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COMPUTATIONALLY EFFICIENT SUBSPACE-BASED METHOD FOR ANGLE ESTIMATION WITHOUT EIGENDECOMPOSITION IN BISTATIC MIMO RADAR

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Abstract. *A computationally efficient angle estimation algorithm with a good statistical performance is attractive in multiple-input multiple-output radar system. In this paper, we propose a new computationally simple subspace-based method for joint estimation of direction of departure (DOD) and direction of arrival (DOA), combining multistage Wiener filter (MSWF) and unitary ESPRIT algorithm. The proposed algorithm provides comparable angle estimation performance with unitary ESPRIT algorithm and much better angle estimation performance than ESPRIT algorithm. Due to no need of eigendecomposition operation, theoretical analysis indicates that the proposed algorithm has lower complexity compared with the unitary ESPRIT and ESPRIT algorithms. Numerical results verify the effectiveness of the proposed method.*

Keywords: Bistatic MIMO radar, Direction of departure (DOD), Direction of arrival (DOA), Multistage Wiener filter (MSWF)

1. **Introduction.** Recently, multiple-input multiple-output (MIMO) radar [1-11] which exploits multiple antennas to simultaneously transmit diverse waveforms and utilises multiple antennas to receive the reflected signals has attracted essential interest in radar community as it has a great deal of potential advantages over conventional phased-array radar, such as high angular estimation accuracy, more degrees of freedom (DOF), extended virtual aperture and strengthening parameter identifiability [2,3]. In general, in terms of configuration of its transmitter and receiver antennas, MIMO radar can be classified into two types. One is collocated radar, which includes bistatic and monostatic MIMO radars, whose transmitters and receivers are closely spaced. The other is statistical radar, whose transmitters and receivers are widely spaced.

For direction of departures (DODs) and direction of arrivals (DOAs) estimation for bistatic MIMO radar, several direction estimation algorithms [4-10] have been developed. The two-dimensional multiple signal classification (2D-MUSIC) algorithm proposed in [4] is computationally expensive due to two-dimensional spatial spectrum peak search. In order to alleviate computational burden, Zhang et al. [5] introduced a reduced-dimensional multiple signal classification (RD-MUSIC) algorithm with angle estimation performance approximating that of the 2D-MUSIC algorithm using the property of the Kronecker product, where only one-dimension search is performed. Chen et al. [6] applied estimation of signal parameters via rotational invariance technique (ESPRIT) to estimating the DODs and DOAs of targets, which uses the rotational invariance property of both transmit and receive arrays and needs additional pairing procedure. Chen et al. [7] investigated the relationship between the two one-dimensional ESPRITs and addressed the problem of automatically paired DODs and DOAs estimation. Bencheik et al. [8] exploited polynomial root-finding technique to estimate the DODs and DOAs without additional pairing procedure. However, the root determination of the polynomial is very time consuming as the number of transmit antennas and receive antennas increases. In [9], a combined ESPRIT-Root MUSIC algorithm is proposed without pairing, which allows great reduction in computing time by comparison with the polynomial root-finding technique. Zheng et al. [10] proposed unitary ESPRIT algorithm with automatic pairing, which affords high angle estimation accuracy and reduces computational complexity due to real-valued processing and doubles the number of data samples.

The aforementioned methods have common features. One is eigen-value decomposition (EVD), which is computationally expensive especially for a large MIMO array, and another is satisfied solution quality only when signal-to-noise ratio (SNR) is moderately high. To tackle the problems, we proposed a computationally efficient algorithm based on multistage Wiener filter (MSWF) [12,13] and unitary ESPRIT algorithm, and the signal subspace can be acquired by the forward recursion of the MSWF without EVD, which only requires matrix-vector products. There are some differences between the proposed algorithm and EVD-based algorithms (ESPRIT and unitary ESPRIT). Firstly, the proposed algorithm does not require its EVD, and only involves real matrix-products. Secondly, the proposed algorithm has much lower computational complexity than EVD-based algorithms.

