

A VALUE ANALYSIS OF SEARCH ENGINE ADVERTISING PAGE BASED ON THE THEORY OF TWO-SIDED MARKETS

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ABSTRACT. *With the fast development of search engine advertising, the value of search engine advertising page has become a hot field of research for scholars. Based on the theory of two-sided markets, this paper considers the competition between pages and the influence of multi-attribute of advertisement on the intrinsic value of advertising position, and concludes that compared with separate pricing of advertising position, the price of advertising position and platform income by comprehensive pricing of advertising position is higher.*

Keywords: Search engine, Advertising, Value analysis, Two-sided markets

1. Introduction. When users are searching for relevant keywords, according to certain correlation algorithm, search engines will display search advertising links of search advertising firms who bought it on the page, which will be browsed by search users. Chinese online advertising market scale reached 209.7 billion Yuan in 2015, increased by 36.1% compared with that of previous year, of which the search engine market reached 80.8 billion Yuan, accounting for 33.8%¹. At the same time, the academic research on search advertising has been widely carried out. Search engine advertising research involves at least two of the three main bodies: search engine, firm and audience. From a research perspective, there are theoretical researches and empirical researches. Studies of search engines are mainly theoretical researches and large number of studies focus on auction of the advertising position. For example: Varian (2007) analyzed the game equilibrium of advertising auction of Google and Yahoo [1]. B. Edelman and M. Schwarz (2007) analyzed the similarities and differences of the online advertising auction between “generalized second price” (GSP) auction and vickrey-Clarke-Groves (VCG) auction mechanism. GSP auction is a strategy which had no advantage. He designed an England auction equivalent to VCG auction [2]. Using the panel data from Google, A. Ghose and S. Yang (2009) built an empirical analysis model about the relationship of different sponsored search measures and the ranking advertising, such as click-through rates, conversion rate and cost per click [3]. From the perspective of firm, researches mainly focus on the firm advertising strategy. For example: W. Dou et al. (2010) analyzed how to use the search engine marketing to brand positioning [4]. H. Jafarzadeh et al. (2015) analyzed the main factors influencing the enterprises in choosing the search engine advertising using theory of TPB (Theory of Planned Behavior), TAM (Technology Acceptance Model) and UTAUT (Unified Theory of Acceptance and Use of Technology) [5]. From the aspect of audience, researches

¹Data sources: IResearch, <http://www.askci.com/news/chanye/2016/01/14/153037jxia.shtml>

focus on the audience advertising search behavior. For example: adopting the experiment method, K. C. Yang (2004) studied how motivation will influence the consumer's behavior of using online advertising [6]. With the deep-going of research, scholars begin to notice that the research of search engine advertising must consider the interaction of search engine, firm and audience. The researches of this field mainly focus on how the search engines, firm and audience together will decide the intrinsic value of the search engine advertising position. L. Z. Xu and A. Whinston (2008), for example, built a game theory model that firm competed for advertising position and for audience through price competition, and obtained the endogenous value of advertising position accordingly [7]. L. Z. Xu et al. (2009) established theory game model consisting of search engine, firm and audience based on the search behavior of user and concluded that direct price competition occurs only between firms adjacent to each other [8]. Using economic model J. Chen and J. Stallaert (2014) analyzed the influence of online advertising on publishers, firms and society after adopting the user positioning technology [9]. The existing literature focuses on 3 problems: (1) the pricing strategy of the advertising position, by auction or by other methods; (2) the advertising strategy of the firms, some considering the audiences searching behavior; (3) the endogenous price of the advertising position.

We can find that the position distribution of search engine advertising is made through the way of auction after the literature research, but firms must estimate the intrinsic value of each advertising position before an auction to determine the bidding strategy. Intrinsic value is determined by interaction of search engine, firm and audience; thus it forms a two-sided market consisting of search engine for mediation, firm and the audience for each side. The endogenous value of the advertising position is the basis to the pricing strategy and advertising strategy, so it is the key of this issue. However, existing studies of the endogenous value of the advertising position do not match the reality. First, the existing literature considers only advertising position competition in the same page by ignoring competition between pages; secondly, the current literature does not consider the network effect which is the nature of the search engine advertising. So the author of this paper tries to analyze the intrinsic value of search engine advertising page by constructing economic theory model while considering the page competition and the network effect of advertisement using the theory of two-sided markets.

