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ON-DEMAND TYPE FEEDBACK CONTROLLER FOR SELF-TUNING GENERALIZED MINIMUM VARIANCE CONTROL IN STATE-SPACE REPRESENTATION

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Abstract. *This paper proposes on-demand type feedback controller for self-tuning generalized minimum variance control (GMVC) in state-space representation using coprime factorization. GMVC can be extended by using coprime factorization, and the extended controller introduces a new design parameter. The parameter can change the characteristics of the extended controller without changing the closed-loop characteristics. In order to design safe systems, strong stability systems are effective because both the closed-loop system and its controller are stable. So the parameter mentioned above is useful to design such a system. Moreover, focusing on feedback signal, the extended controller can adjust the magnitude of the feedback signal. It means that the proposed controller has the ability to make the magnitude of the feedback signal be zero when the control objective was achieved. In other words, the feedback signal of the proposed method emerges on demand of achieving the control objective. Therefore, this paper explores the design method of on-demand type feedback controller using self-tuning GMVC in state-space representation. A numerical example is given in order to check the characteristics of the proposed method.*

Keywords: On-demand type feedback controller, Self-tuning controller, Generalized minimum variance control, Coprime factorization

1. **Introduction.** GMVC has been proposed by Clarke and others [1]. GMVC is one of the control methods for application in industry. This control method can be designed by an evaluation index called as generalized output which is selected to make the closed-loop system stable. The control law is derived by minimizing the variance of generalized output. Once the generalized output is selected, the derived controller cannot be re-designed unless the poles of closed-loop system are changed. In the case of considering the application in industry, it is desirable for both of the closed-loop system and the controller to be stable in view point of safety. That is, even if the closed-loop characteristic has been designed, it is desirable that the flexibility of re-designing the controller characteristic remains because of designing safe systems. Authors have proposed the extended GMVC design method [2, 3]. Compared to conventional GMVC, the extended method newly introduces a design parameter by coprime factorization [4, 5]. In the method, the controller's poles can be re-designed by the parameter without changing the poles of closed-loop system. Therefore, a strong stability system, which means that both of closed-loop system and controller are stable, and can be obtained by re-designing stable controller. Although the authors have proposed such a design method [6] and a concept of strong stability rate [7, 8] by

coprime factorization and showed that strong stability systems can be obtained, and the previous researches have not focused on feedback signal clearly. Under the assumption that the controlled plant is stable, the research about strong stability rate has focused on a stable open-loop output. For example, if the value of strong stability rate becomes 1, the controlled output becomes equal to reference signal in the steady state whether the feedback loop is cut or not. This situation indicates that the control objective is achieved and the feedback signal is not demanded (it means that the feedback signal becomes zero) in the steady state. In other words, new concept controller, whose feedback signal emerges according to the demand to make the controlled output follow the reference signal and disappears if the controlled output becomes equal to the reference signal, can be considered by coprime factorization. Although the authors have proposed on-demand type feedback controller for GMVC using coprime factorization through transfer function approach [9] and state-space representation [10], the self-tuning controller in state-space representation has not been proposed. Therefore, this paper proposes on-demand type feedback controller in state-space representation for self-tuning GMVC. The advantages of the proposed on-demand type feedback controller for self-tuning GMVC in state-space representation compared with previous ones are the facility of extension to multi-input multi-output systems and the adaptation ability to model uncertainty. A numerical example is shown in order to check the characteristics of the proposed controller.

This paper is organized as follows. Section 2 describes problem statement and self-tuning GMVC in state-space representation. Section 3 extends self-tuning GMVC through coprime factorization and gives the proposed controller. Section 4 shows a numerical example to explore the characteristics of on-demand type feedback controller. Section 5 concludes this paper.

Notations. This paper assumes that the controlled plant is stable. z^{-1} means backward shift operator $z^{-1}y(t) = y(t-1)$. $A[z^{-1}]$ and $A(z^{-1})$ mean polynomial and rational function with z^{-1} respectively. Steady state gain $A(1)$ of transfer function is calculated as $z^{-1} = 1$ under the assumption that signals such as input and output for system does not change with regard to time t.

