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OPTIMAL INTERVAL ASSIGNMENT FOR ENERGY HARVESTING COGNITIVE RADIO WITH IMPERFECT SPECTRUM SENSING

Cunzhou Shi, Ming Jin and Youming Li

College of Information Science and Engineering
Ningbo University
No. 818, Fenghua Road, Jiangbei District, Ningbo 315211, P. R. China
cunzhoushi@yahoo.com; { jinming; liyouming }@nbu.edu.cn

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Abstract. *This work focuses on maximizing the ergodic capacity of energy harvesting cognitive radio with unstable energy arriving rate and imperfect spectrum sensing. We formulate the ergodic capacity maximization problem as a joint optimization problem of the harvesting and sensing durations, i.e., the optimization of interval assignment. To lower the computational complexity, an equivalent version of the joint optimization problem is proposed, and it is verified that the equivalent one is concave. With this, we propose a greedy iteration algorithm for achieving the optimal solution to the problem. Finally, numerical results validate the proposed algorithm.*
Keywords: Cognitive radio, Energy harvesting, Interval assignment, Joint optimization, Ergodic capacity maximization

1. **Introduction.** In conventional wireless communications, the spectral recourses are allocated to primary users (PUs), which have the authority to exclusively use the spectral resources. However, field measurement results have shown that the spectral recourses are of great inefficiency [1]. Cognitive radio (CR) is a promising technology to improve the spectral efficiency by opportunistically accessing to the channels of PUs. To avoid interfering the PUs, cognitive users (CUs) have to periodically sense the channels of PUs. A variety of spectrum sensing algorithms, such as the energy detection and cyclostationarity detection [2], have been proposed. Energy detector is widely used for its low complexity, and its performance depends on the sensing duration length. In [3], the sensing duration was optimized for achieving the maximum throughput of CUs. However, energy consumption, which depends on the length of sensing duration, was not considered there. In some scenarios, energy consumption is of great concern from the CU's perspective.

Cognitive radio with energy harvesting [4] has attracted the attention of researchers in recent years. In [5], the authors investigated the optimal energy-saving ratio to maximize the throughput of CUs. However, the sensing error was not considered. In [6], the throughput was optimized based on the multi-slot spectrum sensing with the assumption of stable energy arriving rate. In practice, the energy arriving rate is unstable.

In this paper, we consider the optimization of energy harvesting and spectrum sensing durations for energy harvesting CR with unstable energy arriving rate and imperfect spectrum sensing. Synchronized mode between PU and CU frames is employed [7]. Due to the duplex-constrained hardware [8, 9], the frame structure of CU is harvesting-sensing-transmitting, which consists of energy harvesting, channel sensing and data transmission. The performance of energy harvesting CR depicted by the ergodic capacity is related with the frame structure. From one side, the more time spent on energy harvesting the more energy can be applied to sensing and transmitting, which could potentially improve the sensing accuracy and ergodic capacity. From the other side, even though more time spent on energy harvesting or sensing leads to more energy harvested or more precise sensing outcome, less time or energy is left for transmission, which generates a negative effect on ergodic capacity. Motivated by this contradictory factor, we investigate the optimal saving-ratio and sensing-ratio within a frame to maximize the ergodic capacity.

The remainder of this paper is organized as follows. Section 2 presents the system model. Problem formulation and analysis are illustrated in Section 3. In Section 4, numerical results and discussions of the proposed algorithm are given. Finally, some conclusions are made in Section 5.

The main notations used in this paper are as follows. ρ_1 denotes energy harvesting saving-ratio; ρ_2 stands for spectrum sensing-ratio; χ is the energy harvesting rate; P_d is the detection probability, and P_f represents the false alarm probability; p_s is the power consumption of spectrum sensing.

2. **System Model.** We consider a scenario consisting of primary and cognitive users, where the cognitive user (CU) opportunistically accesses a channel allocated to the primary user (PU). The CU employs the harvesting-sensing-transmitting protocol, and an energy harvesting module is used for supporting the energy consumption of spectrum sensing and data transmission. The frame structure of the CU is given in Figure 1, where each frame consists of an energy harvesting phase with the duration $\rho_1 T$, a spectrum sensing phase with the duration $\rho_2 T$ and a data transmission phase with the duration $(1-\rho_1-\rho_2)\,T$. In our design, the "data transmission" duration will be used for harvesting energy when the spectrum sensing phase makes a decision that the PU is in active state.