This article is structured as follows: first, introduction; second, the basic model assumptions; third, basic model construction and analysis; fourth, a comparative analysis of Model (I) and Model (II); fifth, conclusion.

2. The Basic Model Assumptions. For convenience of analysis, in general, the basic assumptions of the model are as follows. Firstly, we assume that there are two similar search engine advertising pages, page a and page b , respectively. Advertising positions are classified into two categories: the one near the top of the page referred as position i , the one close to the bottom of the page called position j . Secondly, hypotheses about the two-sided markets are as follows: firm side is called side 1 and audience side is called side 2. According to the actual situation, the firm side is single-homing and the audience side is multi-homing. We assume that t_1 is the matching cost of side 1 and t_2 is the matching cost of side 2, and matching cost of side 1 is equal to that of side 2, namely, $t_1 = t_2$. Thirdly, the hypotheses of network effects are as follows: we assume that there are network effects between groups. The strength of network effects for firm side gathering in the position i to audience side is α_{12}^i ; the strength of network effects for firm side gathering in the position j to audience side is α_{12}^j . The strength of network effects for audience side gathering in the position i to firms side is α_{21}^i ; the strength of network effects for audience side gathering in the position j to firm side is α_{21}^j . Thus, the strength of network effects between groups is $\alpha_{12}^i, \alpha_{12}^j, \alpha_{21}^i, \alpha_{21}^j$. On the premise of not influencing the analysis conclusion, we assume that the strength of network effects between groups

are equal to α . Fourth, we assume that the use value of side 1 firm advertising on the position i is \overline{u}_1^i ; the use value of side 1 firm advertising on the position j is \overline{u}_1^j ; the use value of side 2 audience browsing advertisement on the position i is \overline{u}_2^i ; the use value of side 2 audience browsing advertisement on the position j is \overline{u}_2^j . For side 2, the attribute of advertisement is multiple. They are attribute of quality m and the attribute of price k . Accordingly, $\overline{u}_2^i = \overline{u}_{2m}^i + \overline{u}_{2k}^i$, $\overline{u}_2^j = \overline{u}_{2m}^j + \overline{u}_{2k}^j$ (where \overline{u}_{2m}^i is the utility of quality attribute m of side 2 audience browsing advertisement on the position i , \overline{u}_{2m}^j is the utility of quality attribute m of side 2 audience browsing advertisement on the position j ; \overline{u}_{2k}^i is the utility of price attribute k of side 2 audience browsing advertisement on the position i , \overline{u}_{2k}^j is the utility of price attribute k of side 2 audience browsing advertisement on the position j). According to the actual situation, $\overline{u}_{2m}^i > \overline{u}_{2m}^j$, $\overline{u}_{2k}^j > \overline{u}_{2k}^i$. p_1^i, p_1^j represent advertising cost of firm advertising on position i and position j , respectively.

3. Basic Model Construction and Analysis. In this session, this article constructs two models, in which p_1^i, p_1^j are determined endogenously. They are the value of the advertising positions.

3.1. Model (I): Search engine advertising markets which price the advertising position comprehensively. Search engine advertising market is an oligopoly competition market when competition between pages is fierce. For a selected page, Model (I) diagram is shown as in Figure 1. In oligopoly competition market, the firm can choose to advertise on this page, it can also choose to advertise on another page. In this model the firms in $[x_1, x_2]$ do not advertise on the selected page, they advertise on the competitors'.