2. Self-Tuning GMVC in State-Space Representation.

The following single-input single-output system is considered.

$$A\left[z^{-1}\right]y(t) = z^{-k_m}B\left[z^{-1}\right]u(t) + \xi(t) \quad (t = 0, 1, 2, \cdots) \tag{1}$$

where $A\left[z^{-1}\right] = 1 + a_1 z^{-1} + \cdots + a_n z^{-n}$ and $B[z^{-1}] = b_0 + b_1 z^{-1} + \cdots + b_{n-1} z^{-(n-1)}$ are assumed to be coprime. $u(t)$ and $y(t)$ are the input and the output respectively. k_m is the time delay and $\xi(t)$ is a white Gaussian noise with zero mean. The control objective is that the output $y(t)$ has a desirable response to reference signal $r(t)$. To achieve this objective, the conventional GMVC minimizes the objective function $J = E\left[\Phi^2(t+k_m)\right]$. J is averaged over the noise $\xi(t), \xi(t-1), \cdots$. Moreover, $\Phi(t+k_m) = P\left[z^{-1}\right]y(t+k_m) + Q[z^{-1}]u(t) - R[z^{-1}]r(t)$ is named as generalized output. The design polynomials $P[z^{-1}]$, $Q[z^{-1}]$ and $R[z^{-1}]$ are assumed to be constant ($P[z^{-1}] = 1$, $Q[z^{-1}] = q_0$ and $R[z^{-1}] = r_0$) in this paper. In order to obtain GMVC law in state-space representation, Equation (1) is expressed as the following observable canonical form.

$$\boldsymbol{x}(t+1) = \boldsymbol{A}_p \boldsymbol{x}(t) + \boldsymbol{b}_p u(t) - \boldsymbol{a}_p \xi(t) \tag{2}$$

$$y(t) = \boldsymbol{c}_p^{\mathrm{T}} \boldsymbol{x}(t) + \xi(t) \tag{3}$$

$$\boldsymbol{A}_p = \begin{bmatrix} -\boldsymbol{a}_p & \begin{matrix} \boldsymbol{I}_{n+k_m-2} \\ \boldsymbol{0}_{1\times(n+k_m-2)} \end{matrix} \end{bmatrix}, \quad \boldsymbol{a}_p = \left[a_1, \cdots, a_n\ \boldsymbol{0}_{1\times(k_m-1)}\right]^{\mathrm{T}}$$

$$\boldsymbol{b}_p = \left[\boldsymbol{0}_{1\times(k_m-1)}\ b_0, \cdots, b_{n-1}\right]^{\mathrm{T}}, \quad \boldsymbol{c}_p = \left[1,\ \boldsymbol{0}_{1\times(n+k_m-2)}\right]^{\mathrm{T}}$$

From (2) and (3), $y(t+k_m)$ can be given as

$$y(t+k_m) = \boldsymbol{c}_p^{\mathrm{T}} \boldsymbol{A}_p^{k_m} \boldsymbol{x}(t) + b_0 u(t) + \sum_{i=0}^{k_m-1} \boldsymbol{c}_p^{\mathrm{T}} \boldsymbol{A}_p^{k_m-1-i} \boldsymbol{a}_p \xi(t+i) + \xi(t+k_m) \tag{4}$$

The output prediction $\hat{y}(t+k_m|t)$ can be defined by eliminating the future values of noise from (4).

$$\hat{y}(t+k_m|t) = \boldsymbol{c}_p^{\mathrm{T}} \boldsymbol{A}_p^{k_m} \boldsymbol{x}(t) + b_0 u(t) \tag{5}$$

Then the k_m-ahead predicted value $\hat{\Phi}(t+k_m|t)$ of the generalized output is given by

$$\hat{\Phi}(t+k_m|t) = \boldsymbol{c}_p^{\mathrm{T}} \boldsymbol{A}_p^{k_m} \boldsymbol{x}(t) + (b_0 + q_0) u(t) - r_0 r(t) \tag{6}$$

Because $\hat{\Phi}(t+k_m|t)$ and the noise term in $\Phi(t+k_m)$ have no correlation with each other, the control input $u(t)$ which minimizes J is obtained by choosing $u(t)$ as $\hat{\Phi}(t+k_m|t) = 0$.

$$u(t) = \frac{1}{b_0 + q_0} \left(r_0 r(t) - \boldsymbol{c}_p^{\mathrm{T}} \boldsymbol{A}_p^{k_m} \hat{\boldsymbol{x}}(t) \right) \tag{7}$$

