ICIC Express Letters
Volume **10**, Number **11**, November **2016**

ICIC International ©2016 ISSN 1881-803X
pp. 2743–2749

RECEIVED SIGNAL STRENGTH-BASED WIRELESS LOCALIZATION BY CONSIDERING UNKNOWN TRANSMIT POWER

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Received April 2016; accepted July 2016

Abstract. *Due to low complexity and cost in software and hardware implementations, received signal strength (RSS) measurements are widely applied to the wireless localization. When transmit power is unavailable, an accurate location estimation approach for RSS-based wireless localization is proposed. The proposed approach obtains the corresponding best linear unbiased estimate for the source location. The simulations demonstrate the validity of the location estimation approach and test the impacts of noises on localization errors. The performance of designed approach can achieve the position Cramer-Rao low bound (CRLB) of the estimation problem in the small noise condition.*
Keywords: Wireless sensor networks, Received signal strength, Wireless localization, Unknown transmit power

1. **Introduction.** Obtaining accurate positions of sensor nodes in wireless sensor networks is important because the positions of sensor nodes are a critical input to many higher-level networking tasks including the test of ship model self-propulsion, search and rescue when the passenger ship is in the maritime distress. A straightforward solution is to equip each sensor node with a GPS receiver that can accurately provide the sensors with their exact location. However, it is not a feasible solution from an economic perspective since sensor nodes are often deployed in very large numbers and manual configuration is too cumbersome and hence not feasible [1]. Therefore, localization in wireless sensor networks (WSNs) is very challenging. Over the years many research efforts have resulted in a plethora of algorithms to enable the location discovery process in WSNs to be autonomous and able to function independently of GPS and other manual techniques.

In all this literature, the focal point of location discovery has been a set of specialty nodes known as anchor nodes. These anchor nodes know their location, either through a GPS receiver or through manual configuration, which they provide to other source nodes. Using this location of anchor nodes, source nodes compute their location using various ranging techniques, including time of arrival (TOA) [2], time difference of arrival (TDOA) [3], angle of arrival (AOA), and received signal strength (RSS) [4, 5] measurements. Among the different types of measurements, RSS-based technique provides an inherent tradeoff between the accuracy performance and the implementation complexity due to its low software complexity and hardware cost.

To estimate the source location, some algorithms including maximum likelihood (ML), semidefinite programming (SDP) method [6] and linear estimator [7] are proposed to achieve excellent performance. The ML estimator is always solved by the numerical method which requires initial solution to ensure the convergence. When the selected initial solution is far from the actual, it will be trapped in the local optimum. To overcome the shortcoming of the ML estimator, the SDP and linear estimator are proposed to obtain the robust source location estimates. By relaxing the nonconvex optimization into convex problem, the SDP method provides robust solution and improves the performance in the condition of larger noises. However, the complexity of SDP is high. The accuracy performance of SDP cannot achieve the optimal CRLB due to the relaxation. Linear estimator represents the source location estimates as closed-form solution by converting the nonlinear optimization function into linear model. The complexity of the linear estimator is much lower than that of SDP method.

RSS measurement is often modeled as logarithm decay scale and related with the transmit power. However, the transmit power of source node might change with time. In [8] and [9], the SDP methods are proposed by considering the unknown transmit power. However, the proposed algorithms run slowly due to the high computational complexity. When the transmit power is assumed to be unavailable, an accurate linear estimator is proposed by considering the uncertain anchor positions in this paper. Auxiliary variable based algorithm is introduced to formulate the localization problem as a linear least squares estimation and obtains the initial estimate for the source location. By employing the element relationship of the initial estimates, we then present the location refinement approach to achieve the positioning CRLB based on the prior knowledge of the RSS noises variance.

