ICIC Express Letters ICIC International ©2016 ISSN 1881-803X
Volume **10**, Number **11**, November **2016** pp. 2653–2660

STUDY ON APPROXIMATION ABILITY OF GENERAL EXPRESSION OF NONLINEAR AUTOREGRESSIVE MODEL

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Received April 2016; accepted July 2016

Abstract. *Time-series analysis relies on the appropriate statistical modeling, so time-series model is crucial. The existing linear and nonlinear time-series models have certain limitations. So a novel time-series model, General Expression for Nonlinear Autoregressive (GNAR) model, was introduced. The approximation degree of Bernstein polynomial in m-dimension simple space form was discussed based on approximation degree of one dimension Bernstein polynomial to prove approximation ability of GNAR model for systems in mathematics. And the GNAR models were applied to tracking for chaotic system and vibration system. The experiment results show that GNAR model demonstrates good approximation ability.*
Keywords: GNAR model, Nonlinear time series, Approximation ability, Bernstein polynomial

1. **Introduction.** Time series analysis technology is a kind of system identification method which can establish models based on the inherent law of data with no need for system inputs, so it has important applications in natural and social science field of industrial process control, economy and biomedical engineering, etc. [1-3]. Time series analysis depends on the proper statistical modeling; thus time-series model has become a very important issue nowadays. Traditional linear time series models (ARMA model and AR, MA and ARIMA model) have made a lot of progresses in modeling algorithm [4-7]. However, most systems in the practical engineering have the nonlinear characteristics. So the linear models based on the stationarity assumptions are not applicable to identify the nonlinearity and irregularity of data sequences.

The existing modeling methods of nonlinear time series can be divided into two types. The first is traditional time series models combined with some nonlinear algorithms [8,9] and the other applies heuristic techniques, such as genetic algorithm, neural network and support vector machine (SVM), to system modeling and forecasting [10-13]. These heuristic methods can approximate systems accurately, but they still have shortcomings; for example, the genetic algorithm and neural network modeling methods need a great number of training samples, and SVM model parameters are hard to determine.

So a novel time-series model, General Expression for Nonlinear Autoregressive (GNAR) model, is introduced in this paper. First of all, the approximation degree of Bernstein polynomial in m-dimension simple space form is discussed based on approximation degree of one dimension Bernstein polynomial to prove approximation ability of GNAR model for systems in mathematics. And then the GNAR models are applied to tracking for chaotic system and vibration system. Prior knowledge is not essential when adopting GNAR model for system modeling because there are linear and nonlinear terms in this model, so the modeling process is simplified. The experiment results show that the model can accurately approximate the linear and nonlinear time-series data and chaotic sequence.

Therefore, GNAR model proposed can be applied to system identification, data tracking and system prediction, etc.

2. Proving of Approximation Ability of GNAR Model for Systems.

2.1. GNAR model structure. The expression of GNAR model is:

$$\begin{aligned} w_t &= \sum_{i_1=1}^{\infty} \alpha_{i_1} w_{t-i_1} + \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty} \alpha_{i_1,i_2} w_{t-i_1} w_{t-i_2} + \cdots + \sum_{i_1=1}^{\infty} \cdots \sum_{i_p=1}^{\infty} \alpha_{i_1,\cdots,i_p} \prod_{\tau=1}^{\infty} w_{t-i_\tau} + a_t \\ &= \sum_{j=1}^{\infty}\sum_{i_1=1}^{\infty} \cdots \sum_{i_j=1}^{\infty} \alpha_{i_1,\cdots,i_j} \prod_{\tau=1}^{j} w_{t-i_\tau} + a_t \end{aligned} \tag{1}$$

where w_{t-i} presents the system observed data at the moment of $t-i$, $i = 0, 1, 2, \cdots$; α_{i_1}, $\cdots$, α_{i_1,i_2}, $\cdots$ are model parameters; a_t is white noise with zero mean.

