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SELF-REPAIRING CONTROL USING ADAPTIVE ADJUSTING LAWS

Masanori Takahashi

Department of Electrical Engineering and Computer Science
Tokai University
9-1-1, Toroku, Higashi-ku, Kumamoto 862-8652, Japan
masataka@ktmail.tokai-u.jp

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Abstract. *In this paper, a new design method for a self-repairing control system (SRCS) with two adaptive tuners is presented for plants with unknown parameters and lost sensor. The developed SRCS has two automatic mechanisms, which are self-repairing and adaptive parameter tuning mechanisms. The self-repairing function can attain to replace the failed sensor with the backup, and the adaptive tuning function adjusts the controller parameters against unknown plant parameters. This paper shows a theoretical analysis of stability and self-repairing performance, and also explores a simulation to confirm the effectiveness.*
Keywords: Self-repairing, Adaptive and robust control, Dead zone, Sensor failure

1. **Introduction.** Self-repairing control (SRC) is one of the active and intelligent fault tolerant control (FTC) methods [1, 2]. This kind of the FTC can automatically detect the failure and replace the failed control components with the healthy backups. In the previous works [2, 3], for the SRC system (SRCS), the nonlinear and unstable detection filter has been developed that can guarantee exact and early detection against sensor failure of a stuck type. By using this filter, one can specify the maximum detection time in advance. Also, based on high-gain feedback [4], the structure of the SRCS can be simplified, which does not depend on the order of the mathematical model of the plant. However, in designing the controller, it is assumed that the plant parameters are known. As a remedy, an adaptive controller of a switching type has been proposed in [3] where the gain p of the output feedback controller $u = -py$ is adjusted as follows.

$$p(t) = (p_0)^k, \quad t \in [t_k, t_{k+1}), \quad p_0 > 1$$

where $k = 0, 1, \cdots$, is the number of switches, and $t_k \in \mathbb{R}^+$ is the k-th switching time. Of course, the stability of the overall SRCS can be guaranteed theoretically. However, unfortunately, fast-switching often occurs, and the switched gain p sometimes becomes extremely large. For example, if p_0 is set as $p_0 = 10$, then just four switches, $k = 4$, makes p grow to 10000.

As a countermeasure for this problem, this paper proposes a new design method for the SRCS with two adaptive tuners of continuously adjusting laws. By using these tuners, instead of the above-mentioned switched p, it is expected that the feedback gain p can be adequately tuned and one can avoid the excessive adjustment. Furthermore, to cope with the nonlinearity of the detection filter, the well-known dead zone technique [5, 6] is exploited in the adjusting laws.

This paper is organized as follows. In Section 2, the problem on the SRC against sensor failures is described. Section 3 explains a concrete design method for the SRCS with the two adaptive tuners, and also shows the theoretical results on the SRC performance, that is, fault detection, stability and asymptotic convergence of the plant output to the residual region. In Section 4, several numerical simulation results are shown to confirm the effectiveness of the proposed SRCS. Finally, we conclude in Section 5.

2. **Problem Statement.**

Consider a linear time invariant system of the form [4]:

$$\begin{aligned}\Sigma_P : \dot{y} &= ay + bu + \boldsymbol{h}^T \boldsymbol{z} \\ \dot{\boldsymbol{z}} &= \boldsymbol{F}\boldsymbol{z} + \boldsymbol{g}y\end{aligned} \tag{1}$$

where $y \in \mathbb{R}$ is the actual output, $u : \mathbb{R}^+ \to \mathbb{R}$ is the control input, and $\boldsymbol{z} \in \mathbb{R}^{n-1}$ is the state of the plant. Furthermore, $a \in \mathbb{R}$, $b \in \mathbb{R}$, $\boldsymbol{F} \in \mathbb{R}^{n-1\times(n-1)}$ and $\boldsymbol{g} \in \mathbb{R}^{n-1}$ are unknown constants, a matrix and a vector respectively. Here, we assume that the plant Σ_P has minimum-phase characteristic, and the sign of high-frequency gain b is supposed to be positive.

To measure the output y, the two sensors ♯1 (primary) and ♯2 (backup) are exploited. Thus, the measured signal is given by

$$y_S(t) = \begin{cases} y_1(t) & (t \le t_D) \\ y_2(t) & (t > t_D) \end{cases} \tag{2}$$

where $t_D \in \mathbb{R}^+$ is a failure detection time, which will be defined later. Each $y_i \in \mathbb{R}$, $i \in \{1, 2\}$ is the output of the sensor ♯i. If the sensors are healthy, then we have $y_i = y$. From (2), if the failure of the primary sensor ♯1 is detected, then the backup ♯2 is activated.