In order to construct a conventional self-tuning controller, an adaptive observer is introduced, which gives the estimated value of state $\boldsymbol{x}(t)$ and the identified plant parameters. Therefore, the controlled plants (2) and (3) are firstly deformed as follows.

$$\boldsymbol{x}(t+1) = \boldsymbol{F}\boldsymbol{x}(t) + (-\boldsymbol{a}_p + \boldsymbol{f}) y(t) + \boldsymbol{b}_p u(t) \tag{8}$$

$$y(t) = \boldsymbol{c}_p^{\mathrm{T}} \boldsymbol{x}(t) \tag{9}$$

$$\boldsymbol{F} = \begin{bmatrix} -\boldsymbol{f} & \begin{matrix} \boldsymbol{I}_{n+k_m-2} \\ \boldsymbol{0}_{1\times(n+k_m-2)} \end{matrix} \end{bmatrix}, \quad \boldsymbol{f} = [f_1, \cdots, f_n \ \boldsymbol{0}_{1\times(k_m-1)}]^{\mathrm{T}}$$

$\boldsymbol{F}$ is chosen to be stable and the eigenvalues are the roots of the polynomial $f[z^{-1}] = 1 + f_1 z^{-1} + \cdots + f_n z^{-n}$. Next, $\boldsymbol{R}_1(t), \boldsymbol{R}_2(t) \in R^{(n+k_m-1)\times(n+k_m-1)}$ are defined as

$$\boldsymbol{R}_1(t+1) = \boldsymbol{F}\boldsymbol{R}_1(t) + \boldsymbol{I} y(t), \quad \boldsymbol{R}_1(0) = 0$$

$$\boldsymbol{R}_2(t+1) = \boldsymbol{F}\boldsymbol{R}_2(t) + \boldsymbol{I} u(t), \quad \boldsymbol{R}_2(0) = 0$$

Using $\boldsymbol{R}_1(t)$ and $\boldsymbol{R}_2(t)$, (8) and (9) are expressed by the following equations.

$$\boldsymbol{x}(t) = \boldsymbol{R}(t)\boldsymbol{\theta} \tag{10}$$

$$y(t) = \boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\theta} \tag{11}$$

$$\boldsymbol{R}(t) = [\boldsymbol{R}_1(t) \ \boldsymbol{R}_2(t)], \quad \boldsymbol{\zeta}^{\mathrm{T}}(t) = \left[\boldsymbol{c}_p{}^{\mathrm{T}} \boldsymbol{R}_1(t) \ \boldsymbol{c}_p{}^{\mathrm{T}} \boldsymbol{R}_2(t) \right], \quad \boldsymbol{\theta}^{\mathrm{T}} = \left[(-\boldsymbol{a}_p + \boldsymbol{f})^{\mathrm{T}} \ \boldsymbol{b}_p{}^{\mathrm{T}} \right]$$

where the terms with the initial state value $\boldsymbol{x}(0)$ are removed from Equations (10) and (11), since $\boldsymbol{x}(0)$ decreases exponentially.

Replacing the plant parameter vector $\boldsymbol{\theta}$ with the identified plant parameter vector $\hat{\boldsymbol{\theta}}(t)$ in Equations (10) and (11), the adaptive observer is obtained as

$$\hat{\boldsymbol{x}}(t) = \boldsymbol{R}(t)\hat{\boldsymbol{\theta}}(t) \tag{12}$$

$$\hat{y}(t) = \boldsymbol{\zeta}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t) \tag{13}$$

The parameter identification law is given by

$$\hat{\boldsymbol{\theta}}(t) = \hat{\boldsymbol{\theta}}(t-1) - \boldsymbol{\Gamma}(t-1)\boldsymbol{\zeta}(t)\varepsilon(t), \quad \varepsilon(t) = \frac{\boldsymbol{\zeta}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t-1) - y(t)}{1 + \boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\Gamma}(t-1)\boldsymbol{\zeta}(t)}$$

$$\boldsymbol{\Gamma}(t) = \boldsymbol{\Gamma}(t-1) - \frac{\boldsymbol{\Gamma}(t-1)\boldsymbol{\zeta}(t)\boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\Gamma}(t-1)}{\lambda + \boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\Gamma}(t-1)\boldsymbol{\zeta}(t)}$$

where $0 < \lambda \leq 1$, $\boldsymbol{\Gamma}(0) = \alpha \boldsymbol{I}$, $0 < \alpha < \infty$. Then the conventional self-tuning GMVC law in state-space representation using adaptive observer can be given by (7) using $\hat{\boldsymbol{\theta}}(t)$ and $\hat{\boldsymbol{x}}(t)$.