When modeling in engineering, the model order usually is a limited value, so Equation (1) is rewritten as:

$$\begin{aligned} w_t &= \sum_{i_1=1}^{n_1} \alpha_{i_1} w_{t-i_1} + \sum_{i_1=1}^{n_2}\sum_{i_2=1}^{n_2} \alpha_{i_1,i_2} w_{t-i_1} w_{t-i_2} + \cdots + \sum_{i_1=1}^{n_p} \cdots \sum_{i_p=1}^{n_p} \alpha_{i_1,\cdots,i_p} \prod_{\tau=1}^{p} w_{t-i_\tau} + a_t \\ &= \sum_{j=1}^{p}\sum_{i_1=1}^{n_j} \cdots \sum_{i_j=1}^{n_j} \alpha_{i_1,\cdots,i_j} \prod_{\tau=1}^{j} w_{t-i_\tau} + a_t \end{aligned} \tag{2}$$

where p is the polynomial order; $\sum_{i_1=1}^{n_1} \alpha_{i_1} w_{t-i_1}$ is the first order linear term, α_{i_1} ($i_1 = 1, 2, \cdots, n_1$) are linear coefficients; $\sum_{i_1=1}^{n_2}\sum_{i_2=1}^{n_2} \alpha_{i_1,i_2} w_{t-i_1} w_{t-i_2}$ is the second order nonlinear term, α_{i_1,i_2} ($i_1, i_2 = 1, 2, \cdots, n_2$) are the second order nonlinear coefficients; $\sum_{i_1=1}^{n_p} \cdots \sum_{i_p=1}^{n_p} \alpha_{i_1,\cdots,i_p} \prod_{\tau=1}^{p} w_{t-i_\tau}$ is the pth order nonlinear term, $\alpha_{i_1,\cdots,i_p}$ ($i_1, \cdots, i_p = 1, 2, \cdots, n_p$) are the pth order nonlinear coefficients; n_j ($j = 1, 2, \cdots, p$) are the memory steps of every linear or nonlinear term. The model can be written as GNAR (p; $n_1, n_2, \cdots, n_p$) in abbreviation.

2.2. Convergence of GNAR model. For GNAR model shown in Equation (2), if the observation data w_t are bounded, there will be a constant $C > 0$ and $|w_t| \le C$ ($t = 1, 2, \cdots$). Hence:

$$\begin{aligned} & |\mathrm{GNAR}(p; n_1, n_2, \cdots, n_p)| \\ = & \left| \sum_{i_1=1}^{n_1} \alpha_{i_1} w_{t-i_1} + \sum_{i_1=1}^{n_2}\sum_{i_2=1}^{n_2} \alpha_{i_1,i_2} w_{t-i_1} w_{t-i_2} + \cdots + \sum_{i_1=1}^{n_p} \cdots \sum_{i_p=1}^{n_p} \alpha_{i_1,\cdots,i_p} \prod_{\tau=1}^{p} w_{t-i_\tau} \right| \\ \le & \sum_{i_1=1}^{n_1} \alpha_{i_1} |w_{t-i_1}| + \sum_{i_1=1}^{n_2}\sum_{i_2=1}^{n_2} \alpha_{i_1,i_2} |w_{t-i_1}| |w_{t-i_2}| + \cdots + \sum_{i_1=1}^{n_p} \cdots \sum_{i_p=1}^{n_p} \alpha_{i_1,\cdots,i_p} \prod_{\tau=1}^{p} |w_{t-i_\tau}| \\ = & \sum_{j=1}^{p}\sum_{i_1=1}^{n_j} \cdots \sum_{i_j=1}^{n_j} \alpha_{i_1,\cdots,i_j} \prod_{\tau=1}^{j} |w_{t-i_\tau}| \\ \le & \sum_{j=1}^{p}\sum_{i_1=1}^{n_j} \cdots \sum_{i_j=1}^{n_j} \alpha_{i_1,\cdots,i_j} C^j \end{aligned} \tag{3}$$

Equation (3) indicates GNAR model must be convergence with the bounded system outputs. A stable system with bounded inputs will give out bounded outputs in engineering, so GNAR model established for a stable system output sequence can be convergence.

