ICIC Express Letters
Volume **10**, Number **11**, November **2016**

ICIC International ©2016 ISSN 1881-803X
pp. 2727–2733

A NEW INTERCONNECTION NETWORK: EXTENDED EXCHANGED HYPERCUBE

Weixia Gui[1,2] and Jiarong Liang[1,2]

[1]College of Automation Science and Engineering
South China University of Technology
No. 381, Wushan Road, Tianhe District, Guangzhou 510640, P. R. China
wxgui@gxu.edu.cn

[2]College of Computer and Electronic Information
Guangxi University
No. 100, Daxue East Road, Nanning 530004, P. R. China

Received April 2016; accepted July 2016

Abstract. *The exchanged hypercube is one of new interconnection network topologies in recent years. It reduces the cost of topology connecting by removing some links. Based on it, a new fault tolerant interconnection network called extended exchanged hypercube (EEH) is proposed. We show that EEH maintains the appealing properties of the exchanged hypercube and prove that it provides many additional advantages as average distance, reduced diameter, and constant degree of nodes at the same time. Furthermore, the optimal message routing algorithm is proposed, and it is easy and simple to implement.*

Keywords: Interconnection network, Message routing, Extended exchanged hypercube, Diameter

1. **Introduction.** In these past few years, the large-scale parallel computing systems have got more and more attention and increased efforts. The interconnection networks play an important role in this area [1,2]. It is widely known that most of the performances of the interconnection networks are determined by the topology. Hypercube has received much considerable attention because of its attractive features [3]. Variations of this basic topology have been proposed in the literature to overcome the shortcomings and further enhance some features, such as Möbius cubes [4], crossed cube [5], twisted cube [6], exchanged hypercube [7], locally twisted cube [8], fractal cubic network [9]. Especially the exchanged hypercube (EH) highly reduces interconnection complexity and solves the problem of hardware cost by removing some edges from hypercube. Thus some related works on EH have been investigated such as domination number [10], connectivity [11], super connectivity [12], fault-tolerance measurement [13] and diagnosabilities [14]. However, the processor nodes in EH are involved in communicating messages between their neighbors. As we know, an efficient communication scheme is one in which the processor nodes perform more of computation tasks and less of communication tasks. In addition, computing systems should have a good scalability; there should be no changes in the basic node configuration as we increase the number of nodes.

The demand for increasing the efficiency of EH and achieving the truly expandable ability motivates our investigation in proposing a new interconnection network. Inspired by the cube-type networks such as Extended Hypercube [15,16] and Hierarchical Crossed Cube [17], we propose a new hierarchical fault-tolerant interconnection network called Extended Exchanged Hypercube, denoted by EEH.

The proposed interconnection network combines some of the topological features of the architectures proposed in [15,17-19] and at the same time retains the attractive features of EH topology to a large extent. EEH is a hierarchical, expansive and recursive with a constant predefined building block, without changing the hardware configuration of all the nodes whenever the number of nodes grows exponentially.

This paper is organized as follows. The second section presents construction and addressing of EEH. Its various topological properties are discussed in the third section. And then, message routing issue is discussed in Section 4. Finally, we conclude the paper in Section 5.

2. **The Extended Exchanged Hypercube.** In this part, a formal introduction to the EEH architecture is given and a methodology for addressing the nodes of the EEH is also discussed. The EEH architecture is suited for hierarchical expansion of multiprocessor systems.

The k-dimensional EEH having levels of hierarchy defined as EEH(k,l) (l is the degree of the EEH) is a labeled graph which can be defined recursively with two special types of vertices called Network Controller (NC) and Processing Element (PE). The PE performs computational task whereas NC is responsible for communication task.

As shown in Figure 1, NC is at the top, which is at the highest level and PE's are at the zero level. The basic module EEH$(k,1)$ that consists of a k-Dimensional EH and one NC, has two levels of hierarchy: the NC at the first level and the EH at the zero level. In addition, there is an EH of 2^k NC's at the $(l-1)$th level and one NC at the lth level in an EEH(k,l). Again it can be seen that 2^k NC's form an EH at the $(l-1)$th level. The EH consisting of the PE's is referred to as the EEH$(k,0)$. An EEH$(k,2)$ has 2^k EH of PE's at the 0th level, one EH of NC's at the first level, and one NC at the second level. In general any EEH(k,l) can be recursively built from the basic module EEH$(k,1)$.