3. **Extended Self-Tuning GMVC in State-Space Representation Using Coprime Factorization.** In order to introduce a new design parameter, this section gives the coprime factorization of the conventional GMVC in state-space representation and extends it, under the assumption that the identified plant parameters converge on true values.

The coprime factorization approach considers the family of stable rational functions as follows:

$$RH_\infty = \left\{ G\left(z^{-1}\right) = \frac{G_n\left[z^{-1}\right]}{G_d\left[z^{-1}\right]} \right\}$$

where $G_d\left[z^{-1}\right]$ is stable polynomial.

Remark 3.1. *Because $G_d[z^{-1}] = 1$ is stable polynomial, the rational function whose denominator is equal to be 1, that is, the polynomials belong to RH_∞.*

Assuming that $\xi(t) = 0$, the transfer function of (2) and (3) is given by a ratio of rational functions in RH_∞ which is the family of stable rational functions.

$$G\left(z^{-1}\right) = N\left(z^{-1}\right) D^{-1}\left(z^{-1}\right) = \widetilde{D}^{-1}\left(z^{-1}\right) \widetilde{N}\left(z^{-1}\right) = \boldsymbol{c}_p^{\mathrm{T}}(z\boldsymbol{I} - \boldsymbol{A}_p)^{-1}\boldsymbol{b}_p \tag{14}$$

where $N\left(z^{-1}\right), D\left(z^{-1}\right) \in RH_\infty$ are right coprime factorization and $\widetilde{N}(z^{-1}), \widetilde{D}(z^{-1}) \in RH_\infty$ are left coprime factorization. The rational functions $X(z^{-1}), Y(z^{-1}) \in RH_\infty$ are assumed to denote the solutions of Bezout equation $X(z^{-1})N(z^{-1}) + Y(z^{-1})D(z^{-1}) = 1$. Because it is assumed that the identified plant parameters converge on true values, the adaptive observer can be expressed as the following form.

$$\hat{\boldsymbol{x}}(t) = (z\boldsymbol{I} - \boldsymbol{F})^{-1}(-\boldsymbol{a}_p + \boldsymbol{f})y(t) + (z\boldsymbol{I} - \boldsymbol{F})^{-1}\boldsymbol{b}_p u(t) \tag{15}$$

Then the right coprime factorization of $N(z^{-1})$ and $D(z^{-1})$, and the solutions $X(z^{-1})$ and $Y(z^{-1})$ of Bezout equation are given by the controllers (7) and (15) [5].

$$\begin{aligned} N\left(z^{-1}\right) &= \boldsymbol{c}_p^{\mathrm{T}}(z\boldsymbol{I} - \boldsymbol{A}_p + \boldsymbol{b}_p\boldsymbol{L})^{-1}\boldsymbol{b}_p, \quad D\left(z^{-1}\right) = 1 - \boldsymbol{L}(z\boldsymbol{I} - \boldsymbol{A}_p + \boldsymbol{b}_p\boldsymbol{L})^{-1}\boldsymbol{b}_p \\ X\left(z^{-1}\right) &= \boldsymbol{L}(z\boldsymbol{I} - \boldsymbol{F})^{-1}(-\boldsymbol{a}_p + \boldsymbol{f}), \quad Y\left(z^{-1}\right) = 1 + \boldsymbol{L}(z\boldsymbol{I} - \boldsymbol{F})^{-1}\boldsymbol{b}_p \\ \boldsymbol{L} &= \frac{1}{b_0 + q_0}\boldsymbol{c}_p^{\mathrm{T}}\boldsymbol{A}_p^{k_m}, \quad K\left(z^{-1}\right) = \frac{r_0}{b_0 + q_0} \end{aligned}$$

From these equations, the conventional control law (7) can be expressed as the following stabilizing controller through coprime factorization.

$$u(t) = Y^{-1}\left(z^{-1}\right) K\left(z^{-1}\right) r(t) - Y^{-1}\left(z^{-1}\right) X\left(z^{-1}\right) y(t) \tag{16}$$