The failure scenario to be considered here is given as follows.

$$y_1(t) = 0, \quad t \ge t_F \tag{3}$$

where $t_F \in \mathbb{R}^+$ is an unknown failure time.

The SRC problem is to replace the failed sensor automatically so as to maintain the control system stability. The following section will show the concrete design procedure for the SRCS, and analyze the performances theoretically.

3. **Design of the SRCS with the Adaptive Tuners.**

3.1. **Basic structure.**

First of all, we introduce the detection filter Σ_D as follows.

$$\dot{v} = |v| + \gamma + \dot{y}_S + p_1 y_S \tag{4}$$

where $\gamma \in \mathbb{R}^+$ is an arbitrary constant, and $p_1 : \mathbb{R}^+ \to \mathbb{R}$ is an adaptive gain tuned by

$$\dot{p}_1 = D(\xi)(-y_S v) \tag{5}$$

where $D : \mathbb{R}^+ \to \mathbb{R}^+$ is a dead zone given by

$$D(\xi) \triangleq \begin{cases} 1 & (\xi > \delta) \\ 0 & (\xi \le \delta) \end{cases}, \quad \xi \triangleq y_S^2 + v^2 \tag{6}$$

where $\delta \in \mathbb{R}^+$ is an arbitrary small constant, which specifies the radius of the residual region of y and v, that is, the control performance. The details will be discussed later.

Next, the adaptive controller Σ_C is designed as follows.

$$u = -p_2 (y_S + v) \tag{7}$$

The adaptive feedback gain $p_2 : \mathbb{R}^+ \to \mathbb{R}^+$ is tuned by

$$\dot{p}_2 = D(\xi) (y_S + v)^2 \tag{8}$$

On the time period $[0, t_F)$ where the sensor is healthy, i.e., $y_S(t) = y(t)$, $t \in [0, t_F)$, the overall system can be expressed by

$$\begin{aligned}\dot{y} &= -(bp_2^* - a) y + \boldsymbol{h}^T \boldsymbol{z} - bp_2^* v + b\Delta_2 (y + v) \\ \dot{\boldsymbol{z}} &= \boldsymbol{F}\boldsymbol{z} + \boldsymbol{g}y \\ \dot{v} &= -bp_2^* v + |v| + \gamma - (bp_2^* - p_1^* - a) y + \boldsymbol{h}^T \boldsymbol{z} + b\Delta_2 (y + v) - \Delta_1 y\end{aligned} \tag{9}$$

where $\Delta_1 \triangleq p_1^* - p_1$ and $\Delta_2 \triangleq p_2^* - p_2$, and $p_1^* \in \mathbb{R}^+$ and $p_2^* \in \mathbb{R}^+$ are some positive constants to be estimated by p_1 and p_2 respectively.

From this result, we obtain the following lemma on the system stability.

Lemma 3.1. *On the time period $[0, t_F)$ where the sensor is healthy, all the signals, y, $\boldsymbol{z}$ and v in the overall control system are bounded.*

Proof: On the period, $[0, t_F)$, we consider the following positive function $S : \mathbb{R}^+ \to \mathbb{R}^+$:

$$S = \frac{1}{2}\left\{y^2 + \boldsymbol{z}^T\boldsymbol{P}\boldsymbol{z} + v^2 + \Delta_1^2 + b\Delta_2^2\right\} \tag{10}$$

where $\boldsymbol{P} \in \mathbb{R}^{(n-1)\times(n-1)}$ is a positive definite matrix satisfying

$$\boldsymbol{F}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{F} = -2\boldsymbol{Q} \tag{11}$$

for arbitrarily given, positive definite, $\boldsymbol{Q} \in \mathbb{R}^{(n-1)\times(n-1)}$. The detail of $\boldsymbol{Q}$ will be determined later.