2.3. **Approximation ability of GNAR model.** Approximation degree of one dimension Bernstein polynomial is as follows [14]:

$$\max_{0\le x\le 1}|g(x)-B_n(x)|\le K\omega\left(\frac{1}{\sqrt{n}}\right)\quad n=1,2,\cdots \tag{4}$$

where $g(x)$ is a kind of continuous function defined on $[0,1]$; $B_n(x)$ is Bernstein polynomial, $B_n(x)=\sum\limits_{i=0}^{n}g(\frac{i}{n})C_n^ix^i(1-x)^{n-i}$; $K=\frac{4306+837\sqrt{6}}{5832}=1.089887\cdots$; $\omega(\delta)$ is the functional moduli of continuity for $g(x)$, $\omega(\delta)=\max\limits_{|x-y|\le\delta}|g(x)-g(y)|\,(x,y\in[0,1])$.

According to the time series analysis modeling strategy, the system outputs w_t can be expressed as follows:

$$w_t=f(w_{t-1},w_{t-2},w_{t-3},\cdots) \tag{5}$$

Suppose the observation sequence length is m, then f in Equation (5) indicates a function in m-dimension simple space form.

Suppose without loss of generality (original data can be normalized):

$$\begin{cases}\sum\limits_{t=1}^{m}w_t\le 1\\ w_t\ge 0\end{cases}\quad(t=1,2,\cdots,m) \tag{6}$$

$B_n(f,w)$ presents Bernstein polynomial in m-dimension simple space form:

$$B^f_{n_1,n_2,\cdots,n_m}(w_{t-1},w_{t-2},\cdots,w_{t-m})=\sum_{i_1=0}^{n_1}\cdots\sum_{i_m=0}^{n_m}f\left(\frac{i_1}{n_1},\frac{i_2}{n_2},\cdots,\frac{i_m}{n_m}\right)p^{i_1,i_2,\cdots,i_m}_{n_1,n_2,\cdots,n_m}W \tag{7}$$

where $p^{i_1,i_2,\cdots,i_m}_{n_1,n_2,\cdots,n_m}=C^{i_1}_{n_1}C^{i_2}_{n_2}\cdots C^{i_m}_{n_m}$; $W=w^{i_1}_{t-1}(1-w_{t-1})^{n_1-i_1}w^{i_2}_{t-2}(1-w_{t-2})^{n_2-i_2}\cdots w^{i_m}_{t-m}(1-w_{t-m})^{n_m-i_m}$.

Approximation degree can be obtained:

$$\begin{aligned}&\left|B^f_{n_1,n_2,\cdots,n_m}(w_{t-1},w_{t-2},\cdots,w_{t-m})-f(w_{t-1},w_{t-2},\cdots,w_{t-m})\right|\\ \le\ &\sum_{i_1=0}^{n_1}\cdots\sum_{i_m=0}^{n_m}\left|f\left(\frac{i_1}{n_1},\cdots,\frac{i_m}{n_m}\right)-f(w_{t-1},\cdots,w_{t-m})\right|p^{i_1,i_2,\cdots,i_m}_{n_1,n_2,\cdots,n_m}W\\ \le\ &\sum_{i_1=0}^{n_1}\cdots\sum_{i_m=0}^{n_m}(1+\lambda_1+\cdots+\lambda_m)\omega(\delta_1,\delta_2,\cdots,\delta_m)p^{i_1,i_2,\cdots,i_m}_{n_1,n_2,\cdots,n_m}W\\ \le\ &\omega(\delta_1,\delta_2,\cdots,\delta_m)\left(1+\sum_{l=1}^{m}\sum_{i_1=0}^{n_1}\cdots\sum_{i_m=0}^{n_m}\lambda_lp^{i_1,i_2,\cdots,i_m}_{n_1,n_2,\cdots,n_m}W\right)\end{aligned} \tag{8}$$

where $\lambda_l=\lfloor(w_{t-l}-i_l/n_l)/\delta_l\rfloor)$ $(l=1,2,\cdots,m)$, and $\lfloor\cdot\rfloor$ presents taking the maximum positive integer less than the value in this symbol; $\omega(\delta_1,\delta_2,\cdots,\delta_m)$ presents the functional moduli of continuity for $f(w_{t-1},w_{t-2},\cdots,w_{t-m})$.