Then the extended controller of the conventional control law (7) can be given as the following equation.

$$u(t) = \frac{1}{b_0 + q_0}\left(r_0 r(t) - \boldsymbol{c}_p^{\mathrm{T}}\boldsymbol{A}_p^{k_m}\hat{\boldsymbol{x}}(t)\right) + U\left(z^{-1}\right)\left(\boldsymbol{c}_p^{\mathrm{T}}\hat{\boldsymbol{x}}(t) - y(t)\right) \tag{17}$$

where $U(z^{-1}) \in RH_\infty$ is a new design parameter introduced in the conventional GMVC and $\hat{\boldsymbol{x}}(t)$ is the estimated value of $\boldsymbol{x}(t)$ obtained by the adaptive observer. Moreover, the controller (17) has the following form in coprime factorization.

$$u(t) = C_1\left(z^{-1}\right) r(t) - C_2\left(z^{-1}\right) y(t) \tag{18}$$

$$\begin{aligned} C_1\left(z^{-1}\right) &= \left(Y\left(z^{-1}\right) - U\left(z^{-1}\right)\widetilde{N}\left(z^{-1}\right)\right)^{-1} K\left(z^{-1}\right) \\ C_2\left(z^{-1}\right) &= \left(Y\left(z^{-1}\right) - U\left(z^{-1}\right)\widetilde{N}\left(z^{-1}\right)\right)^{-1}\left(X\left(z^{-1}\right) + U\left(z^{-1}\right)\widetilde{D}\left(z^{-1}\right)\right) \end{aligned}$$

where $\widetilde{N}(z^{-1}) = \boldsymbol{c}_p(z\boldsymbol{I} - \boldsymbol{F})^{-1}\boldsymbol{b}_p$ and $\widetilde{D}(z^{-1}) = 1 - \boldsymbol{c}_p(z\boldsymbol{I} - \boldsymbol{F})^{-1}(-\boldsymbol{a}_p + \boldsymbol{f})$ are the left coprime factorization of the systems (2) and (3).

In order to design on-demand type feedback controller, the extended controller (17) is deformed by defining $u_1(t) \triangleq \frac{1}{b_0+q_0} r_0 r(t) - U(z^{-1})y(t)$ and $u_2(t) \triangleq \frac{1}{b_0+q_0}\boldsymbol{c}_p^{\mathrm{T}}\boldsymbol{A}_p^{k_m}\hat{\boldsymbol{x}}(t) - U(z^{-1})\boldsymbol{c}_p^{\mathrm{T}}\hat{\boldsymbol{x}}(t)$,

$$\begin{aligned} u(t) &= u_1(t) - u_2(t) \\ &= \left(\frac{1}{b_0 + q_0} r_0 r(t) - U\left(z^{-1}\right) y(t)\right) - \left(\frac{1}{b_0 + q_0}\boldsymbol{c}_p^{\mathrm{T}}\boldsymbol{A}_p^{k_m} - U\left(z^{-1}\right)\boldsymbol{c}_p^{\mathrm{T}}\right)\hat{\boldsymbol{x}}(t) \end{aligned} \tag{19}$$

$u_1(t)$ is generated from the reference signal and the measured output, and it is named as pseudo-feedforward signal in this paper. $u_2(t)$ is state feedback signal and it is given from the adaptive observer. The on-demand type feedback controller, which means that the feedback signal becomes zero when the controlled output becomes equal to the reference signal, can be obtained if $u_2(t)$ becomes zero and the steady state gain of closed-loop system $N(1)K(1)$ is designed to be 1 (this means that the controlled output becomes equal to the reference signal). So this paper calculates r_0 as follows, which is one of the GMVC's parameters.

$$r_0 = \frac{b_0 + q_0}{\boldsymbol{c}_p^{\mathrm{T}}\left(\boldsymbol{I} - \boldsymbol{A}_p + \boldsymbol{b}_p\boldsymbol{L}\right)^{-1}\boldsymbol{b}_p}$$

Then the steady state gain of closed-loop system becomes $N(1)K(1) = 1$. Moreover, because the closed-loop system $y(t) = N(z^{-1})K(z^{-1})r(t)$ is independent of $U(z^{-1})$, and $u_2(t)$ becomes zero if an appropriate $U(z^{-1})$ is given, that is, the on-demand type feedback controller is obtained.