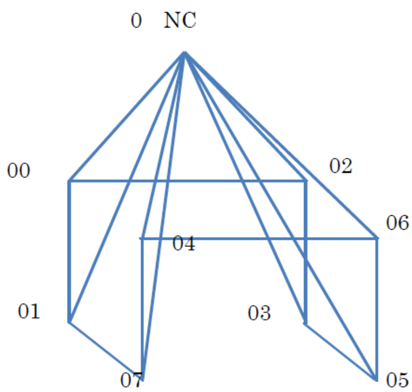

FIGURE 1. Basic module of EEH, e.g., EEH$(3,1)$

For example, the NC's of each EEH$(k,2)$ can be built from EEH$(k,1)$ and this procedure can be repeated hierarchically to build the required size of EEH. The basic module of the EEH is a constant predefined building block and the node remains the same configuration regardless of the dimension of the EEH. As shown in Figure 1, the PE's of the basic module or EEH$(k,1)$ can be addressed as $0,1,\ldots,M$ ($M=2^k-1$). Now if the NC of EEH$(k,1)$ is identified by N, then the PE's of this EEH$(k,1)$ are addressed as N_0, N_1, N_2, $\ldots$, N_M. The address of the NC precedes the address of the PE. In general, the address for an arbitrary node at the 0th level can be written as $D_lD_{l-1}D_{l-2}D_{l-3}\ldots D_0$, each D_i ($0\leq i\leq l$) is a k-bit mod 2^k number. Here D_0 corresponds to the address of a node in EEH$(k,0)$, D_1 corresponds to the address of the node (NC) at the first level (to which the first node is connected), D_2 corresponds to the address of the node (NC) at the second level, and so on. Consider an EEH of degree "l", a node at the 0th level will have a $(l+1)$ digit address, a node at the first level will have l digit address, a node at the second level will have a $(l-1)$ digit address, etc. The solitary node at the lth level has one digit address, that is 0.

The NC's at the $(l-1)$th level of EEH$(k,1)$ are addressed by 0. The EH at the lth level consisting of 2^k NC's have ids as 00, 01, ..., $0M$ $(M = 2^k - 1)$. The id of the NC precedes the node id of PE's. Thus the PE's connected to the NC's $0i$ $(0 \leq i \leq m)$ have addresses $0i0$, $0i1$, ..., $0iM$.

In an EEH, the NC's are used as communication processor for global communication for level to level communication and also for local communication between two different basic modules. However, the NC's are not used for the communication between two nodes of the same basic module. There are $(k+1)$ parallel paths between any two nodes of the EEH(k,l), k-path contributed by the k-edges of EH and one path due to the NC.

3. Topological Properties of EEH.

3.1. Degree.

Degree of a node or connectivity of a node determines the hardware complexity of the network. The higher the connectivity, the higher is the hardware complexity and the cost of the network. A constant connectivity implies extendibility without change in the hardware structure of each node.

In an EEH(k,l) the PE's are at the 0th level of hierarchy. Each PE's belonging to EEH$(k,1)$ is directly connected to k neighboring PE's of the same EH and to an NC at the next higher level. Thus the degree of PE in EEH(k,l) is $(k+1)$. However, for an NC, where NC is connected to 2^k PE's at its just lower level, k NC's at its level and one NC at its next higher level. Therefore the degree of an NC other than the highest NC is $(2^k + k + 1)$. As an NC at highest level is connected to 2^k NC's at its just lower level, the degree connectivity of the NC at highest level is 2^k.

3.2. Diameter.

Theorem 3.1. *The diameter of EEH(k,l) denoted by $D(G)$ is $k + 2l - 1$.*

Proof: Considering two nodes N_1 and N_2, they are either in the same EH or in different EHs.

Case 1. Suppose N_1 and N_2 are in the same k-EH. Then the distance between N_1 and N_2 is at most $k+1$.

Case 2. Suppose N_1 and N_2 are in different EHs. Let us choose a node N_0 in the EH that contains N_1. By previous case the distance between N_1 and N_0 is at most $k+1$. N_2 and N_0 can be connected by links through NC's at 1, 2, 3, ..., $(l-1)$ levels. Thus the shortest path between N_2 and N_0 has a distance $2(l-1)$. Hence the distance between N_1 and N_2 is at most $k + 1 + 2(l-1) = k + 2l - 1$. So the diameter of EEH$(k,l)$ is $k + 2l - 1$.