Suppose $l=1$:

$$\begin{aligned}&\sum_{i_1=0}^{n_1}\cdots\sum_{i_m=0}^{n_m}\lambda_1p^{i_1,i_2,\cdots,i_m}_{n_1,n_2,\cdots,n_m}W\\ =\ &\sum_{i_1=0}^{n_1}\cdots\sum_{i_m=0}^{n_m}\lfloor(w_{t-1}-i_1/n_1)/\delta_1\rfloor C^{i_1}_{n_1}C^{i_2}_{n_2}\cdots C^{i_m}_{n_m}w^{i_1}_{t-1}(1-w_{t-1})^{n_1-i_1}\cdots w^{i_m}_{t-m}(1-w_{t-m})^{n_m-i_m}\\ =\ &\sum_{\lambda_1\ge1}\lfloor(w_{t-1}-i_1/n_1)/\delta_1\rfloor C^{i_1}_{n_1}w^{i_1}_{t-1}(1-w_{t-1})^{n_1-i_1}\\ \le\ &\sum_{|w_{t-1}-i_1/n_1|>1/\sqrt{n_1}}\sqrt{n_1}|w_{t-1}-i_1/n_1|C^{i_1}_{n_1}w^{i_1}_{t-1}(1-w_{t-1})^{n_1-i_1}\quad(\text{let }\delta_1=1/\sqrt{n_1})\\ \le\ &K-1\\ \le\ &0.089887\cdots\end{aligned} \tag{9}$$

By that analogy, setting $\delta_2 = 1/\sqrt{n_2}$, $\delta_3 = 1/\sqrt{n_3}$, $\cdots$, $\delta_m = 1/\sqrt{n_m}$, the following equation can be obtained:

$$\begin{aligned}&\left|B^f_{n_1,n_2,\cdots,n_m}(w_{t-1}, w_{t-2}, \cdots, w_{t-m}) - f(w_{t-1}, w_{t-2}, \cdots, w_{t-m})\right| \\ \leq\ & [1 + m(K-1)]\omega\left(\frac{1}{\sqrt{n_1}}, \cdots, \frac{1}{\sqrt{n_m}}\right)\end{aligned} \tag{10}$$

The functional moduli of continuity has the character that if $\delta \to 0$, f tends to be constant and $\omega(\delta_1, \delta_2, \cdots, \delta_m) \to 0$. So we can conclude:

$$\lim_{n_1,n_2,\cdots,n_m\to\infty}\left|B^f_{n_1,n_2,\cdots,n_m}(w_{t-1}, w_{t-2}, \cdots, w_{t-m}) - f(w_{t-1}, w_{t-2}, \cdots, w_{t-m})\right| = 0 \tag{11}$$

Equation (11) shows that Bernstein polynomial can be utilized to approximate the functions in multidimension simple space form and Equation (10) gives its approximation degree.

Expand $W = w^{i_1}_{t-1}(1-w_{t-1})^{n_1-i_1}w^{i_2}_{t-2}(1-w_{t-2})^{n_2-i_2}\cdots w^{i_m}_{t-m}(1-w_{t-m})^{n_m-i_m}$ in $B_n(f, w)$, merge the similar items and set coefficients of w_{t-i} as α_{i_1} $(i_1 = 1, 2, \cdots, n_1)$, coefficients of w_{t-i} as α_{i_1} $(i_1 = 1, 2, \cdot\cdot\cdot, n_1)$, $\cdots$, then GNAR model expressed in Equation (2) can be obtained. Therefore, we can rewrite Equation (11) as follows:

$$\lim_{n_j\to\infty}\left|\sum_{j=1}^{p}\sum_{i_1=1}^{n_j}\cdots\sum_{i_j=1}^{n_j}\alpha_{i_1,\cdots,i_j}\prod_{\tau=1}^{j}w_{t-i_\tau} - f(w_{t-1}, w_{t-2}, \cdots, w_{t-m})\right| = 0 \tag{12}$$

3. Numerical Example.

3.1. Approximation for chaotic system.

Chaos describes a kind of deterministic dynamic system with random characteristics which appears frequently in nature and human activity. Hence here gives two GNAR model tracking experiments of chaotic time series.

(1) ICMIC mapping

The expression of ICMIC mapping is:

$$w_t = \sin\left(\frac{a}{w_{t-1}}\right) \quad a \in (0, \infty) \quad t = 1, 2, \cdots \tag{13}$$

Control parameters a and initial values w_1 of ICMIC mapping are generated randomly as 0.06/0.160 or 0.06/0.165, and the system output sequences are illustrated in Figure 1. The GNAR model tracking results are presented in Table 1 and Table 2.