From (9), the time derivative of S can be expressed as follows.

$$\begin{aligned}\dot{S} = &-(bp_2^* - a)\,y^2 + \boldsymbol{h}^T\boldsymbol{z}y - bp_2^*vy + b\Delta_2(y+v)\,y - \boldsymbol{z}^T\boldsymbol{Q}\boldsymbol{z} + \boldsymbol{g}^T\boldsymbol{P}\boldsymbol{z} - bp_2^*v^2 + |v|v \\ &+\gamma v - (bp_2^* - p_1^* - a)\,yv + \boldsymbol{h}^T\boldsymbol{z}v + b\Delta_2(y+v)\,v - \Delta_1 yv - \Delta_1\dot{p}_1 - b\Delta_2\dot{p}_2\end{aligned} \tag{12}$$

Here, we consider the constants p_1^* and p_2^* satisfying

$$bp_2^* - p_1^* - a > 0 \tag{13}$$

Then, it follows that

$$\begin{aligned}\dot{S} \le &-\frac{1}{2}\left(p_1^* - a - 1 - \|\boldsymbol{P}\|^2\right)y^2 - \left(\lambda_{\min}[\boldsymbol{Q}] - \|\boldsymbol{g}\|^2 - \|\boldsymbol{h}\|^2\right)\|\boldsymbol{z}\|^2 \\ &-\frac{1}{2}\left(p_1^* - a - \frac{\gamma^2}{\lambda} - 3\right)v^2 + \lambda + \Delta_1(-yv - \dot{p}_1) + \Delta_2\left\{(y+v)^2 - \dot{p}_2\right\}\end{aligned} \tag{14}$$

For analysis, we shall consider the two modes, that is, one is the case where $\xi > \delta$ (Mode I), and the other is the case where $\xi \le \delta$ (Mode II).

Mode I: Because $\xi > \delta$, we have $D(\xi) = 1$. Hence, from (5) and (8), it follows that

$$\begin{aligned}\dot{S} \le &-\underbrace{\frac{1}{2}\left(p_1^* - a - 1 - \|\boldsymbol{P}\|^2\right)}_{\alpha_1}y^2 - \underbrace{\left(\lambda_{\min}[\boldsymbol{Q}] - \|\boldsymbol{g}\|^2 - \|\boldsymbol{h}\|^2\right)}_{\alpha_2}\|\boldsymbol{z}\|^2 \\ &-\underbrace{\frac{1}{2}\left(p_1^* - a - \frac{\gamma^2}{\lambda} - 3\right)}_{\alpha_3}v^2 + \lambda\end{aligned} \tag{15}$$

where $\lambda \in \mathbb{R}^+$ is a constant introduced for analysis, which is not a designed parameter.

Now, we choose p_1^* and $\boldsymbol{Q}$ such that $\alpha_i > 0$, $\forall i$. Then, the time derivative of S can be evaluated by

$$\dot{S} \le -\alpha\xi + \lambda, \quad \alpha = \min_{i=1,2,3}\{\alpha_i\}. \tag{16}$$

Therefore, by setting $\lambda = \delta\alpha$, we can get $\dot{S} < 0$ and

$$S < S(t_0) \tag{17}$$

where $t_0 \in \mathbb{R}^+$ is the initial time which the system enters the Mode I.

Mode II: Because of the minimum-phase property of the plant Σ_P, the state $\boldsymbol{z}$ can be evaluated by

$$\|\boldsymbol{z}\| \le \|\boldsymbol{z}(t_1)\|\exp\{-\alpha(t-t_1)\} + \frac{\sqrt{\delta}\|\boldsymbol{g}\|}{\lambda_F}\left[1 - \exp\{-\alpha(t-t_1)\}\right]$$

$$\leq \max\left\{\|\boldsymbol{z}(t_1)\|,\ \frac{\sqrt{\delta}\|\boldsymbol{g}\|}{\lambda_F}\right\} \tag{18}$$

where $\lambda_F \in \mathbb{R}^+$ is a positive constant determined by eigenvalues of the matrix $\boldsymbol{F}$, and $t_1 \in \mathbb{R}^+$ is the initial time at which the system enters the Mode II.

In addition, clearly, in this mode, the adaptive gains p_1 and p_2 (also, the derivations, Δ_1 and Δ_2) do not change because $D(\xi) = 0$ ($\xi \leq \delta$). Hence, it is found that S does not go beyond the initial value $S(t_1)$ or a constant which does not depend on time t.

From the discussions in the Modes I and II, we can conclude that S is bounded, and so all the signals are bounded on the time period $[0, t_F)$. □

3.2. **Failure detection.** From Lemma 3.1, because of boundedness of v, there is a finite constant $\Gamma \in \mathbb{R}^+$ so that $|v(t)| < \Gamma$, $t \in [0, t_F)$.