3.3. Node.

Theorem 3.2. *The total number of nodes in EEH(k,l) is given by $p = 2^{kl} + (2^{kl} - 1)/(2^k - 1)$.*

Proof: There are 2^l number of EH of PE's. Thus in the EEH(k,l), the total number of PE's denoted by N is $2^k * 2^l = 2^{kl}$. For $0 < j \leq l$, the number of NC's at level j is $(2^k)^{l-j}$; thus, the total number of NC's denoted by $M = \sum_{j=1}^{l} (2^k)^{l-j} = (2^{kl} - 1)/(2^k - 1)$, so the total number of nodes in EEH(k,l) is given by $p = 2^{kl} + (2^{kl} - 1)/(2^k - 1)$.

3.4. Cost of the network.

Theorem 3.3. *The cost of EEH(k,l) is given by*

$$C = (k + 2l - 1) * \left((k+1) * 2^{kl} + \left(2^k + k + 1\right)\left(2^{kl} - 1\right) / \left(2^k - 1\right)\right) / \left(2^{kl} + \left(2^k + k + 1\right)\left(2^{kl} - 1\right)\right)$$

Proof: Cost of a network is given by the product of node degree and diameter. In an EEH(k,l), the degrees of PE and the NC are different. The average degree of node is

$$D_{avg} = \left((k+1) * 2^{kl} + \left(2^k + k + 1\right)\left(2^{kl} - 1\right) / \left(2^k - 1\right)\right) / \left(2^{kl} + \left(2^k + k + 1\right)\left(2^{kl} - 1\right)\right)$$

and the diameter is $k + 2l - 1$. Hence the cost is given by

$$C = \ (k+2l-1) * \left((k+1) * 2^{kl} + \left(2^k + k + 1\right)\left(2^{kl} - 1\right) / \left(2^k - 1\right)\right) / \left(2^{kl} + \left(2^k + k + 1\right)\left(2^{kl} - 1\right)\right)$$

3.5. **Global average distance.** The global average distance conveys the actual performance of the network. The summation of distances of all nodes from a given node over the total number of nodes gives the average distance of the network.

Theorem 3.4. *The global average distance of EEH(k,l) is given by* $\overline{d} = \sum_{EEH(k,l)} dN_d/(N+M)$ *where* $N = 2^{kl}$ *and* $M = (2^{kl} - 1)/(2^k - 1)$ *and* N_d *is the number of processors at a distance d from the source node.*

N is the total number of PE's and M is the total number of NC's. The global average distance is dependent on the degree of EEH(k,l) and increases with it.

3.6. **Links and message traffic density.** The total number of links in an EEH(k,l) is given by

$$E = 2^k * (k/2 + 1) * 2^{k(l-1)}[(1 - 2^{-kl})/(1 - 2^{-k})]$$

The message traffic density in EEH(k,l) is given by $\rho =$ (Average message distance $*$ Number of nodes)/(Number of links) $= \overline{d}(N+M)/E$, where E is the total number of links. Assuming each node is sending one message to a node at distance $\overline{d}$ on the average and considering the availability of n links to accommodate such a traffic, ρ can be a good measure to estimate the message traffic in the network. $(N+M)$ is the total number of nodes consisting of PE's and NC's.

3.7. **Extensibility and fault tolerance.** Extensibility is the property which facilitates constructing large-sized systems out of small-sized systems with minimum changes in the configuration of the nodes of the system. The EEH(k,l) is hierarchical in nature and can be built by extension of the number of levels without affecting the basic structure. The most important advantage of this property is that the degree of a node remains the same, independent of the total number of nodes and hence allows for further expansion. Thus the architecture of EEH(k,l) is well suited for hierarchical expansion of multiprocessor system.

Fault tolerance of a network is an important characteristic in parallel computing environment. For a graph, it is defined as the maximum number of vertices that can be removed from it provided that the graph is still connected. Hence the fault tolerance of a graph is defined to be one less than its connectivity. As discussed in [20], a system is said to be k-fault tolerant if it can sustain up to k number of edge faults without disturbing the network. For symmetric interconnection networks the connectivity is equal to the node degree. For the EEH(k,l) which is a hierarchical network of EH with all processing elements at the lowest level and all communication processors at the highest level the node degree is $(k+1)$. So EEH(k,l) can tolerate up to k faults.

