w_n}\left(\mathbf{R}^{k-1\mathrm{H}}\mathbf{R}^{k-1}\right)^n\mathbf{R}^{k-1\mathrm{H}},\end{aligned} \tag{11}$$

where $w_0, \ldots, w_{J-1}$ are scalar coefficients.

Now we examine the computational complexity of the detection. Left-multiplying (6) by (11) yields

$$\tilde{\mathbf{r}}^k = \mathbf{W}^{k-1}\hat{\mathbf{r}}^k = \sum_{n=0}^{J-1} w_n\left(\mathbf{R}^{k-1\mathrm{H}}\mathbf{R}^{k-1}\right)^n\mathbf{R}^{k-1\mathrm{H}}\hat{\mathbf{r}}^k. \tag{12}$$

If $n = 0$, i.e., $J = 1$, (12) becomes $\tilde{\mathbf{r}}^k = w_0\mathbf{R}^{k-1\mathrm{H}}\hat{\mathbf{r}}^k$, which results in a maximal ratio combining (MRC) detector. We consider the complexity calculation of the detection as the count of total number of complex multiplications, complex summations, and square-root operations. Since the complexity calculation is counted as $N_t^2 + N_t$ at each receiving antenna in this case [9], the detector thus requires $N_t^2N_r + N_tN_r$ complexity calculations.

If $1 \le n \le J-1$, denote

$$\bar{\mathbf{r}}_n^k = \left(\mathbf{R}^{k-1\mathrm{H}}\mathbf{R}^{k-1}\right)^n\mathbf{R}^{k-1\mathrm{H}}\hat{\mathbf{r}}^k. \tag{13}$$

Then (12) becomes

$$\tilde{\mathbf{r}}^k = \sum_{n=0}^{J-1} w_n\bar{\mathbf{r}}_n^k. \tag{14}$$

From (7), we know that $\mathbf{R}^{k-1\mathrm{H}}\mathbf{R}^{k-1} = \mathbf{R}^{k-1}\mathbf{R}^{k-1\mathrm{H}}$. Thus, (13) is simplified as

$$\begin{aligned}\bar{\mathbf{r}}_n^k &= \left(\mathbf{R}^{k-1\mathrm{H}}\mathbf{R}^{k-1}\right)\left(\mathbf{R}^{k-1\mathrm{H}}\mathbf{R}^{k-1}\right)^{n-1}\mathbf{R}^{k-1\mathrm{H}}\hat{\mathbf{r}}^k \\ &= \left(\mathbf{R}^{k-1\mathrm{H}}\mathbf{R}^{k-1}\right)\bar{\mathbf{r}}_{n-1}^k \\ &= \mathbf{R}^{k-1}\left(\mathbf{R}^{k-1\mathrm{H}}\bar{\mathbf{r}}_{n-1}^k\right).\end{aligned} \tag{15}$$

Hence the detection of transmitted signals in (15) can be conducted in iterations, which enables the parallel computing over multi-core processing in mobile terminals. Further, for $1 \le n \le J-1$, each iteration needs $2N_t^2 + N_t$ complexity calculations. Then the complexity calculation is counted as $(J-1)2N_t^2 + (J-1)N_t$ for each receiving antenna [6,7]. The entire computational complexity is thus $(J-1)2N_t^2N_r + (J-1)N_tN_r$ calculations for all receiving antennas. For comparison, the complexity calculation for each receiving antenna at the ordinary detector as in (8) for our constructed VBLAST code is counted and is shown as $2N_t^3 + 4N_t^2$ according to [9]. The entire complexity calculation is thus $2N_t^3N_r + 4N_t^2N_r$ calculations for all receiving antennas.

Based on the above analysis, Table 1 summarizes the complexity calculations in terms of the total number of complex multiplications, complex summations, and square-root operations on the detectors we have discussed. It shows that our proposed TPE-based scheme with $J = 1$, $J = \min(N_r, N_t) = N_t$, and the ordinary detector in (8) are listed in ascending order of their computational complexities.

4. **Numerical Results.** The numerical simulations are conducted to evaluate the performance of the proposed TPE-based differential detection algorithm. We set $N_r = 50$ and $N_t = 5$, and use QPSK constellation in the simulations. Figure 1 illustrates the BER

TABLE 1. Computational complexity calculations

Algorithms	Computational complexity
TPE, $J = 1$	$N_t^2 N_r + N_t N_r$
TPE, $J = N_t$	$2N_t^3 N_r - N_t^2 N_r - N_t N_r$
Ordinary detector in (8)	$2N_t^3 N_r + 4N_t^2 N_r$