2. **Problem Specification.** In a 2-dimensional geographical area, a sensor network of size $M+N$ is deployed. Let $\mathbf{x}_i = [x_i \quad y_i]^T$ $(i = 1, 2, \ldots, N)$ be the unknown coordinates of the source node to be determined. The known coordinates of M anchor nodes are denoted as $\mathbf{x}_j = [x_j \quad y_j]^T$, $j = 1, 2, \ldots, M$. When $\mathbf{x}_j^o$ represents the true position of anchor node j, $\mathbf{x}_j = \mathbf{x}_j^o + \Delta\mathbf{x}_j$. Here the position of anchor node j is assumed to include error $\Delta\mathbf{x}_j$ due to the inaccurate measurements. The received power (in dB) at the node j, $p_{i,j}$, under log-normal shadowing is modeled as

$$p_{i,j} = p_{i,0} - 10\beta\log_{10}\frac{d_{i,j}}{d_{i,0}} + n_{i,j} \tag{1}$$

where $p_{i,0}$ is the reference power at distance $d_{i,0}$ from the transmit source i ($p_{i,j}$ depends on the transmit power $p_{i,0}$). Without loss of generality, $d_{i,0}$ can be set to 1 m. β is called as path loss exponent. Usually the path-loss exponent values vary in different environments, and can be obtained in advance for a given environment. The noises $n_{i,j}$ are the log-normal shadowing terms modeled as independent and identically distributed zero-mean Gaussian random variables with shadow fading $\delta_{i,j}^2$. The variance of the shadowing term is constant with distance and only depends on the environment of the deployed nodes. The distance between source node i and anchor j, denoted by $d_{i,j}$, is

$$d_{i,j} = \sqrt{(x_i - x_j^o)^2 + (y_i - y_j^o)^2} \tag{2}$$

The source nodes can not only connect with anchor nodes but also measure the RSS between themselves. We assume that each RSS measurement received at the jth node can be correctly associated to the ith node. Denote the positions of the source nodes, and transmit powers in vector form by

$$\mathbf{x} = \begin{bmatrix}\mathbf{x}_1^T & \mathbf{x}_2^T & \ldots & \mathbf{x}_N^T\end{bmatrix}^T \tag{3}$$

$$\mathbf{p}_0 = \begin{bmatrix}p_{1,0} & p_{2,0} & \ldots & p_{N,0}\end{bmatrix}^T \tag{4}$$

The source transmit powers are considered as nuisance parameters and estimated jointly with the source node locations. The goal of the proposed algorithm is to estimate $\mathbf{x}$ based on the ranging model depicted as (1) where the transmit power $\mathbf{p}_0$ is unavailable.

3. **Algorithm Design.** In this section we assume that the anchor positions include Gaussian errors, the transmit powers are considered as nuisance parameters and estimated jointly with the source locations. The localization algorithm is designed with a two-step method: initial estimation and location refinement.

3.1. **Initial estimation.** We start with rewriting (2) as

$$d_{i,j}^2 = 10^{\frac{p_{i,0}-p_{i,j}+n_{i,j}}{5\beta}} \quad j \in \mathcal{A}_i \tag{5}$$

where $i = 1, 2, \ldots, N$. $\mathcal{A}_i$ is the set of the indices of the source node i connected to the anchor nodes. $j \in \mathcal{A}_i$ represents that anchor node j is connected to source node i. For sufficiently small noise, the right-hand side of (5) can be approximated using the first-order Taylor series expansion as

$$d_{i,j}^2 = \lambda_{i,j} + \frac{\lambda_{i,j}\ln 10}{5\beta} n_{i,j} \tag{6}$$

where $\lambda_{i,j} = 10^{\frac{p_{i,0}-p_{i,j}}{5\beta}}$. $\frac{\lambda_{i,j}\ln 10}{5\beta} n_{i,j}$ is a zero-mean Gaussian random variable with variance $\frac{\lambda_{i,j}^2(\ln 10)^2\delta_{i,j}^2}{25\beta^2}$. Let $\mathbf{x}_j = \mathbf{x}_j^o + \Delta\mathbf{x}_j$, $d_{i,j} = \sqrt{\left(x_i - x_j^o\right)^2 + \left(y_i - y_j^o\right)^2}$, and then (6) can also be represented as

$$-2x_jx_i - 2y_jy_i + x_i^2 + y_i^2 = -x_j^2 - y_j^2 + \lambda_{i,j} + \frac{\lambda_{i,j}\ln 10}{5\beta} n_{i,j} + 2\left(x_i - x_j\right)\Delta x_j + 2\left(y_i - y_j\right)\Delta y_j \tag{7}$$