3.8. **Performance evaluation.** We compare the proposed EEH(k,l) with Hypercube (HQ$_n$), Extended Hypercube (EH(k,l)) and Exchanged Hypercube (EH(s,t), $s+t+1=n$) for the same dimension. Seeing Table 1 the comparison is based on the following parameters: number of links, network diameter, node degree, cost factor, expandability and decomposition, which have significant impact on the performance of a parallel computing system. To facilitate comparisons, asymptotic values are used where necessary.

TABLE 1. Comparison of various networks ($n = s + t + 1$, $n = k * l$)

Network	HQ_n	$\mathrm{EH}(k, l)$	$\mathrm{EH}(s, t)$	$\mathrm{EEH}(k, l)$
Links	$n2^{n-1}$	$\rightarrow (k+2)2^{n-1}$	$(n+1)2^{n-2}$	$\rightarrow (k/2+1)2^{kl}$
Diameter	n	$k + 2(l-1)$	$n+1$	$k + 2l - 1$
Node Degree	n	$\rightarrow k+2$	$s+1$ or $t+1$	$2^k + k + 1$
Cost	n^2	$\rightarrow k^2 + 2n$	$\rightarrow n^2/2$	$\rightarrow k^2 + 2l$
Expandability	Sub-optimal	Optimal	Sub-optimal	Optimal
Decomposition	Complex	Simple	Complex	Simple

4. **Message Routing in EEH.** An optimal routing algorithm is to find a shortest path between two communicating nodes. Using the NC, the inter-processor message traffic of a module gets redistributed into two categories, that is, local communication and global communication [15]. Communication among the PE's belonging to the same EH is classified as local communication. Communication between the PE's of different basic modules via the network controller is called global communication.

For local communication the message is routed within the same EH without going to the NC and this can be done by employing EH message passing algorithms. The algorithm always finds a shortest path between source and destination nodes in $\mathrm{O}(n)$ time. The NC's are involved in global communication. The highest NC transmits the message from source to destination PE's via the network of NC's. The message passing operation in the global communication involves 1) the source PE, 2) up to $2(l-1)$ NC's and 3) the destination PE. The transfer of message between two nodes at different levels of hierarchy is referred to as the vertical shift [15]. Routing in EEH (k, l) involves two vertical shifts for level to level communication and a cube shift for movement in EH. The algorithm first checks whether it is a local communication or a global one. The message routing for different source and destination pairs has been given in Table 2 for illustration.

Following procedure describes the message routing procedure. Let x be the source node with node id $D_l^s D_{l-1}^s D_{l-2}^s \ldots D_0^s$, and y be the destination node with node id $D_l^d D_{l-1}^d$ $D_{l-2}^d \ldots D_0^d$.

The procedure for routing a message from the host system to a PE involves a vertical shift from D_l^s to $D_l^d D_{l-1}^d D_{l-2}^d \ldots D_0^d$ via $D_l^d D_{l-1}^d$, $D_l^d D_{l-1}^d D_{l-2}^d$, $D_l^d D_{l-1}^d D_{l-2}^d \ldots D_1^d$, where $D_l^s = D_l^d$ (as illustrated in Table 2 for routing).

Algorithm: msgrouting (x, y)
Begin
  If $[D_l^s D_{l-1}^s D_{l-2}^s \ldots D_0^s] = [D_l^d D_{l-1}^d D_{l-2}^d \ldots D_0^d]$
    Then destination is source; terminate.
  Else set $j = 0$;
  While $j = 0$ do
   for $i = 1$ to l
    Begin
     If $D_{l-i}^s = D_{l-i}^d$ then $j = i$;
    End
    Begin
     Vertical shift from $D_l^s D_{l-1}^s D_{l-2}^s \ldots D_0^s$ to $D_l^s D_{l-1}^s D_{l-2}^s \ldots D_j^s$;
     Cube shift from $D_l^s D_{l-1}^s D_{l-2}^s \ldots D_j^s$ to $D_l^d D_{l-1}^d D_{l-2}^d \ldots D_j^d$;
     Vertical shift from $D_l^d D_{l-1}^d D_{l-2}^d \ldots D_j^d$ to $D_l^d D_{l-1}^d D_{l-2}^d \ldots D_0^d$;
    End
End

Table 2. Message routing sequence

Distance	Destination	Routing Sequence From 043
1	041	043-041
2	040	043-041-040
3	060	043-04-06-060
4	021	043-04-0-02-021