2. **Data Model.**

Consider a bistatic MIMO radar system composed of an M-element transmitting array and an N-element receiving array, both of which are half-wavelength spaced uniform linear arrays [5-7]. The transmitting antennas transmit orthogonal waveforms, which have identical bandwidth and center frequency. Suppose that there are K uncorrelated targets located at the same range. The DOD and DOA of the kth target with respect to the transmitting array normal and the receiving array normal are signified by θ_k and ϕ_k $(k = 1, \ldots, K)$, respectively. Then, the output of the all matched filters in all receivers can be formulated as [5-7]

$$\mathbf{y}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_K]$ is an $MN \times K$ matrix composed of the K steering vectors, $\mathbf{a}_k = \mathbf{a}_r(\phi_k) \otimes \mathbf{a}_t(\theta_k)$ denotes the Kronecker product of the transmit and the receive steering vectors for the kth; $\mathbf{a}_r(\phi_k) = [1,\ \exp(j\pi v_k), \ldots, \exp(j\pi(N-1)v_k)]^T$, $\mathbf{a}_t(\theta_k) = [1,\ \exp(j\pi u_k), \ldots, \exp(j\pi(N-1)u_k)]^T$, where $v_k = \sin\phi_k$ and $u_k = \sin\theta_k$; $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_K]^T$ is the transmit signal vector, where $s_k(t) = \alpha_k e^{j2\pi f_{dk} t}$, α_k and f_k being the amplitude and Doppler frequency, respectively [5,6]; $\mathbf{n}(t)$ is an $MN \times 1$ complex white noise vector with zeros mean and covariance matrix $\sigma^2\mathbf{I}_{MN}$.

3. **The Proposed Angle Estimation Algorithm.**

3.1. **Real processing.**

Suppose that $\mathbf{Y}$ is expressed as the data matrix composed of L snapshots $\mathbf{y}(t_l)$, $1 \le l \le L$. The extended data matrix which is centro-Hermitian can be defined as $\mathbf{Z} = [\mathbf{Y}\mathbf{\Pi}_{MN}\mathbf{Y}^*\mathbf{\Pi}_L]$, where $\mathbf{\Pi}_{MN}$ signifies the exchange matrix with K ones on its anti-diagonal and zeros elsewhere and $(\cdot)^*$ denotes the complex conjugate. Then, the real-valued extended data matrix $\mathbf{\Gamma}$ is obtained by applying unitary transformation on the complex-valued extended data matrix as the following [10]

$$\mathbf{\Gamma} = \mathbf{Q}_{MN}^H \mathbf{Z} \mathbf{Q}_{2L} \in R^{MN \times 2L} \tag{2}$$

where $(\cdot)^H$ represents the complex conjugate transpose, and $\mathbf{Q}_K$ denotes the unitary transformation matrix, expressed as

$$\mathbf{Q}_K = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_K & j\mathbf{I}_K \\ \mathbf{\Pi}_K & -j\mathbf{\Pi}_K \end{bmatrix}, \quad \mathbf{Q}_{2K+1} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_K & 0 & j\mathbf{I}_K \\ \mathbf{0}^T & \sqrt{2} & \mathbf{0}^T \\ \mathbf{\Pi}_K & 0 & -j\mathbf{\Pi}_K \end{bmatrix} \tag{3}$$

Then, the real-valued covariance matrix can be estimated as

$$\hat{\mathbf{R}}_{real} = \frac{1}{(2L)} \mathbf{\Gamma}\mathbf{\Gamma}^H \tag{4}$$

3.2. **Signal subspace estimation.** Multistage Wiener filter was proposed by Goldstein [12] as an effective reduced-dimension method, which can solve the Wiener-Hopf equation without inverse of the covariance matrix, and an efficient forward recursion of the multistage Wiener filter based on data-level lattice structure [13] by the set of recursions can be expressed as