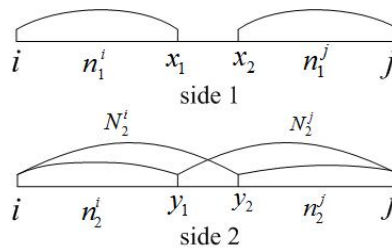


FIGURE 1. A search engine advertising market which prices advertising position comprehensively

Description of the notations in Figure 1: i , the advertising position i ; j , the advertising position j ; n_1^i , the firms who advertise on i position and x_1 is the edge firm; n_1^j , the firms who advertise on j position and x_2 is the edge firm; n_2^i , the audiences who browse the i position only and y_1 is the edge audience; N_2^i , the audiences who browse the i position and browse the i and j position and y_2 is the edge audience; n_2^j , the audiences who browse the j position only and y_2 is the edge audience; N_2^j , the audiences who browse the j position and browse the i and j position and y_1 is the edge audience.

The utility of firm advertising on position i is

$$u_1^i = \overline{u}_1^i - p_1^i - tx + \alpha y_2 \tag{1}$$

The utility of firm is 0 if firm advertises between x_1 and x_2 and firm will advertise on other platform. So,

$$\overline{u}_1^i - p_1^i - tx_1 + \alpha y_2 = 0 \tag{2}$$

The utility of firm advertising on position j is

$$u_1^j = \overline{u}_1^j + \alpha(1 - y_1) - t(1 - x) - p_1^j \tag{3}$$

The utility of firm is 0 if firm advertises between x_1 and x_2 and firm will advertise on other platform. So,

$$\overline{u_1^j} + \alpha(1 - y_1) - t(1 - x_2) - p_1^j = 0 \tag{4}$$

The utility of audience browsing advertisement on position i is

$$u_2^i = \overline{u_2^i} + \alpha x_1 - ty \tag{5}$$

The utility of audience browsing advertisement both on position i and j is

$$u_2^{ij} = \overline{u_{2m}^i} + \overline{u_{2k}^j} + \alpha[x_1 + (1 - x_2)] - t \tag{6}$$

The utility of audience browsing advertisement on position j is

$$u_2^j = \overline{u_{2m}^j} + \overline{u_{2k}^j} + \alpha(1 - x_2) - t(1 - y) \tag{7}$$

Let: $V_1^d = \overline{u_1^i} - \overline{u_1^j}$, $V_2^d = \overline{u_{2k}^j} - \overline{u_{2k}^i} + \overline{u_{2m}^i} - \overline{u_{2m}^j}$, $V_{2k}^d = \overline{u_{2k}^j} - \overline{u_{2k}^i}$, $V_{2m}^d = \overline{u_{2m}^i} - \overline{u_{2m}^j}$.

Profit function is

$$\begin{aligned} \pi &= n_1^i \cdot p_1^i + n_1^j \cdot p_1^j \\ &= \frac{t}{t^2 - \alpha^2} \left[\overline{u_1^i} p_1^i - (p_1^i)^2 + \frac{\alpha V_{2m}^d}{t} p_1^i + \overline{u_1^j} p_1^j - (p_1^j)^2 + \frac{\alpha V_{2k}^d}{t} p_1^j \right] \end{aligned} \tag{8}$$

π is the function of (p_1^i, p_1^j) , take the first order derivative of p_1^i and p_1^j and the reaction function is

$$\frac{\partial \pi}{\partial p_1^i} = \frac{t}{t^2 - \alpha^2} \left(\overline{u_1^i} - 2p_1^i + \frac{\alpha V_{2m}^d}{t} \right), \quad \frac{\partial \pi}{\partial p_1^j} = \frac{t}{t^2 - \alpha^2} \left(\overline{u_1^j} - 2p_1^j + \frac{\alpha V_{2k}^d}{t} \right)$$

Let the reaction function be zero, and the Bertrand-Nash equilibrium price is

$$p_1^i = \frac{1}{2} \overline{u_1^i} + \frac{\alpha}{2t} V_{2m}^d, \quad p_1^j = \frac{1}{2} \overline{u_1^j} + \frac{\alpha}{2t} V_{2k}^d$$

When $t > \alpha$, $\frac{\partial \pi^2}{\partial (p_1^i)^2} < 0$, $\frac{\partial \pi^2}{\partial (p_1^j)^2} < 0$, so p_1^i and p_1^j is the maximum.