When the transmit powers are unavailable, (7) is represented as

$$\begin{aligned} &-2x_jx_i - 2y_jy_i + x_i^2 + y_i^2 - \mu_{i,j}\rho_i \\ &= -x_j^2 - y_j^2 + \frac{\lambda_{i,j}\ln 10}{5\beta} n_{i,j} + 2(x_i - x_j)\Delta x_j + 2(y_i - y_j)\Delta y_j \qquad j \in \mathcal{A}_i \end{aligned} \tag{8}$$

where $\mu_{i,j} = 10^{-\frac{p_{i,j}}{5\beta}}$, $\rho_i = 10^{\frac{p_{i,0}}{5\beta}}$ and $\lambda_{i,j} = \mu_{i,j}\rho_i$. Let $\mathbf{u}_i = [x_i \quad y_i \quad x_i^2 + y_i^2 \quad \rho_i]^T$ be the unknown vector to be estimated. Then (8) can be expressed in matrix form as

$$\mathbf{C}_i\mathbf{u}_i = \mathbf{d}_i + \alpha_i \tag{9}$$

where the row vectors of $\mathbf{C}_i$, $\mathbf{d}_i$ and α_i are equal to $[-2x_j \quad -2y_j \quad 1 \quad -\mu_{i,j}]$, $[-x_j^2 - y_j^2]$ and $\left[\frac{\lambda_{i,j}\ln 10}{5\beta} n_{i,j} + 2(x_i - x_j)\Delta x_j + 2(y_i - y_j)\Delta y_j\right]$, respectively. Since $n_{i,j}$ is independent for different anchors, the covariance matrix of error α_i, $\mathbf{\Sigma}_i$ can be written as

$$\mathbf{\Sigma}_i = \operatorname{diag}\left\{\frac{\lambda_{i,j}^2 \ln 10^2}{25\beta^2}\delta_{i,j}^2 + \mathbf{e}_{i,j}\Sigma_j\mathbf{e}_{i,j}^T\right\} \tag{10}$$

where $\mathbf{e}_{i,j} = [2(x_i - x_j) \quad 2(y_i - y_j)]$, $\mathbf{\Sigma}_j$ represents the covariance of anchor position error $\Delta\mathbf{x}_j$. So the WLS solution to (9) is

$$\mathbf{u}_i = \left(\mathbf{C}_i^T\mathbf{\Sigma}_i^{-1}\mathbf{C}_i\right)^{-1}\mathbf{C}_i^T\mathbf{\Sigma}_i^{-1}\mathbf{d}_i \tag{11}$$

$\mathbf{\Sigma}_i$ relies on the estimate $\lambda_{i,j}$ which is determined by the transmit power $p_{i,0}$ and not available. We preliminarily consider $\mathbf{\Sigma}_i$ as unit matrix $\mathbf{I}$. Then putting the estimated initial $p_{i,0}$ into (10) gives an approximated $\mathbf{\Sigma}_i$ and using it in (11) would produce a better solution of the vector $\mathbf{u}_i$.

The covariance of $\mathbf{u}_i$ is denoted as $\mathbf{\Sigma}_i^u$, which also can be written as

$$\mathbf{\Sigma}_i^u = \left(\mathbf{C}_i^T\mathbf{\Sigma}_i^{-1}\mathbf{C}_i\right)^{-1} \tag{12}$$

Extracting from $\mathbf{u}_i$ we obtain the initial estimated position $\mathbf{x}_i^e = \mathbf{u}_i(1:2)$, $\rho_i^e = \mathbf{u}_i(4)$ and the corresponding transmit power $p_{i,0}^e$ in the primitive stage. The covariance of $\mathbf{x}_i^e$ is represented as $\mathbf{\Sigma}_i^e = \mathbf{\Sigma}_i^u(1:2, 1:2)$. Similarly, by stacking in an ascending order of i, the initial position estimate $\mathbf{x}^e = [\mathbf{x}_1^{eT} \quad \mathbf{x}_2^{eT} \quad \ldots \quad \mathbf{x}_N^{eT}]^T$ is obtained. By taking this relation between elements of $\mathbf{u}_i$ into account and refining the position for wireless localization, we can obtain the more accurate source location.