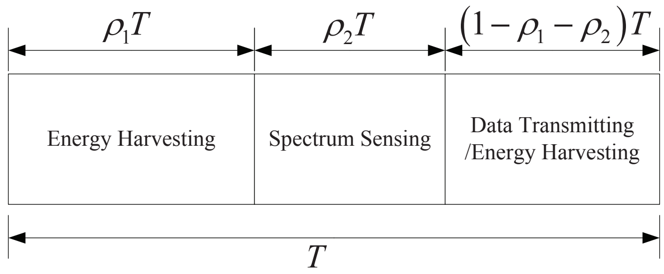

FIGURE 1. Frame structure of the energy harvesting cognitive user

In the energy harvesting phase, the energy harvesting module harvests the energy from the ambient environment. Without loss of generality, it is assumed that the arrival rate of harvested energy χ follows the Gamma distribution [8], whose probability density function is given by $f(\chi) = \frac{\chi^{a-1}}{b^a\Gamma(a)} e^{-\frac{\chi}{b}}$, $\chi > 0$ where a, b, $\Gamma(\cdot)$ denote the shape parameter, scale parameter and Gamma function, respectively. We also assume that χ keeps constant within a time frame, which is rational for the reason that the length of a time frame is relatively short.

During the spectrum sensing phase, energy detection is employed. With the binary hypothesis, the received signal $x(t)$ is given by

$$x(t) = \begin{cases} \omega(t), & \mathcal{H}_0 \\ g * s(t) + \omega(t), & \mathcal{H}_1 \end{cases} \tag{1}$$

where $\mathcal{H}_0$ and $\mathcal{H}_1$ denote the PU in the state of silent and active; $\omega(t)$ is the additive white Gaussian noise and assumed to be with mean zero and variance σ_ω^2; $s(t)$ is the primary signal assumed to have mean zero and variance σ_s^2, and g stands for the gain of the primary-cognitive link. The test-statistic of energy detector is given by

$$\Lambda = \frac{1}{\rho_2 T} \int_{\rho_1 T}^{\rho_1 T + \rho_2 T} x^2(t)dt\,. \tag{2}$$

By comparing Λ with a decision threshold λ_{th}, CU makes a decision on the active/silent state of the PU. The definitions of P_d and P_f are, respectively, given by

$$P_d = P\left(\Lambda \geq \lambda_{th} | \mathcal{H}_1\right) = Q\left(\sqrt{\frac{\rho_2 T f_s}{2}} \times \frac{\lambda_{th} - (g^2\sigma_s^2 + \sigma_\omega^2)}{(g^2\sigma_s^2 + \sigma_\omega^2)}\right)$$

$$P_f = P\left(\Lambda \geq \lambda_{th} | \mathcal{H}_0\right) = Q\left(\sqrt{\frac{\rho_2 T f_s}{2}} \times \frac{\lambda_{th} - \sigma_\omega^2}{\sigma_\omega^2}\right)$$

where λ_{th} can be derived from taking the inverse of P_d, given as

$$\lambda_{th} = \sqrt{\frac{2}{\rho_2 T f_s}} \left(g^2\sigma_s^2 + \sigma_\omega^2\right) Q^{-1}(P_d) + \left(g^2\sigma_s^2 + \sigma_\omega^2\right) \tag{3}$$

and f_s is the sampling frequency, $Q(x)$ is the complementary Gaussian function, i.e.,

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-\frac{t^2}{2}} dt \tag{4}$$

and define $\gamma = \frac{g^2\sigma_s^2}{\sigma_\omega^2}$ as the PU's signal to noise ratio (SNR) at the CU.

3. **Problem Formulation and Analysis.**

3.1. **Problem formulation.** Denote $P_{\mathcal{H}_0}$ and $P_{\mathcal{H}_1}$ to be the corresponding probability of PU in the states of $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. In our design, the "data transmission" duration would be used for harvesting energy, if the spectrum sensing result indicates that the PU is active. Define $(\mathcal{H}_j \mid \mathcal{H}_i)$, $i, j \in \{0, 1\}$ as the sensing result $\mathcal{H}_j$ under the actual state $\mathcal{H}_i$ of the PU. Hence, there are four different cases, and their probabilities are given by

$$P(\mathcal{H}_j \mid \mathcal{H}_i) = \begin{cases} 1 - P_f, & j = 0,\ i = 0 \\ P_f, & j = 0,\ i = 1 \\ 1 - P_d, & j = 1,\ i = 0 \\ P_d, & j = 1,\ i = 1\,. \end{cases} \tag{5}$$

In the following, we give detailed description about these four cases.