Let Formula (5) equal Formula (6), and we can get y_1 , $N_2^j = 1 - y_1$; let Formula (6) equal Formula (7), we can get y_2 , $N_2^i = y_2$; let y_2 substitute into Formula (2), and we can get $x_1 n_1^i = x_1$; let y_1 substitute into Formula (4), and we can get x_2 , $n_1^j = 1 - x_2$.

Lemma 3.1. *The user scale of firm advertising on position i is $n_1^i = \frac{t}{2(t^2 - \alpha^2)} \left[\overline{u_1^i} + \frac{\alpha}{t} V_{2m}^d \right]$, the user scale of firm advertising on position j is $n_1^j = \frac{t}{2(t^2 - \alpha^2)} \left[\overline{u_1^j} + \frac{\alpha}{t} V_{2k}^d \right]$, the user scale of audience browsing advertisement on position i is $N_2^i = \frac{V_{2m}^d}{t} + \frac{\alpha}{2(t^2 - \alpha^2)} \left[\overline{u_1^i} + \frac{\alpha}{t} V_{2m}^d \right]$, the user scale of audience browsing advertisement on position j is $N_2^j = \frac{V_{2k}^d}{t} + \frac{\alpha}{2(t^2 - \alpha^2)} \left[\overline{u_1^j} + \frac{\alpha}{t} V_{2k}^d \right]$, the price of position i of platform is $p_1^i = \frac{1}{2} \overline{u_1^i} + \frac{\alpha}{2t} V_{2m}^d$, the price of position j of platform is $p_1^j = \frac{1}{2} \overline{u_1^j} + \frac{\alpha}{2t} V_{2k}^d$, and the total income of platform is $\pi = \frac{t}{4(t^2 - \alpha^2)} \left[\left(\overline{u_1^i} + \frac{\alpha}{t} V_{2m}^d \right)^2 + \left(\overline{u_1^j} + \frac{\alpha}{t} V_{2k}^d \right)^2 \right]$.*

3.2. Model (II): Search engine advertising markets which price the advertising position respectively. When x_1 overlaps with x_2 , the utility of firm advertising on position i is

$$u_1^i = \overline{u_1^i} - p_1^i - tx + \alpha N_2^i \tag{9}$$

The utility of firm advertising on position j is

$$u_1^j = \overline{u_1^j} - p_1^j - t(1 - x) + \alpha N_2^j \tag{10}$$

The utility of audience browsing advertisement on position i is

$$u_2^i = \overline{u_2^i} + \alpha n_1^i - ty \tag{11}$$

The utility of audience browsing advertisement both on position i and j is

$$u_2^{ij} = \overline{u_{2m}^i} + \overline{u_{2k}^j} + \alpha - t \tag{12}$$

The utility of audience browsing advertisement on position j is

$$u_2^j = \overline{u_2^j} + \alpha n_1^j - t(1 - y) \tag{13}$$

Profit functions are

$$\begin{aligned} \pi^i &= n_1^i \cdot p_1^i \\ &= \left[\frac{1}{2} + \frac{tV_1^d}{2(t^2 - \alpha^2)} + \frac{\alpha}{2(t^2 - \alpha^2)} (V_{2m}^d - V_{2k}^d) + \frac{p_1^j t}{2(t^2 - \alpha^2)} - \frac{p_1^i t}{2(t^2 - \alpha^2)} \right] \cdot p_1^i \end{aligned} \tag{14}$$

$$\begin{aligned} \pi^j &= n_1^j \cdot p_1^j \\ &= \left[\frac{1}{2} - \frac{t}{2(t^2 - \alpha^2)} V_1^d - \frac{\alpha}{2(t^2 - \alpha^2)} (V_{2m}^d - V_{2k}^d) - \frac{p_1^j t}{2(t^2 - \alpha^2)} \right. \\ &\quad \left. + \frac{p_1^i t}{2(t^2 - \alpha^2)} \right] \cdot p_1^j \end{aligned} \tag{15}$$