As can be seen from the tables, GNAR model can fit ICMIC mapping well, and when the control parameter is unchanged, GNAR model parameters are basically stable. GNAR

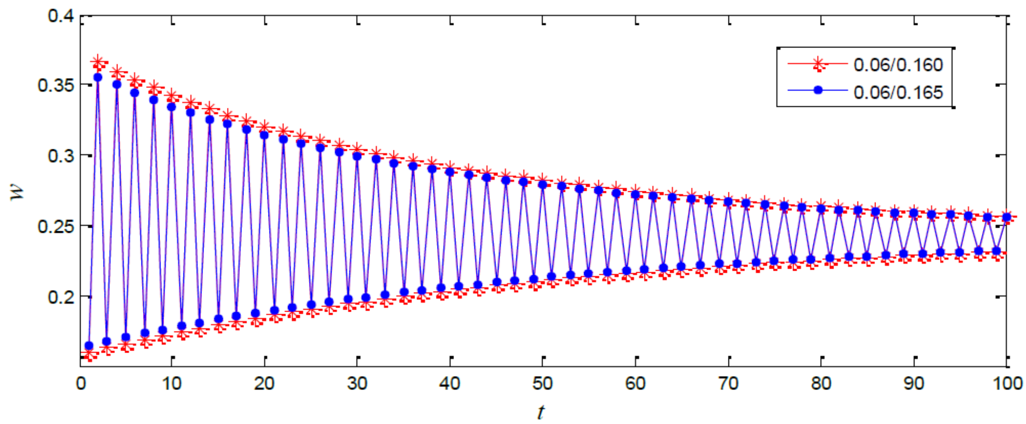

Figure 1. ICMIC sequences

TABLE 1. Fitting GNAR model for ICMIC sequences (a/w_1 is 0.06/0.160)

	GNAR model	Residual sum of squares
GNAR(2;1,1)	$w_t = -7.186w_{t-1}^2 + 2.853w_{t-1}$	9.35×10^{-2}
GNAR(2;2,1)	$w_t = -0.107w_{t-1}^2 + 0.062w_{t-1} + 0.964w_{t-2}$	2.76×10^{-6}
GNAR(2;2,2)	$w_t = -0.041w_{t-1}^2 - 0.417w_{t-1}w_{t-2} - 0.207w_{t-2}^2 + 0.060w_{t-1} + 1.102w_{t-2}$	1.03×10^{-10}
GNAR(3;1,1,1)	$w_t = 41.548w_{t-1}^3 - 29.492w_{t-1}^2 + 5.741w_{t-1}$	7.80×10^{-3}
GNAR(3;2,1,1)	$w_t = 0.810w_{t-1}^3 - 0.671w_{t-1}^2 + 0.170w_{t-1} + 0.946w_{t-2}$	2.11×10^{-7}

TABLE 2. Fitting GNAR model for ICMIC sequences (a/w_1 is 0.06/0.165)

	GNAR model	Residual sum of squares
GNAR(2;1,1)	$w_t = -7.316w_{t-1}^2 + 2.874w_{t-1}$	7.02×10^{-2}
GNAR(2;2,1)	$w_t = -0.107w_{t-1}^2 + 0.062w_{t-1} + 0.964w_{t-2}$	1.74×10^{-6}
GNAR(2;2,2)	$w_t = -0.041w_{t-1}^2 - 0.414w_{t-1}w_{t-2} - 0.206w_{t-2}^2 + 0.060w_{t-1} + 1.103w_{t-2}$	4.82×10^{-11}
GNAR(3;1,1,1)	$w_t = 42.561w_{t-1}^3 - 29.964w_{t-1}^2 + 5.766w_{t-1}$	5.10×10^{-3}
GNAR(3;2,1,1)	$w_t = 0.815w_{t-1}^3 - 0.670w_{t-1}^2 + 0.168w_{t-1} + 0.947w_{t-2}$	1.15×10^{-7}

model fitting precision of ICMIC mapping is greatly influenced by memory step length, so the memory step length of linear and nonlinear terms is at least 2.