Initialization: $\mathbf{d}_0(t)$ and $\mathbf{x}_0(t) = \hat{\mathbf{R}}_{real}$
Forward recursion: For $i = 1, 2, \ldots, K$

$$\mathbf{h}_i = \mathbf{E}[\mathbf{x}(t)_{i-1}\mathbf{d}_{i-1}^*(t)] / \left\|\mathbf{x}(t)_{i-1}\mathbf{d}_{i-1}^*(t)\right\|_2$$

$$\mathbf{d}(t) = \mathbf{h}_i^H \mathbf{x}_{i-1}(t),\ \mathbf{x}_i(t) = \mathbf{x}_{i-1}(t) - \mathbf{h}_i\mathbf{d}_i(t)$$

where $\mathbf{d}_0(t)$ is mean of $\hat{\mathbf{R}}_{real}$ line, and $\hat{\mathbf{R}}_{real}$ is estimation real-value covariance matrix. Performing the algorithm above, we can obtain the pre-filtering matrix as $\mathbf{T}_K = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K]$. It has been proven in [13,16] that the matched filters $\mathbf{h}_i$ are orthonormal, and the signal subspace $\mathbf{E}_s$ can be spanned by these matching filters. Then we have $\mathbf{E}_s = \text{span}[\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K]$.

3.3. **Angle estimation.** Then, the real-valued invariance relation can be given as [10]

$$\mathbf{K}_2^\theta \mathbf{d}_k = \tan(\frac{\pi u_p}{2}) \mathbf{K}_1^\theta \mathbf{d}_k \tag{5}$$

where $\mathbf{K}_1^\theta = \text{Re}\left\{\mathbf{Q}_{(M-1)N}^H diag^N \left\{\mathbf{J}_2^\theta\right\} \mathbf{Q}_{MN}\right\}$ and $\mathbf{K}_2^\theta = \text{Im}\left\{\mathbf{Q}_{(M-1)N}^H diag^N \left\{\mathbf{J}_2^\theta\right\} \mathbf{Q}_{MN}\right\}$ are real-valued matrices. $\mathbf{d}_k = \mathbf{Q}_{MN}^H \mathbf{a}_k$ denotes a real-valued steering vector. Thus, the real-valued invariance equation for the transmitting array is given as [10] $\mathbf{K}_2^\theta \mathbf{E}_s = \mathbf{K}_1^\theta \mathbf{E}_s \mathbf{\Psi}_\theta$ where $\mathbf{\Psi}_\theta = \mathbf{T}^{-1}\mathbf{\Phi}_\theta \mathbf{T}$ and $\mathbf{\Phi}_\theta = diag[\tan(\pi u_1/2), \tan(\pi u_2/2), \ldots, \tan(\pi u_k/2)]$ denotes a real-valued diagonal matrix whose diagonal components contain the desired DOD information. Likewise, the real-valued invariance equation for the receiving array can be obtained by

$$\mathbf{K}_2^\phi \mathbf{E}_s = \mathbf{K}_1^\phi \mathbf{E}_s \mathbf{\Psi}_\phi \tag{6}$$

where

$$\mathbf{K}_1^\phi = \text{Re}\left\{\mathbf{Q}_{(M-1)N}^H \mathbf{J}_2^\phi \mathbf{Q}_{MN}\right\} \text{ and } \mathbf{K}_2^\phi = \text{Im}\left\{\mathbf{Q}_{(M-1)N}^H \mathbf{J}_2^\phi \mathbf{Q}_{MN}\right\},$$

$$\mathbf{J}_1^\phi = [\mathbf{I}_{M(N-1)\times M(N-1)} \mathbf{0}_{M(N-1)\times M}] \text{ and } \mathbf{J}_2^\phi = [\mathbf{0}_{M(N-1)\times M} \mathbf{I}_{M(N-1)\times M(N-1)}].$$

$\mathbf{\Psi}_\phi = \mathbf{T}^{-1}\mathbf{\Phi}_\phi \mathbf{T}$, $\mathbf{\Phi}_\phi = diag[\tan(\pi v_1/2),\ \tan(\pi v_2/2), \ldots, \tan(\pi v_k/2)]$ denotes a real-valued diagonal matrix whose diagonal components contain the desired DOA information. Note that $\mathbf{\Psi}_\theta + j\mathbf{\Psi}_\phi$ can be spectrally decomposed as

$$\mathbf{\Psi}_\theta + j\mathbf{\Psi}_\phi = \mathbf{T}^{-1} \left\{\mathbf{\Phi}_\theta + j\mathbf{\Phi}_\phi\right\} \mathbf{T} \tag{7}$$

Then, the DODs and DOAs can be obtained by

$$\hat{\theta}_k = \arcsin\left\{2\arctan([\mathbf{\Phi}_\theta]_{kk})/\pi\right\},\ \hat{\phi}_k = \arcsin\left\{2\arctan([\mathbf{\Phi}_\phi]_{kk})/\pi\right\},\ k = 1, \ldots, K \tag{8}$$

Till now, we have achieved the proposal for a computationally efficient angle estimation algorithm in bistatic MIMO radar.