3.2. **Location refinement.** The refinement method is also based on the initial estimate in (11). Considering the impacts of the source-source measurements, the optimal estimate of ρ_i should also be refined. The error of ρ_i is denoted as $\Delta\rho_i$. So let $\mathbf{x}_i = \mathbf{x}_i^e + \Delta\mathbf{x}_i$ and $\rho_i = \rho_i^e + \Delta\rho_i$, (8) is rewritten as

$$\begin{aligned} &2\left(x_i^e - x_j\right)\Delta x_i + 2\left(y_i^e - y_j\right)\Delta y_i - \mu_{i,j}\Delta\rho_i \\ &= \lambda_{i,j}^e - x_j^2 - y_j^2 - x_i^{e2} - y_i^{e2} + 2x_i^e x_j + 2y_i^e y_j + \frac{\lambda_{i,j}\ln 10}{5\beta} n_{i,j} \qquad j \in \mathcal{A}_i \end{aligned} \tag{13}$$

where $i = 1, 2, \ldots, N$, $\lambda_{i,j}^e = \mu_{i,j}\rho_i^e$. Similarly if there are totally L source-anchor measurements, the global matrix form can be written as

$$\mathbf{J}_1\Delta\theta = \mathbf{q}_1 + \varphi_1 \tag{14}$$

where $\Delta\theta \in \mathbb{R}^{3N}$, $\mathbf{J}_1 \in \mathbb{R}^{L\times 3N}$, $\mathbf{q}_1 \in \mathbb{R}^L$ and $\varphi_1 \in \mathbb{R}^L$. $\Delta\theta = [\Delta\mathbf{x}^T \quad \Delta\rho^T]^T$ denotes the refined vector, $\Delta\rho = [\Delta\rho_1 \quad \Delta\rho_2 \quad \ldots \quad \Delta\rho_N]^T$ and $\Delta\rho_i$ is the refined error in ρ_i, $i = 1, 2, \ldots, N$. The row vectors of $\mathbf{J}_1$, $\mathbf{q}_1$ and φ_1 are respectively equal to $[\mathbf{0}_{1\times 2(i-1)} \quad 2(x_i^e - x_j) \quad 2(y_i^e - y_j) \quad \mathbf{0}_{1\times(2N-i-1)} \quad -\mu_{i,j} \quad \mathbf{0}_{1\times(N-i)}]$, $\left[\lambda_{i,j}^e - x_j^2 - y_j^2 - x_i^{e2} - y_i^{e2} + 2x_i^e x_j + 2y_i^e y_j\right]$ and $\left[\frac{\lambda_{i,j}\ln 10}{5\beta} n_{i,j}\right]$.

Then by considering the source-source measurements, (8) is rewritten as

$$2\mathbf{x}^{eT}\mathbf{Q}\Delta\mathbf{x} - \mu_{i,j}\Delta\rho_i = \lambda_{i,j}^e - \mathbf{x}^{eT}\mathbf{Q}\mathbf{x}^e + \frac{\lambda_{i,j}\ln 10}{5\beta} n_{i,j} \qquad j \in \mathcal{B}_i \tag{15}$$

where $\mathcal{B}_i$ is the set of the indices of the source node i connected to the source node j. $j \in \mathcal{B}_i$ represents that source node i is connected to source node j. Here $\mathbf{Q} \in \mathbb{R}^{2N\times 2N}$ is represented as a sparse matrix, the non-zero elements of which are given by

$$\begin{cases} \mathbf{Q}_{[2i-1,2i-1]} = \mathbf{Q}_{[2i,2i]} = 1 \\ \mathbf{Q}_{[2j-1,2j-1]} = \mathbf{Q}_{[2j,2j]} = 1 \\ \mathbf{Q}_{[2i,2j]} = \mathbf{Q}_{[2j,2i]} = -1 \\ \mathbf{Q}_{[2i-1,2j-1]} = \mathbf{Q}_{[2j-1,2i-1]} = -1 \end{cases} \tag{16}$$

If there are K source-source measurements, the matrix form of (15) is

$$\mathbf{J}_2\Delta\theta = \mathbf{q}_2 + \varphi_2 \tag{17}$$

where $\mathbf{J}_2 \in \mathbb{R}^{K\times 3N}$, $\mathbf{q}_2 \in \mathbb{R}^K$ and $\varphi_2 \in \mathbb{R}^K$. The row vectors of $\mathbf{J}_2$, $\mathbf{q}_2$ and φ_2 are respectively equal to $[2\mathbf{x}^{eT}\mathbf{Q} \quad -\mu_{i,j}]$, $[\lambda_{i,j}^e - \mathbf{x}^{eT}\mathbf{Q}\mathbf{x}^e]$ and $\left[\frac{\lambda_{i,j}\ln 10}{5\beta} n_{i,j}\right]$.