(2) Chebyshev mapping

The expression of Chebyshev mapping is:

$$w_t = \cos(k \cos^{-1} w_{t-1}) \quad k = 4 \quad w \in [-1, 1] \quad t = 1, 2, \cdots \tag{14}$$

Initial values w_1 of Chebyshev mapping are generated randomly as 0.360 or 0.365 and the system output sequences are illustrated in Figure 2.

The tracking curves of GNAR(3;10,6,6) for Chebyshev mapping which have the residual sum of squares of 3.444 and 1.494 are shown in Figure 3 and Figure 4.

According to real function analysis, one-dimension chaotic systems including Logistic, Sine, ICMIC, Tent, Bernouilli shift and Chebyshev mapping, etc. can be regarded as real functions with the value domain of w_t and definitional domain of w_{t-1} ($t = 2, 3, \cdots$). Based on real function characteristics, chaotic mapping can be expanded into power series

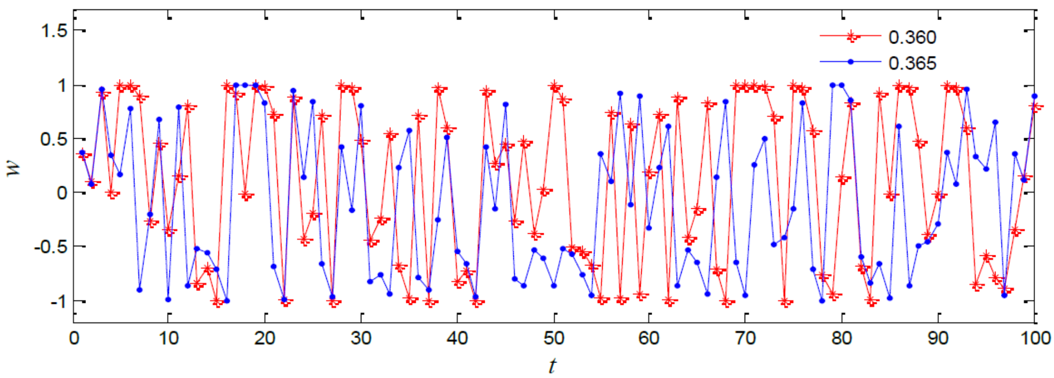

FIGURE 2. Chebyshev sequences

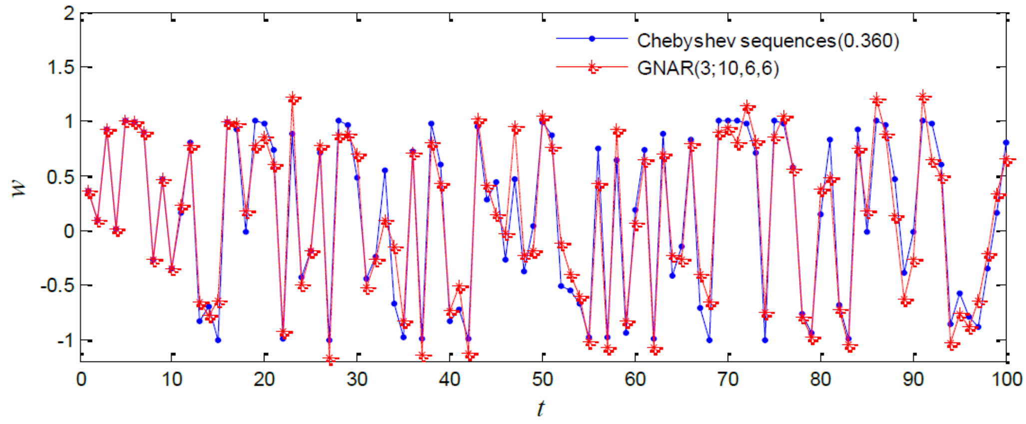

FIGURE 3. GNAR(3;10,6,6) fitting of Chebyshev sequences (initial value is 0.360)