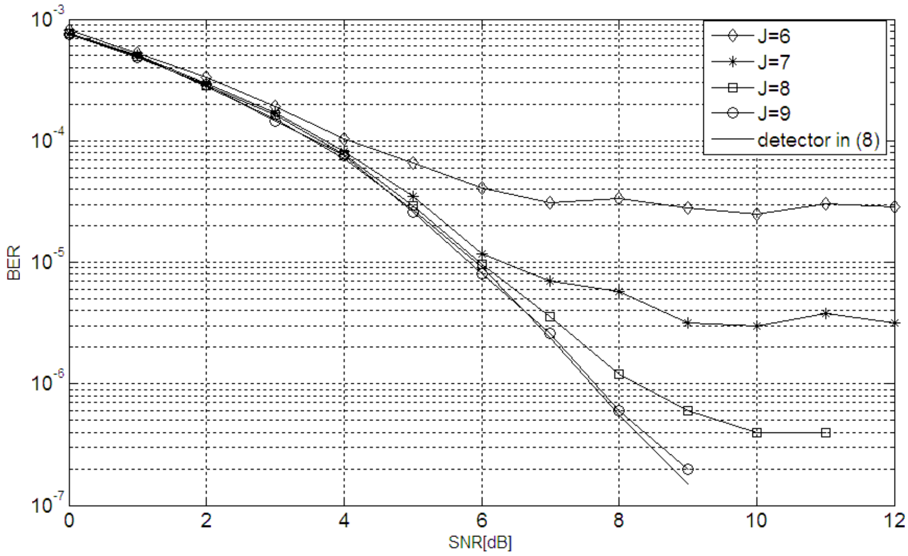

FIGURE 1. BER vs. SNR with different order of J $(N_t = 5, N_r = 50)$

performance of the proposed differential detection algorithm with several TPE order values of J for our constructed VBLAST code, which is also compared with the ordinary detection algorithm in (8) in the same figure.

From the figure, we see that the larger value of J is chosen, the better BER performance is achieved by the proposed TPE-based differential detection algorithm for our constructed VBLAST code. Specifically, the BER performance of our proposed scheme is almost the same as the ordinary VBLAST MIMO detection in (8) when $J = 9$. Furthermore, for any TPE order of J, the proposed TPE-based detection and the ordinary detection in (8) perform almost identically at the low SNR values. However, the performance gap between our scheme with lower TPE order and the ordinary detection algorithm in (8) increases with SNR. Hence the trade-off between the BER performance and the computational complexity needs to be considered when the proposed TPE-based differential detection algorithm for our constructed VBLAST code is applied in massive MIMO systems.

5. **Conclusions.** In the paper we propose an improved differential detection algorithm with reduced linear complexity when neither the transmitter nor the receiver has knowledge of the channel state information in massive MIMO systems. To reduce the computational complexity, we utilize a truncated polynomial expansion to avoid large-dimensional matrix inversion in detection for our constructed VBLAST code. Analysis and numerical results show that the proposed scheme can achieve significant reduction in computational complexity compared with the ordinary VBLAST MIMO detector, while maintaining good BER performance.

For further work, we plan to improve our proposed scheme by considering the integration of coherent detection [8]. The channel estimation will be also exploited for this improvement of massive MIMO detection.

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REFERENCES

[1] S. Noh, M. D. Zoltowski, Y. Sung and D. J. Love, Pilot beam pattern design for channel estimation in massive MIMO systems, *IEEE Journal of Selected Topics in Signal Processing*, vol.8, no.5, pp.787-801, 2014.

[2] N. Shariati, E. Björnson, M. Bengtsson and M. Debbah, Low-complexity polynomial channel estimation in large-scale MIMO with arbitrary statistics, *IEEE Journal of Selected Topics in Signal Processing*, vol.8, no.5, pp.815-830, 2014.

[3] Y. Zhu and H. Jafarkhani, Differential modulation based on quasi-orthogonal codes, *IEEE Trans. Wireless Communications*, vol.4, no.6, pp.3018-3030, 2005.

[4] S. J. Alabed, J. M. Paredes and A. B. Gershman, A low complexity decoder for quasi-orthogonal space time block codes, *IEEE Trans. Wireless Communications*, vol.10, no.3, pp.988-994, 2011.

[5] A. Kammoun, A. Müller, E. Björnson and M. Debbah, Linear precoding based on polynomial expansion: Large-scale multi-cell MIMO systems, *IEEE Journal of Selected Topics in Signal Processing*, vol.8, no.5, pp.861-875, 2014.

[6] A. Müller, A. Kammoun, E. Björnson and M. Debbah, Efficient linear precoding for massive MIMO systems using truncated polynomial expansion, *IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM)*, 2014.

[7] A. Müller, A. Kammoun, E. Björnson and M. Debbah, Linear precoding based on polynomial expansion: Reducing complexity in massive MIMO, *EURASIP Journal on Wireless Communications and Networking*, 2016.

[8] K. A. Alnajjar, P. J. Smith and G. K. Woodward, Low complexity V-BLAST for massive MIMO, *Australian Communicaitons Theory Workshop (AusCTW)*, 2014.

[9] R. Hunger, Floating point operations in matrix-vector calculus, *Technical Report V1.3*, Munich University of Technology, 2007.