By combining (14) with (17), the matrix form can be rewritten as

$$\mathbf{J}\Delta\theta = \mathbf{q} + \varphi \tag{18}$$

where $\mathbf{J} = [\mathbf{J}_1^T \quad \mathbf{J}_2^T]^T$, $\mathbf{q} = [\mathbf{q}_1^T \quad \mathbf{q}_2^T]^T$ and $\varphi = [\varphi_1^T \quad \varphi_2^T]^T$. $\mathbf{J} \in \mathbb{R}^{(L+K)\times 2(N-M)}$, $\mathbf{q} \in \mathbb{R}^{L+K}$ and $\varphi \in \mathbb{R}^{L+K}$. The covariance matrix for φ, denoted by $\mathbf{\Sigma}$, is

$$\mathbf{\Sigma} = \operatorname{diag}\left\{\frac{\lambda_{i,j}^2 \ln 10^2}{25\beta^2}\delta_{i,j}^2\right\} \tag{19}$$

Then the best linear unbiased estimate of refined position $\Delta\theta$ is

$$\Delta\theta = \left(\mathbf{J}^T\mathbf{\Sigma}^{-1}\mathbf{J}\right)^{-1}\mathbf{J}^T\mathbf{\Sigma}^{-1}\mathbf{q} \tag{20}$$

Extract from $\Delta\theta$ to obtain $\Delta\mathbf{x}$, which is added to $\mathbf{x}^e$ for giving a refined version of the source location vector. In summary, the algorithm first estimates the initial source location by evaluating (11) and then is applied (20) to refining the source location.

4. **Cramer-Rao Low Bound.** The CRLB defines a lower bound on the variance of any unbiased estimator and is employed as a benchmark for evaluating the performance of estimators. The CRLB of the unknown parameters is the diagonal elements of the inverse of the Fisher information matrix (FIM). Since the transmit power of the source nodes is not available to the estimator, it should also be taken into account as an unknown parameter. Let us recall the vector of unknown parameter $\mathbf{\Phi} = [\mathbf{x}^T \quad \mathbf{p}_0^T]^T$. Here when the transmit power $\mathbf{p}_0$ is unknown, the FIM is denoted as $\mathbf{F}_u$, which is also written as

$$\mathbf{F}_u = -\frac{\partial^2 \ln P(\mathbf{p}|\mathbf{\Phi})}{\partial \mathbf{\Phi}^T \partial \mathbf{\Phi}} \tag{21}$$

where $P(\mathbf{p}|\mathbf{\Phi}) = \prod_{i=1}^{N} \prod_{j=1}^{M+N} \frac{1}{\sqrt{2\pi}\delta_{i,j}} \exp\left\{-\frac{(p_{i,j}-p_{i,0}+10\beta\log_{10}d_{i,j})^2}{2\delta_{i,j}^2}\right\}$, $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, M+N$ and $i < j$, $\mathbf{F}_u \in \mathbb{R}^{3N\times 3N}$. Therefore, $\mathbf{F_u}$ can be further represented as

$$\mathbf{F_u} = \begin{bmatrix} \mathbf{F} & \mathbf{U} \\ \mathbf{U}^T & \mathbf{V} \end{bmatrix} \tag{22}$$

where

$$\mathbf{F} = -\frac{\partial^2 \ln P(\mathbf{p}|\mathbf{x})}{\partial \mathbf{x}^T \partial \mathbf{x}} \tag{23}$$

The elements of matrix $\mathbf{F} \in \mathbb{R}^{2N\times 2N}$ can be further represented as