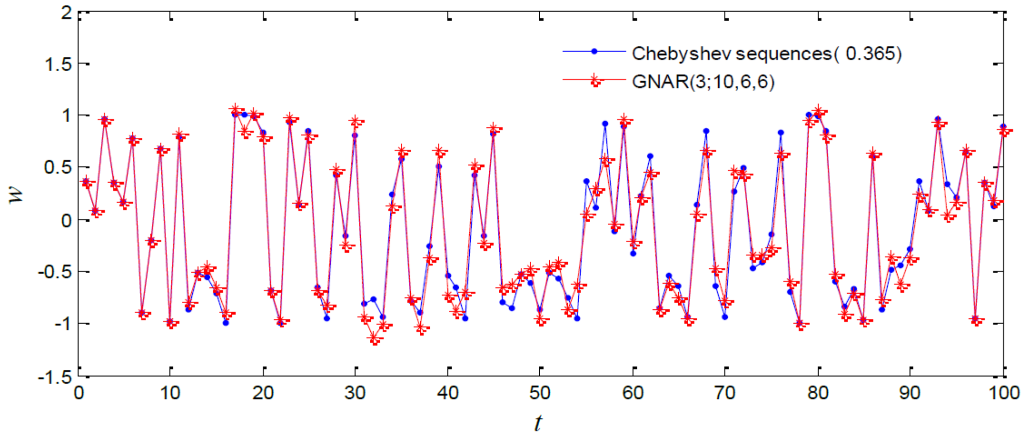

FIGURE 4. GNAR(3;10,6,6) fitting of Chebyshev sequences (initial value is 0.365)

of w_{t-1} whose expression is consistent with GNAR model. This also illustrates GNAR model can approximate one-dimension chaotic systems accurately on the other hand. Other examples can be found in reference [15].

3.2. **Approximation for vibration system.** Vibration signals contain abundant information about the status of the mechanical system. They are widely used in parameter testing, quality evaluation, condition monitoring and fault diagnosis for they have the advantages including easy obtaining, wide range diagnostic and convenient to set up online monitoring system. So here gives experiment to test GNAR model approximation ability for vibration system.

Taking the vibration system excited by square wave for an example, its system dynamics equation is as follows:

$$\ddot{x} + 2\mu\dot{x} + \omega_0^2 x + \varepsilon\omega_0^2 x^3 = F(t) \tag{15}$$

A model describing the Equation (15) is established in MATLAB Simulink platform, as shown in Figure 5 and the square wave impulse is shown in Figure 6. Setting the system parameters $\mu = 0.2$, $\omega_0 = 1.5$ and $\varepsilon = 10.5$, the time series can be obtained shown in Figure 7 with sampling length of 50 s and sampling frequency of 100 Hz.

GNAR(3;2,0,1) is adopted to fit the system outputs and the model tracking curve is shown in Figure 8. The experiment result displays a high fitting precision of GNAR model and the average modeling relative error is 0.4203%.