4. **Numerical Example.** In this section the controlled plant $A[z^{-1}] = 1 + 0.6z^{-1} + 0.7z^{-2}$, $B[z^{-1}] = 0.5 - 1.5z^{-1}$, $k_m = 1$ and the generalized output $\Phi(t + k_m) = y(t + 1) + 0.8u(t) - r_0 r(t)$ are given. The white Gaussian noise $\xi(t)$ is with zero mean and the variance 0.01^2. The adaptive observer's poles are set to -0.9 and 0.6, and the nominal values of the controlled plant parameters are set to be $0.9 \times$ true values. The parameters of parameter identification law are set to be $\lambda = 1$ and $\boldsymbol{\Gamma}(0) = \boldsymbol{I}$. The reference signal $r(t)$ is a rectangular signal with amplitude 1 from the beginning of simulation to the 150th step and 1.5 after the 151st step. The total number of simulation steps is 300. The closed-loop's poles when the identified parameters converge on true values are $|0.3923 \pm 0.5262i| < 1$ and stable. Figure 1 shows the plant output by the proposed method using the self-tuning controller (19) with adaptive observer, where the proposed controller uses the newly introduced parameter $U(z^{-1})$ as $U(z^{-1}) = 1.6542$. On the other hand, the conventional controller can be derived by selecting the parameter $U(z^{-1})$ as $U(z^{-1}) = 0$ (the extended controller (19) becomes equal to the conventional controller (7) when $U(z^{-1}) = 0$). Figure 2 shows the control input by the proposed self-tuning controller (19). This figure shows (a) control input to the controlled plant, (b) pseudo-feedforward signal defined in this paper, which means $u_1(t)$ in (19), and (c) state feedback signal generated by adaptive observer, which means $u_2(t)$ in (19). From Figure 2, it can find that the state feedback signal in the proposed method emerges in order to follow the reference signal, and its signal becomes almost zero when the controlled output becomes equal to the reference signal. When the reference signal is changed from 1 to 1.5 after the 151st step, the state feedback signal emerges until the controlled output becomes equal to the reference signal. Therefore, on-demand type feedback controller, which means that the feedback signal emerges on demand of making the controlled output follow the reference signal, can be obtained. Figure 3 and Figure 4 show the estimation errors of state variables and the identified plant parameters respectively. Their figures indicate that the proposed method can estimate both the state variables and the plant parameters.

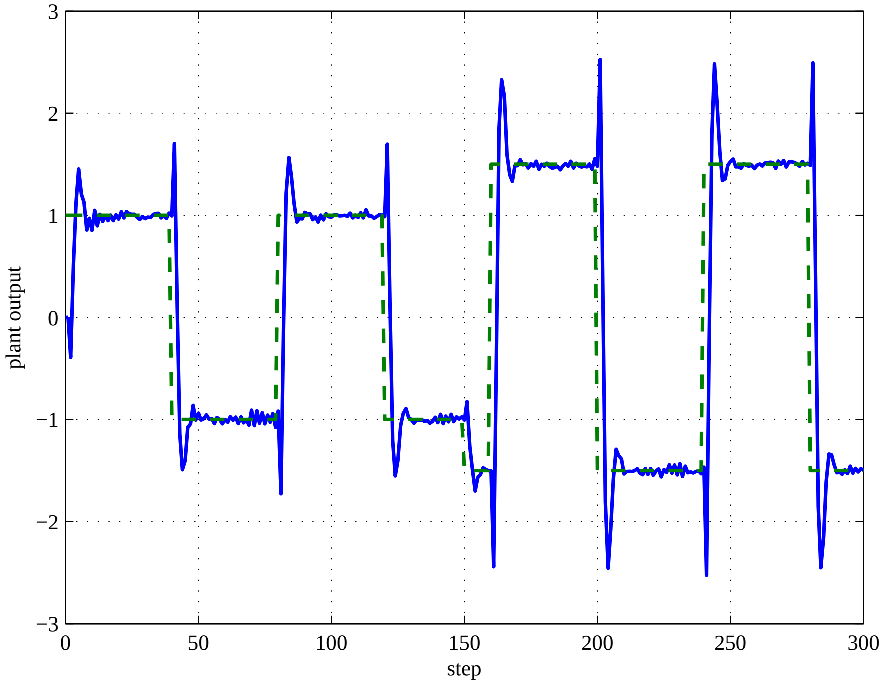

FIGURE 1. The proposed method (output)