However, if the sensor ♯1 fails, then the detection filter Σ_D can be expressed as

$$\dot{v} = |v| + \gamma \geq \gamma > 0, \quad t \geq t_F \tag{19}$$

Hence, we have $v \geq \gamma(t - t_F) + v(t_F) \triangleq \tilde{v}$. Then, taking this unstable behavior of v into consideration, we define the detection time t_D as follows.

$$t_D \triangleq \min\{t \mid |v(t)| \geq \Gamma\} \tag{20}$$

3.3. **Main results.** The control performances of the self-repairing control system can be summarized in the following theorem.

Theorem 3.1. *Consider the self-repairing control system constructed by (1), (2), (4)-(8). Then, the control system has the following properties.*
(P1) If the sensor ♯1 fails, then the detection time t_D exists, and satisfies

$$t_D \leq t_F + \frac{2\Gamma}{\gamma}. \tag{21}$$

(P2) All the signals, y, $\boldsymbol{z}$ and v, are bounded over $[0, \infty)$.
(P3) Regarding the output y, for arbitrarily given, small δ, we have

$$\lim_{t\to\infty} D(\xi)|y(t)| = 0 \tag{22}$$

which means that y asymptotically enters a small region of arbitrarily given radius $\sqrt{\delta}$.

Proof: As mentioned above, if the sensor failure (3) occurs, then there is a time $t_B > t_F$ such that $|\tilde{v}(t_B)| = \Gamma$. Because $|v| > |\tilde{v}|$, the detection time t_D exists so that

$$t_D \leq t_B \leq t_F + \frac{\Gamma + |v(t_F)|}{\gamma} \tag{23}$$

Thus, (P1) is true.

To prove (P2), we shall consider the boundedness of all the signals on the anxious period $[t_F, t_D]$ where the failed sensor ♯1 is still activated. On this period, v is bounded because v does not hit the threshold Γ. Hence, the control input u is also bounded. Furthermore, for bounded u, the plant Σ_P does not have a finite escape time. Therefore, all the signals y, $\boldsymbol{z}$ and v, are bounded on $[t_F, t_D]$.

After the detection time, i.e., $t > t_D$, the sensor ♯1 is replaced, that is, the healthy sensor ♯2 is activated. By the same discussion as Lemma 3.1, all the signals are bounded on the period (t_D, ∞) where the sensor ♯2 is healthy. Thus, (P2) is true.

From (5) and (8), it follows that

$$p_2 + 2p_1 = p_2(0) + 2p_1(0) + \int_0^t D(\xi)\left(y^2 + v^2\right) d\tau \tag{24}$$

From (P2) discussed above, p_1 and p_2 are bounded and so $p_2 + 2p_1$ is also bounded on $[0, \infty)$. Therefore, we have

$$\int_0^{\infty} D(\xi)\left(y^2 + v^2\right) d\tau < \infty \tag{25}$$

Hence, it is found that $\lim_{t\to\infty} D(\xi)\left(y^2 + v^2\right) = 0$, which shows that (P3) holds.

Thus, the proof of Theorem 3.1 is completed. □

Remark 3.1. *Regarding to selection of* Γ*, from (17), the largest candidate for* Γ *can be chosen as* $\Gamma \geq \sqrt{2S(t_0) + \delta}$*. Fortunately, if it is assumed that the failure time* t_F *is sufficiently large (late), then one can choose a smaller candidate* Γ *such that* $\Gamma > \sqrt{\delta}$ *because of the property (P3). In addition, the parameter* γ *is independent of the preceding* Γ*. Hence, from (P1), for given* Γ*,* γ *can be chosen so large that the maximum detection time can be shortened arbitrarily.*

4. **Numerical Example.** This section shows the numerical simulation results to confirm the effectiveness of the proposed method.

Consider the following unstable second order system:

$$\begin{aligned} \Sigma_P : \dot{y} &= y + u + z, \quad y(0) = 1 \\ \dot{z} &= -z + y, \quad z(0) = -0.5 \end{aligned} \tag{26}$$

Suppose that the failure time t_F is set as

$$t_F = 25.0 \text{ [s]}$$

The design parameters are given by

$$\gamma = 1, \quad \Gamma = 2, \quad \delta = 0.5, \quad p_1(0) = p_2(0) = 0$$

Hence, the maximum detection time is estimated as follows.

$$t_D \leq t_F + \frac{2\Gamma}{\gamma} = 29.0 \text{ [s]} \tag{27}$$

The simulation results are shown in Figures 1 and 2. In Figure 1, the actual output y (top) and the absolute value of the filtered signal v (bottom) are shown. Figure 2 shows the time response of the adaptive gains p_1 and p_2. From these results, it can be shown that the unstable plant (26) can be stabilized before and after the failure time. The output y remains the small region of radius $\sqrt{\delta} \simeq 0.7$, and we can find $y(50) \simeq -0.49$. Also, the