4. **Computational Complexity.** Compared with EVD-based methods (unitary ESPRIT and ESPRIT), the computational load of the presented method is dominated by obtaining the signal subspace. All the operations of the MSWF only involve real matrix-vector products and the computational complexity of each matched filter h_i, $i = (1, 2, \ldots, P)$ is $o(M^2N^2)$. Thus, the major computational complexity of the presented method is only $o(PM^2N^2)$. However, EVD-based methods' major computational complexity is $o(M^3N^3)$. Moreover, the proposed algorithm has the following advantages: (a) it has much lower computational complexity than the ESPRIT [6] and unitary ESPRIT algorithms [10]; (b) it enjoys the comparable angle estimation accuracy with unitary ESPRIT and much better angle estimation performance than ESPRIT algorithm; (c) it is effective in the case of small snapshots.

5. **Simulation Results.** In this section, a host of computer simulation results are shown to verify the angle estimation performance of the presented method, compared with the unitary ESPRIT [10] and ESPRIT [6] algorithms. In all following simulations, the bistatic MIMO radar is considered and 100 Monte-Carlo simulations are used in the experiments. Assume that there exist three uncorrelated targets, which are located at angle $(\theta_1, \phi_1) = (10^\circ, -8^\circ)$, $(\theta_2, \phi_2) = (0^\circ, 0^\circ)$ and $(\theta_3, \phi_3) = (-10^\circ, 8^\circ)$, respectively, and root mean squared error (RMSE) is exploited, which is defined in [10].

Figure 1 shows angle estimation results of the proposed algorithm with $M = 8$, $N = 8$, $L = 100$ and $SNR = 10$dB. It is indicated in Figure 1 that the transmit angles and receive angles can be clearly observed and are correctly paired. Thus, the usefulness of the proposed method is verified. Figure 2 presents the angle estimation performance comparison, where we compare the proposed algorithm with unitary ESPRIT algorithm [10], ESPRIT [6] algorithm, and CRB [14,15]. From Figure 2, we can find that the proposed algorithm has much better angle estimation performance than the ESPRIT algorithm, and the proposed algorithm has very close angle estimation performance to unitary ESPRIT algorithm. Figure 3 proposes the complexity comparison with $L = 200$, $K = 3$. From Figure 3, we observe that the proposed algorithm has much less computational complexity than EVD-based methods, especially when the number of transmit array elements and receive array elements is large. Moreover, we can see from Figure 4 that the runtime of the proposed method is much shorter than EVD-based methods, especially in the case of large number of transmit and receive antennas. Thus, the proposed algorithm is much more computationally efficient than the ESPRIT and unitary ESPRIT algorithms.