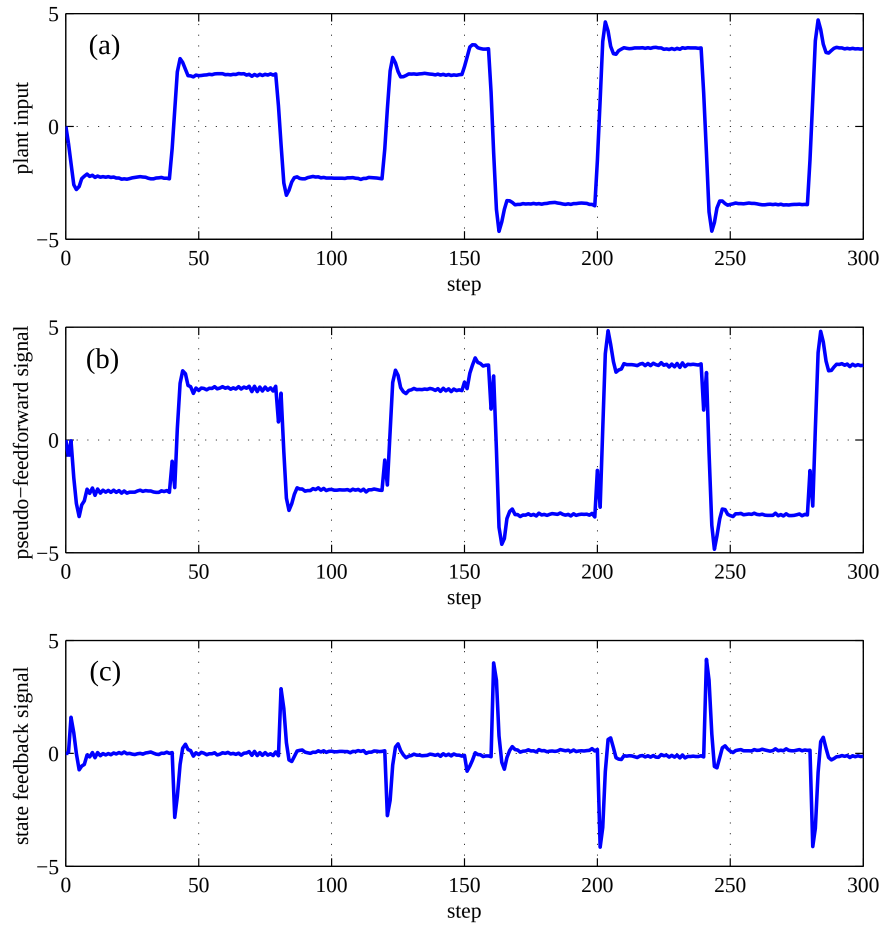

FIGURE 2. The proposed method (input): (a) control input, (b) pseudo-feedforward signal generated by reference signal and measured output, (c) state feedback signal generated by adaptive observer