π^i is the function of p_1^i , and take the first order derivative of p_1^i ; π^j is the function of p_1^j , take the first order derivative of p_1^j , and the reaction function is

$$\begin{aligned} \frac{\partial \pi_1}{\partial p_1^i} &= \frac{1}{2} + \frac{t}{2(t^2 - \alpha^2)} V_1^d + \frac{\alpha}{2(t^2 - \alpha^2)} (V_{2m}^d - V_{2k}^d) + \frac{p_1^j t}{2(t^2 - \alpha^2)} - \frac{p_1^i t}{(t^2 - \alpha^2)} \\ \frac{\partial \pi_2}{\partial p_1^j} &= \frac{1}{2} - \frac{t}{2(t^2 - \alpha^2)} V_1^d - \frac{\alpha}{2(t^2 - \alpha^2)} (V_{2m}^d - V_{2k}^d) + \frac{p_1^i t}{2(t^2 - \alpha^2)} - \frac{p_1^j t}{(t^2 - \alpha^2)} \end{aligned}$$

Let the reaction functions be zero respectively, and the Bertrand-Nash equilibrium price is

$$p_1^i = \frac{t^2 - \alpha^2}{t} + \frac{1}{3} V_1^d + \frac{\alpha}{3t} (V_{2m}^d - V_{2k}^d), \quad p_1^j = \frac{t^2 - \alpha^2}{t} - \frac{1}{3} V_1^d - \frac{\alpha}{3t} (V_{2m}^d - V_{2k}^d)$$

When $t > \alpha$, $\frac{\partial \pi^i}{\partial (p_1^i)^2} < 0$, $\frac{\partial \pi^j}{\partial (p_1^j)^2} < 0$, so p_1^i and p_1^j are the maximum.

Let Formula (11) equal Formula (12), and we can get y_1 , $N_2^j = 1 - y_1$; let formula (12) equal Formula (13), and we can get y_2 , $N_2^i = y_2$; let Formula (9) equal Formula (10), and we can get n_1^i , $n_1^j = 1 - n_1^i$.

Lemma 3.2. *The user scale of firm advertising on position i is $n_1^i = \frac{1}{2} + \frac{t}{6(t^2 - \alpha^2)} V_1^d + \frac{\alpha}{6(t^2 - \alpha^2)} (V_{2m}^d - V_{2k}^d)$, the user scale of firm advertising on position j is $n_1^j = \frac{1}{2} - \frac{t}{6(t^2 - \alpha^2)} V_1^d - \frac{\alpha}{6(t^2 - \alpha^2)} (V_{2m}^d - V_{2k}^d)$, the user scale of audience browsing advertisement on position i is $N_2^i = \frac{\alpha}{2t} + \frac{V_{2m}^d}{t} + \frac{\alpha}{6(t^2 - \alpha^2)} V_1^d + \frac{\alpha^2}{6t(t^2 - \alpha^2)} (V_{2m}^d - V_{2k}^d)$, the user scale of audience browsing advertisement on position j is $N_2^j = \frac{\alpha}{2t} + \frac{V_{2k}^d}{t} - \frac{\alpha}{6(t^2 - \alpha^2)} V_1^d - \frac{\alpha^2}{6t(t^2 - \alpha^2)} (V_{2m}^d - V_{2k}^d)$, the price of position i of platform is $p_1^i = \frac{t^2 - \alpha^2}{t} + \frac{1}{3} V_1^d + \frac{\alpha}{3t} (V_{2m}^d - V_{2k}^d)$, the price of position j of platform is $p_1^j = \frac{t^2 - \alpha^2}{t} - \frac{1}{3} V_1^d - \frac{\alpha}{3t} (V_{2m}^d - V_{2k}^d)$, the total income of platform is $\pi = \frac{(t^2 - \alpha^2)}{t} + \frac{t}{9(t^2 - \alpha^2)} [V_1^d + \frac{\alpha}{t} (V_{2m}^d - V_{2k}^d)]^2$.*

4. A Comparative Analysis of Model (I) and Model (II). Comparing the platform price of Model (I) and Model (II) under different market structures, the use value of firm \overline{u}_1^i advertising on position i is much bigger than the difference of utility V_1^d between firms advertising on position i and j , and the use value of firm \overline{u}_1^i advertising on position i is also much bigger than the difference of utility V_2^d between audience browsing on position i and j . So we get Proposition 4.1.