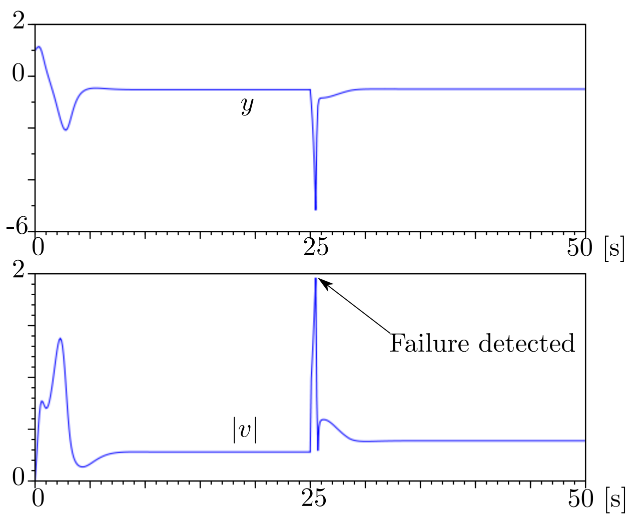

FIGURE 1. Simulation results: y (top) and $|v|$ (bottom)

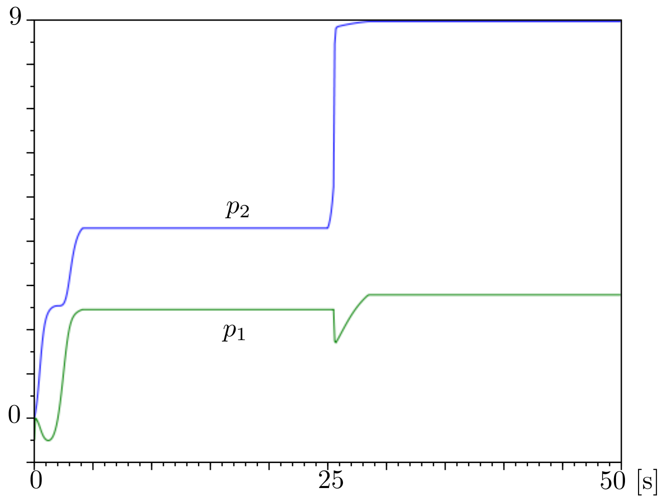

FIGURE 2. Simulation results: adaptive gains p_1 and p_2

failed sensor can be successfully detected and replaced after the failure. The detection time is $t_D \simeq 26.0$ [s]. This is much faster than the maximum time 29.0 [s] estimated by (27). Early and exact fault detection can be attained.

5. **Concluding Remarks.** This paper has presented the SRCS with the adaptive tuner for plants with unknown parameters and sensor failures. The failure scenario considered here, is supposed to be only the loss of effect (LOE), that is, the output of the sensor is stuck at zero. Fortunately, according to [3], if the sign of the gain p_1 does not change during the maximum detection time $2\Gamma/\gamma$, then the modification of the detection filter may make it possible to find the stuck failures of more general types, i.e., $y_1(t) = \varphi$, $t \geq t_F$ where $\varphi \in \mathbb{R}^+$ is the unknown stuck value. However, the concrete design method for such a filter is not clarified theoretically. This problem is still left in the future works.

REFERENCES

[1] Y. Zhang and J. Jiang, Bibliographical review on reconfigurable fault-tolerant control systems, *Annual Reviews in Control*, vol.21, no.2, pp.229-252, 2008.
[2] M. Takahashi, Self-repairing and adaptive control of MIMO systems with faulty sensors, *International Journal of Innovative Computing, Information and Control*, vol.5, no.10(B), pp.3573-3582, 2009.
[3] M. Takahashi, A design method for a self-repairing and adaptive control system based on backstepping, *ICIC Express Letters*, vol.9, no.4, pp.1043-1048, 2015.
[4] A. Ilchmann and E. P. Ryan, Universal λ-tracking for nonlinearly-perturbed systems in the presence of noise, *Automatica*, vol.30, no.2, pp.337-346, 1994.
[5] Y. Miyasato, Model reference adaptive control for nonlinear systems with unknown orders – General case, *Trans. SICE*, vol.31, no.3, pp.324-333, 1995.
[6] P. A. Ioannou and J. Sun, *Robust Adaptive Control*, Prentice-Hall, 1996.