$$\begin{cases} [\mathbf{F}]_{2i-1:2i,2i-1:2i} = \sum_j \frac{100\beta^2}{(\ln 10)^2\delta_{i,j}^2} \frac{(\mathbf{x}_i-\mathbf{x}_j)^T(\mathbf{x}_i-\mathbf{x}_j)}{d_{i,j}^4} & j \in \mathcal{A}_i \cup \mathcal{B}_i \\ [\mathbf{F}]_{2i-1,2j-1} = [\mathbf{F}]_{2j-1,2i-1} = \frac{-100\beta^2}{(\ln 10)^2\delta_{i,j}^2} \frac{(x_i-x_j)^2}{d_{i,j}^4} & j \in \mathcal{A}_i \\ [\mathbf{F}]_{2i-1,2j} = [\mathbf{F}]_{2j,2i-1} = \frac{-100\beta^2}{(\ln 10)^2\delta_{i,j}^2} \frac{(x_i-x_j)(y_i-y_j)}{d_{i,j}^4} & j \in \mathcal{A}_i \\ [\mathbf{F}]_{2j-1,2i} = [\mathbf{F}]_{2i,2j-1} = \frac{-100\beta^2}{(\ln 10)^2\delta_{i,j}^2} \frac{(x_i-x_j)(y_i-y_j)}{d_{i,j}^4} & j \in \mathcal{A}_i \\ [\mathbf{F}]_{[2j,2i]} = [\mathbf{F}]_{2i,2j} = \frac{-100\beta^2}{(\ln 10)^2\delta_{i,j}^2} \frac{(y_i-y_j)^2}{d_{i,j}^4} & j \in \mathcal{A}_i \end{cases} \tag{24}$$

The elements of matrices $\mathbf{U} \in \mathbb{R}^{N\times 2N}$ and $\mathbf{V} \in \mathbb{R}^{N\times N}$ are rewritten as

$$\begin{cases} [\mathbf{U}]_{2i-1:2i} = \frac{10\beta}{(\ln 10)\delta_{i,j}^2} \frac{(\mathbf{x}_i-\mathbf{x}_j)}{d_{i,j}^3} & j \in \mathcal{A}_i \cup \mathcal{B}_i \\ [\mathbf{V}]_i = \sum_j \frac{1}{\delta_{i,j}^2} & j \in \mathcal{A}_i \cup \mathcal{B}_i \end{cases} \tag{25}$$

So the CRLB of source locations is denoted as $\text{CRLB}([\mathbf{x}]_r)$, which can be calculated by

$$\text{CRLB}([\mathbf{x}]_r) = [\mathbf{F} - \mathbf{U}\mathbf{V}^{-1}\mathbf{U}^T]_{r,r}^{-1} \tag{26}$$

where $r = 1, 2, \ldots, 2N$.

5. **Evaluation.**

5.1. **Impacts of noises.** To test the performance of location refinement, we conduct a group of simulations with 12 nodes deployed in a 100 m × 100 m square region. We randomly select 6 nodes as anchors which are used to locate other 6 source nodes. When the range between nodes is less than 80 m, the RSS is considered as measurable. The transmit power p_0 is set at -45 dB and assumed to be unknown. Path loss exponent β is set to 2. All results are averages of 1000 independent Monte Carlo (MC) runs.

When the shadow fading δ^2 is varied from 0.02^2 to 0.2^2, Figure 1 plots the performance with different methods under different shadow fading δ^2. Only source-anchor measurements are involved into the noncooperative refinement, so the mean square error (MSE) in log scale of noncooperative refinement is larger than that of the cooperative refinement which utilizes not only the source-anchor measurements but also the source-source measurements. It can be seen from Figure 1 that the position MSE in log scale is approximately linear with the shadow fading δ^2 in log scale. The MSE performance degrades