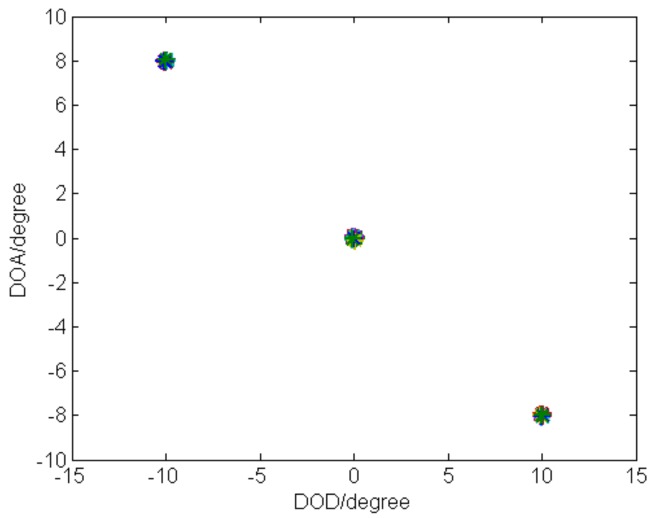

FIGURE 1. Angle estimation results

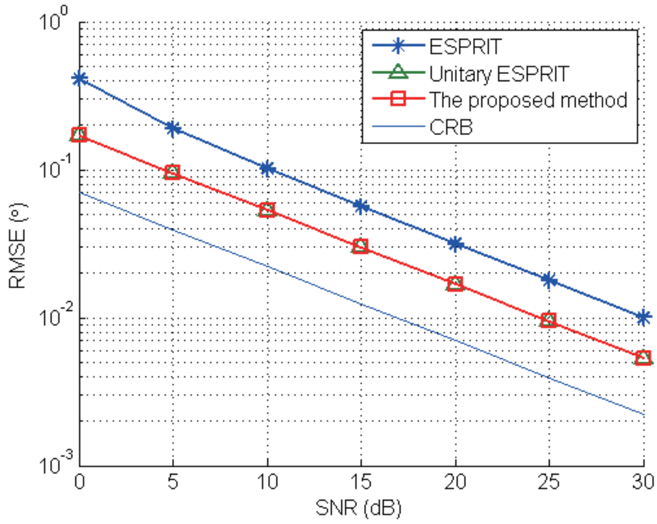

FIGURE 2. Estimation performance comparison

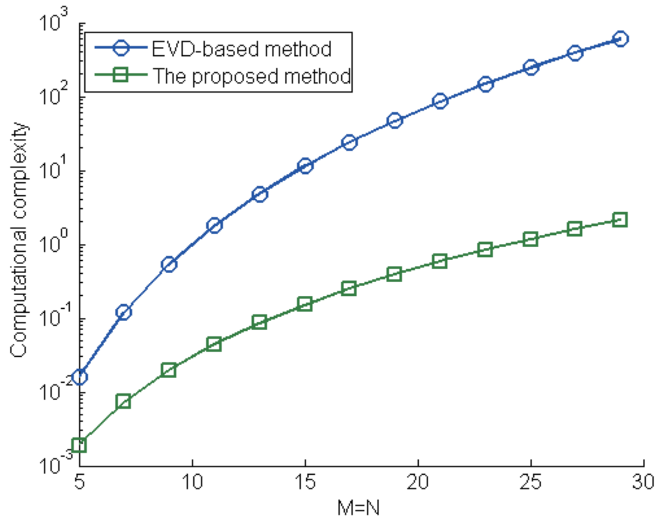

FIGURE 3. Complexity comparison against $M = N$

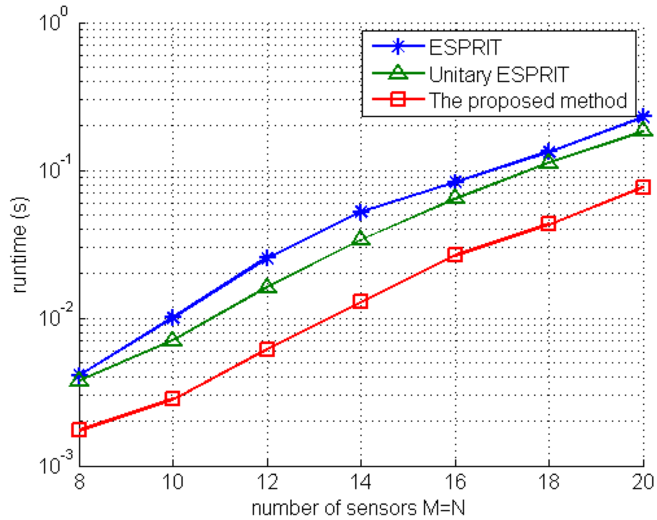

FIGURE 4. Runtime of three algorithms against $M = N$ with $L = 200$, $SNR = 10$dB

6. **Conclusions.** In this paper, we proposed a new computationally efficient angle estimation algorithm for joint DOD and DOA estimation in bistatic MIMO radar with uniform linear arrays, which does not require computationally cumbersome EVD for covariance matrix and an additional pairing operation between the target's DOD and DOA. Furthermore, the real-valued covariance matrix can be obtained by unitary transformation. Thus, the proposed algorithm is much more computationally efficient than the ESPRIT and unitary ESPRIT algorithms, especially for a large MIMO array. Several simulation results indicate that angle estimation performance of the proposed algorithm is very similar to that of unitary ESPRIT algorithm and much better than that of ESPRIT algorithm.

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