5. **Conclusions.** We have presented a new kind of interconnection network: the Extended Exchanged Hypercube EEH(k,l). Compared to its parent topologies exchanged hypercube, EEH(k,l) not only holds many desirable properties such as low diameter and node degree, but also has more scalability than EH. The use of network controller nodes ensures efficient communication of messages via the interconnections among the network controllers. We showed that the proposed network performs well in terms of fault tolerance, cost factor. Furthermore, the various topological properties of the EEH(k,l) are analyzed and evaluated, an optimal message routing algorithm is developed. Extensive comparisons of EEH(k,l) with other hypercubes are included, which proves that EEH(k,l) is applicable to large-scale parallel computing system very well. However, there are many interesting problems such as wide-diameter, fault-diameter, connectivity and its graph embedding capability. We aim to investigate those problems in our future work.

Acknowledgment. This work is supported by Natural Science Foundation of China (Grant No. 61363002 and No. 61462006). The authors would like to thank the reviewers and the editors for their helpful comments and suggestions, which have improved the quality of this paper.

REFERENCES

[1] T. Feng, A survey of interconnection networks, *IEEE Computers*, no.1, pp.12-27, 1981.
[2] L. N. Bhuyan and D. P. Agrawal, Performance of multiprocessor interconnection network, *IEEE Computers*, no.22, pp.25-37, 1989.
[3] Y. Saad and M. H. Schultz, Topological properties of hypercube, *IEEE Trans. Computers*, no.37, pp.867-872, 1988.
[4] S. M. Larson and P. Cull, The Möbius cubes, *IEEE Trans. Computers*, no.44, pp.647-659, 1995.
[5] K. Efe, The crossed cube architecture for parallel computation, *IEEE Trans. Parallel and Distributed Systems*, no.3, pp.513-524, 1992.
[6] P. A. J. Hilbers, M. R. J. Koopman and J. L. A. van de Snepscheut, The twisted cube, *Parallel Architectures and Languages Europe*, Springer Berlin Heidelberg, 1987.
[7] P. K. K. Loh, J. H. Wen and P. Yi, The exchanged hypercube, *IEEE Trans. Parallel and Distributed Systems*, no.16, pp.866-874, 2005.
[8] X. F. Yang, D. J. Evans and G. M. Megson, The locally twisted cubes, *International Journal of Computer Mathematics*, no.82, pp.401-413, 2005.
[9] A. Karci and B. Selçuk, A new hypercube variant: Fractal cubic network graph, *Engineering Science and Technology*, no.18, pp.32-41, 2015.
[10] S. Klavžar and M. J. Ma, The domination number of exchanged hypercubes, *Information Processing Letters*, no.114, pp.159-162, 2014.
[11] M. J. Ma, The connectivity of exchanged hypercube, *Discrete Mathematics, Algorithms and Applications*, no.2, pp.51-57, 2010.
[12] M. J. Ma and L. Y. Zhu, The super connectivity of exchanged hypercubes, *Information Processing Letters*, no.111, pp.360-364, 2011.
[13] X. J. Li and J. M. Xu, Generalized measures of fault tolerance in exchanged hypercubes, *Information Processing Letters*, no.113, pp.533-537, 2013.
[14] J. R. Liang, Y. Huang and L. C. Ye, Diagnosabilities of exchanged hypercube networks under the pessimistic one-step diagnosis strategy, *Journal of Systems Engineering and Electronics*, no.26, pp.415-420, 2015.
[15] J. M. Kumar and L. M. Pattnaik, Extended hypercube: A hierarchical interconnection network of hypercubes, *IEEE Trans. Parallel and Disrtibuted Systems*, no.3, pp.44-57, 1992.

[16] B. Ray and A. R. Tripathy, Extended hypercube with cross-connection – A new interconnection fault tolerant network for parallel computers, *IEEE International Advance Computing Conference*, New York, pp.513-518, 2009.
[17] K. Ghose and K. R. Desai, A versatile interconnection network based on cubes, *IEEE Proc. of Supercomputing*, New York, pp.426-435, 1989.
[18] N. Adhikari and C. R. Tripathy, Extended crossed cube: An improved fault tolerant interconnection network, *The 5th International Joint Conference on INC*, pp.86-91, 2009.
[19] C. R. Tripathy and R. K. Dash, Extended varietal hypercube – A fault tolerant interconnection topology for parallel systems, *Proc. of the 8th International Conference on Information Technology*, pp.214-219, 2005.
[20] A. Avizienis, Fault tolerant system, *IEEE Trans. Computer*, no.25, pp.1304-1312, 1976.