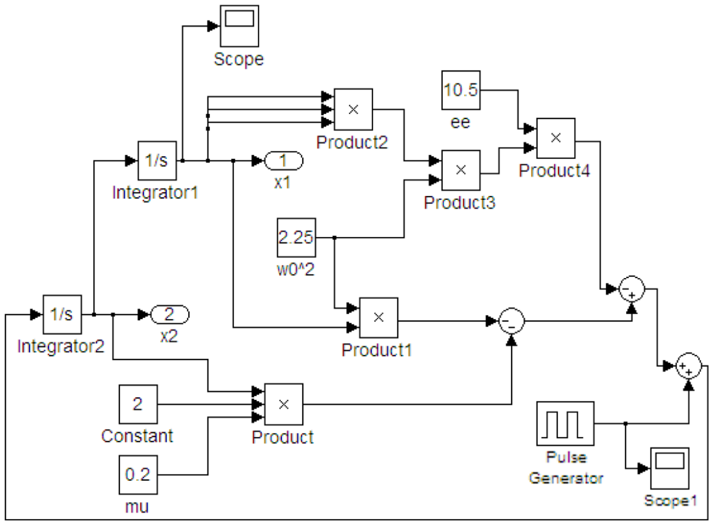

FIGURE 5. Model describing Equation (15) in MATLAB

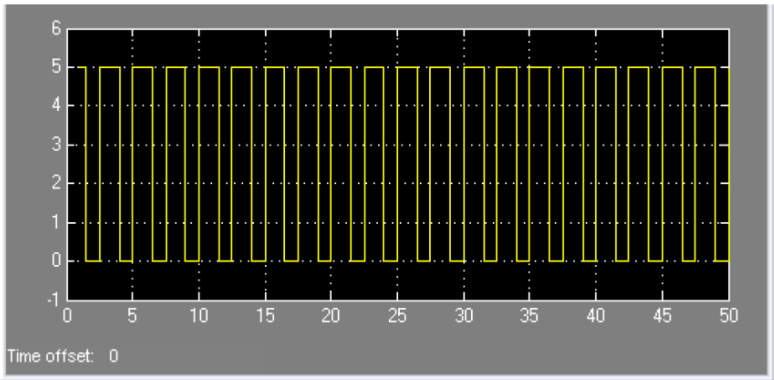

FIGURE 6. Square wave impulse

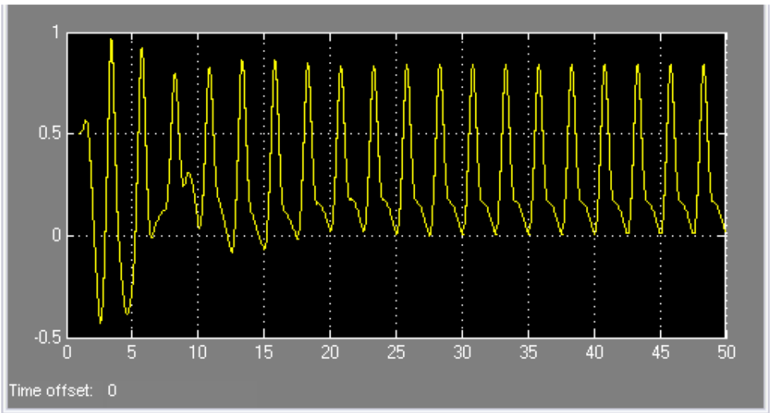

FIGURE 7. Time series in oscilloscope of MATLAB Simulink platform

4. **Conclusions.** In this paper, the approximation ability of GNAR model for systems is proved in mathematics based on approximation degree of Bernstein polynomial in m-dimension simple space form. GNAR model can reflect the internal motion system, and it is easy to interpret and understand. Tracking experiments of chaotic system and vibration system show its good approximation ability.

Because of the complexity of the nonlinear system, there are no uniform and normative theory and index to evaluate nonlinear system modeling so far. As a novel nonlinear

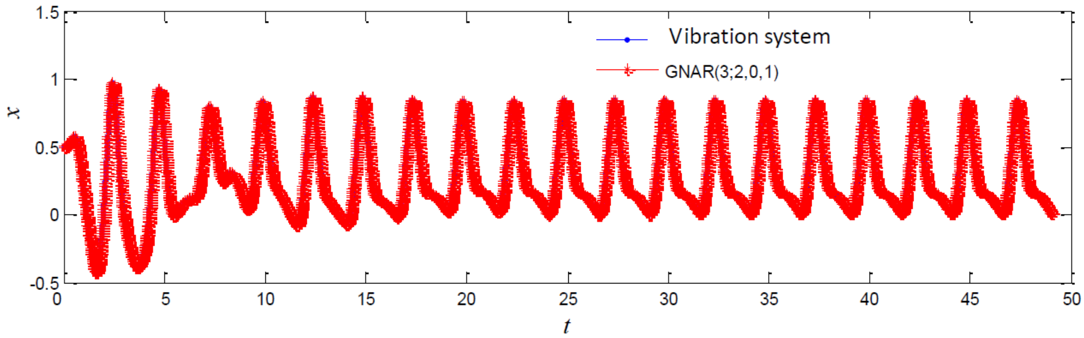

FIGURE 8. GNAR(3;2,0,1) fitting of vibration system

mathematical model, GNAR model still has to be discussed in modeling process, such as how to determine the model orders, parameters estimation and model adaptability testing or goodness discrimination. These are our further research contents.

Acknowledgment. This work is supported by the National Natural Science Foundation of China (51305194, 51405221), Qing Lan Project (2016) and Jiangsu Province Science and Technology Support project (BE2014142). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

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