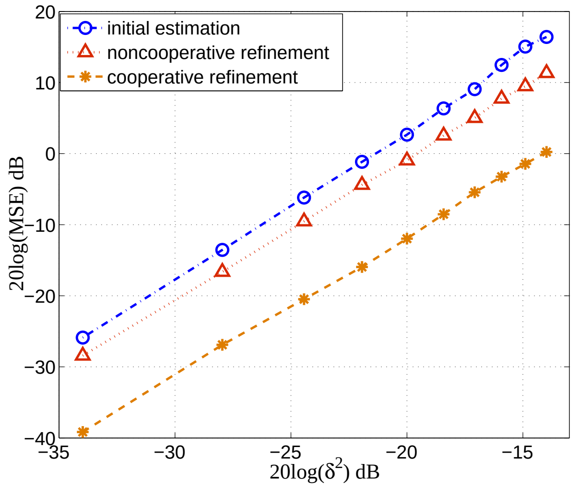
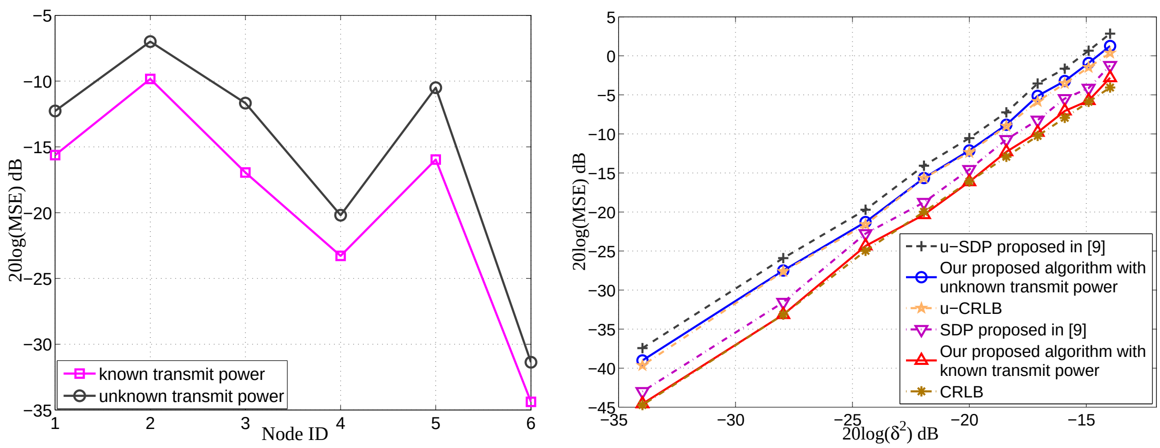


FIGURE 1. Impacts of noises



(a) MSE comparison in different 6 source nodes (b) Comparison of different approaches

FIGURE 2. Performance comparison of different methods

as the shadow fading δ^2 increases. For instance, when the shadow fading δ^2 in log scale is set to -34 dB, the position MSE in log scale of initial estimation is about -26.4 dB. When the shadow fading δ^2 in log scale is set to -14 dB, the position MSE in log-scale of initial estimation is 17.2 dB.

5.2. **Comparison with different methods.** In this subsection, the transmit power p_0 is set at -45 dB and assumed to be known or unknown. When δ^2 is also set to 0.1^2, Figure 2(a) plots the performance comparison of different conditions with known or unknown transmit power. The MSE in log scale of each located node under known transmit power is always less than that of unknown transmit power. For example, when the transmit power is assumed to be known, the MSE in log scale of node 1 is –15.6 dB, which is less than –12.3 dB under unknown transmit power.

Similar to the impact of received signal noises, the simulations are conducted to test the performance comparison under different methods. When $\boldsymbol{\Sigma}_j$ is set to $[0.1^2 \quad 0; 0 \quad 0.1^2]$ and δ^2 is varied from 0.02^2 to 0.2^2 (The shadow fading δ^2 in log scale is varied from -34 dB to -14 dB), Figure 2(b) plots the performance comparison of different conditions as the received signal noises increase. u-CRLB represents the CRLB of unknown transmit power which is calculated according to [9]. When the noises are all set to 0.02^2 and the transmit power is assumed to be known, the MSE in log scale is -44.6 dB, which is approximately identical to the position CRLB. When the transmit power is considered as unknown, the MSE in log scale is −39.0 dB, which is close to −39.6 dB of u-CRLB. The performance of SDP algorithm is denoted as u-SDP by considering the transmit power as unknown. Compared with that of u-SDP, the MSE in log scale of our proposed algorithm is reduced by about 2 dB with unknown transmit power.

6. **Conclusion.** The location estimation approach is proposed for RSS-based wireless localization by considering the unknown transmit power. In the stage of initial estimation, the preliminary source locations are obtained by converting the nonlinear equation into the linear equation. Then the location refinement technique is introduced to improve the preliminary locations by utilizing the source-source RSS measurements. Compared with the SDP method, our proposed refinement algorithm can improve localization accuracy significantly when the noises are small. However, the performance of the proposed location estimation approach is not very good in the larger noise condition. The next works are focused on the robustness of the location estimation approach especially when the noises are large.

Acknowledgments. This work is partially supported by Zhejiang provincial Natural Science Foundation LY16F020036 and ZAFU Scientific Research Development Foundation Project 2013FR086.

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