Proposition 4.1. *The price and income of search platform which prices the advertising position comprehensively is higher than that when pricing the advertising position respectively.*

When search engine advertising platform prices the advertising position, it should not pursue the maximum income of only one advertising position blindly, but it should consider all positions. This ensures that the search engine advertising platform will get overall maximum advertising revenue. For example, Baidu prices the advertising position respectively; Sogou prices the advertising position comprehensively. In 2015, the revenue per market share of Baidu is 0.815 billion dollars compared to the Sogou's 0.866. (Data source: 2015 annual global digital advertising Market Research Report)

Proposition 4.2. *When the intensity of cross network effects between the two-sided users increase, the price of advertising position and the total income of the platform will be increased accordingly.*

Proof:
$$\frac{\partial p_1^i}{\partial \alpha} = \frac{V_{2m}^d}{2t} > 0, \quad \frac{\partial p_1^j}{\partial \alpha} = \frac{V_{2k}^d}{2t} > 0,$$

$$\frac{\partial \pi}{\partial \alpha} = \frac{\left(\overline{tu}_1^i + \alpha V_{2m}^d\right) \left(\alpha \overline{u}_1^i + tV_{2m}^d\right) + \left(\overline{tu}_1^j + \alpha V_{2k}^d\right) \left(\alpha \overline{u}_1^j + tV_{2k}^d\right)}{2(t^2 - \alpha^2)^2} > 0$$

So search engine advertising platform can take measures to increase the intensity between cross network effects of two-sided users in order to increase the viscosity between two-sided users and to improve the service experience of two-sided users.

When the difference of quality utility V_{2m}^d and price utility V_{2k}^d for audience browsing position i and j increases, the price and total income of platform will increase accordingly. So we get Proposition 4.3.

Proposition 4.3. *When the difference of quality utility and price utility increases, the price of advertising position and total income of search engine platform will increase.*

Proof:
$$\frac{\partial p_1^i}{\partial V_{2m}^d} = \frac{\alpha}{2t} > 0, \quad \frac{\partial p_1^j}{\partial V_{2k}^d} = \frac{\alpha}{2t} > 0,$$

$$\frac{\partial \pi}{\partial V_{2m}^d} = \frac{t\alpha \overline{u}_1^i + \alpha^2 V_{2m}^d}{2t(t - \alpha)(t + \alpha)}, \quad \frac{\partial \pi}{\partial V_{2k}^d} = \frac{t\alpha \overline{u}_1^j + \alpha^2 V_{2k}^d}{2t(t - \alpha)(t + \alpha)}$$

When $t > \alpha$, $\frac{\partial \pi}{\partial V_{2m}^d} > 0$, $\frac{\partial \pi}{\partial V_{2k}^d} > 0$.

Search engine advertising platform can arrange advertisement with bigger difference of quality on different advertising positions, so the platform can get more income. For example, compared with advertising only shoes on different advertising positions and advertising shoes and socks on different advertising positions, the quality utility of the latter is bigger and search engine advertising platform can get more income.

5. Conclusion. Based on the two-sided markets this paper built two theoretical models to analyze the endogenous value of the advertising position of search engine advertising page. Through the equilibrium results of comparative static analysis, this article draws the following conclusions.

(1) Comparing with search engine advertising markets which price advertising position respectively, the price and income of platform will be higher when it prices advertising position comprehensively.

(2) The strength of network effects between groups will affect the revenue of the platform. The greater the network effect is, the greater the revenue of the advertising platform will be.

(3) When audiences browse the advertising position, the difference of the quality utility and the difference of the price utility impact the revenue of the search engine advertising platform. The greater the difference in the quality utility or price utility is, the greater the income of the search engine advertising platform will be.

In this article another advertising page is set exogenous; in the future research we can make it endogenous in the model. Another possible direction is to verify the model proposed in this article through simulation.

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