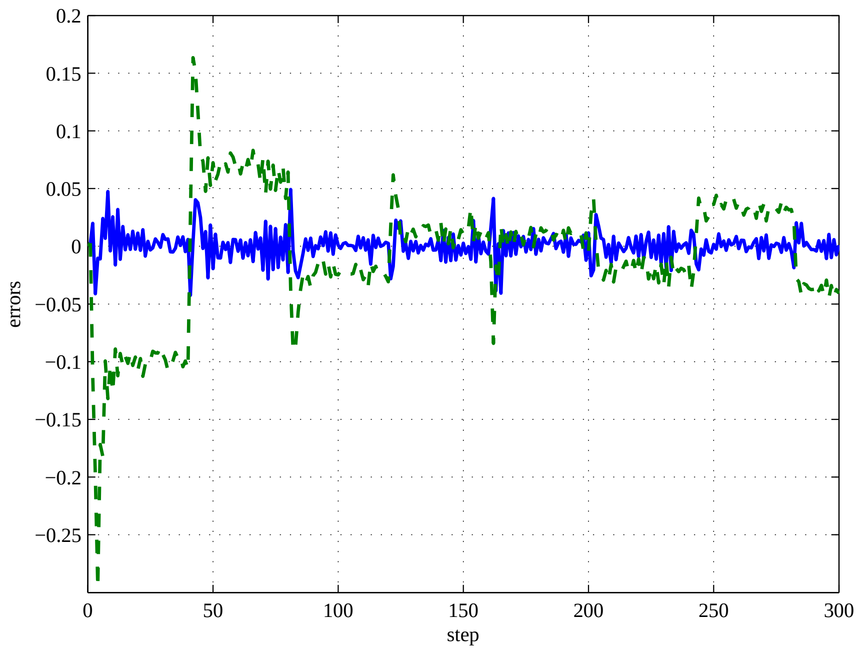

Figure 3. Estimation errors of state variables in the proposed method

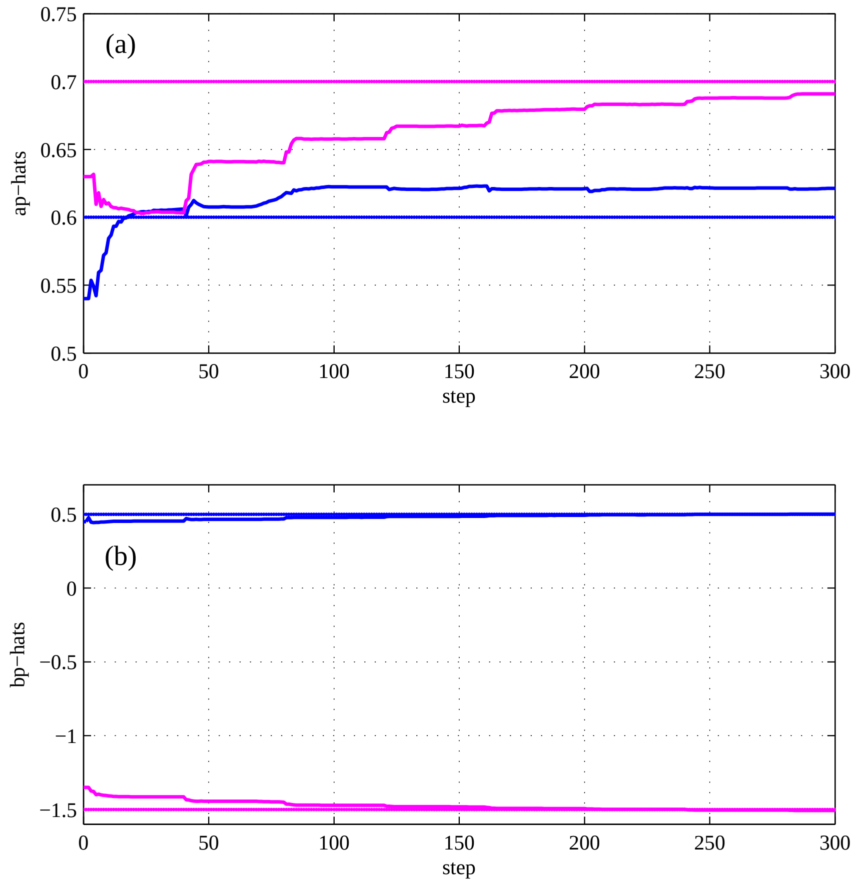

Figure 4. Identified plant parameters in the proposed method: (a) identified parameters $\hat{a}_1(t)$ and $\hat{a}_2(t)$, (b) identified parameters $\hat{b}_0(t)$ and $\hat{b}_1(t)$

5. **Conclusion.** In this paper, on-demand type feedback controller for self-tuning GMVC in state-space representation was proposed. The proposed controller can make the state feedback signal be almost zero when the controlled output becomes almost equal to the reference signal through the new design parameter $U(z^{-1})$. If the reference signal is changed, the state feedback signal emerges on demand of making the controlled output follow the reference signal. The proposed self-tuning controller was derived by extending the conventional self-tuning GMVC in state-space representation through coprime factorization and adaptive observer. The parameter $U(z^{-1})$ was introduced to the extended GMVC. As future works, the formulation of the design parameter $U(z^{-1})$ is needed, as with the formulation derived in transfer function based GMVC [9]. Moreover, there is an extension to multi-input multi-output systems of this method.

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