Case I $(\mathcal{H}_0 \mid \mathcal{H}_0)$: In this case, the PU is silent, and the CU correctly detects its state with the probability of $P_{\mathcal{H}_0} P(\mathcal{H}_0 \mid \mathcal{H}_0)$. The energy available for data transmitting is given by $E_t = E_s + \chi\rho_1 T - p_s\rho_2 T$, where E_s denotes the remained energy from previous frames, $\chi\rho_1 T$ is the harvested energy in the current frame, and $p_s\rho_2 T$ is the consumed energy in spectrum sensing phase. Hence, the number of transmitted bits in this case is $P_{\mathcal{H}_0}(1 - P_f)(1 - \rho_1 - \rho_2)T\log_2\left(1 + \frac{h^2 E_t}{\sigma_\omega^2(1-\rho_1-\rho_2)T}\right)$, where h denotes the channel gain of the cognitive-cognitive link.

Case II $(\mathcal{H}_1 \mid \mathcal{H}_1)$: In this case, the PU is active, and the PU correctly detects its state with the probability of $P_{\mathcal{H}_1} P(\mathcal{H}_1 \mid \mathcal{H}_1)$. To avoid the collision with PU transmission, the CU keeps silent, but proceeds energy harvesting instead. Apart from the energy $p_s\rho_2 T$ consumed in spectrum sensing, an expected amount of $P_{\mathcal{H}_1} P(\mathcal{H}_1 \mid \mathcal{H}_1)(\chi(1-\rho_2)T - p_s\rho_2 T)$ energy can finally be reserved in the current frame.

Case III $(\mathcal{H}_1 \mid \mathcal{H}_0)$: In this case, the PU is silent, but the PU in silent state is falsely detected active with the probability of $P_{\mathcal{H}_0} P(\mathcal{H}_1 \mid \mathcal{H}_0)$, which results in false alarm and wasting access opportunity. Subsequently, the CU regards that the PU's state is active. Instead of to transmit, CU proceeds to harvest energy. By the end of the current frame, it is expected to reserve $P_{\mathcal{H}_0} P(\mathcal{H}_1 \mid \mathcal{H}_0)(\chi(1 - \rho_2)T - p_s\rho_2 T)$ amount of energy.

Case IV $(\mathcal{H}_0 \mid \mathcal{H}_1)$: In this case, the PU is active, while the CU falsely detects its state silent with the probability of $P_{\mathcal{H}_1} P(\mathcal{H}_0 \mid \mathcal{H}_1)$, which results in miss-detection. Then, the

CU initiates data transmission with exhausting all stored energy. However, the data transmission is invalid due to the transmission collision with that of the PU, which means not only does the CU consume energy into transmission in vain, but also generate interference to primary user.

Based on the analysis above, we can obtain the ergodic capacity (in bits/s/Hz) of the CU as

$$R(\rho_1, \rho_2) = P_{\mathcal{H}_0}(1-P_f)(1-\rho_1-\rho_2)\mathbb{E}\left[\log_2\left(1+\frac{h^2E_t}{\sigma_\omega^2(1-\rho_1-\rho_2)T}\right)\right] \tag{6}$$

where $\mathbb{E}[\cdot]$ represents the expectation operation. The energy reserved in frames of Cases II and III will be consumed by the frame of Case IV with probability of $P_{\mathcal{H}_1}P(\mathcal{H}_0 \mid \mathcal{H}_1)$. Hence, the reserved energy can be used for frame of Case I is given by $E_s = (1-P_{\mathcal{H}_1}P(\mathcal{H}_0 \mid \mathcal{H}_1))\left(P_{\mathcal{H}_1}P(\mathcal{H}_1 \mid \mathcal{H}_1) + P_{\mathcal{H}_0}P(\mathcal{H}_1 \mid \mathcal{H}_0)\right)(\chi(1-\rho_2)T - p_s\rho_2T)$.

Our objective is to design the optimal energy harvesting saving-ratio ρ_1 and spectrum sensing-ratio ρ_2 to maximize the ergodic capacity under the condition that the collision probability with PU transmission should be less than $P_{\mathcal{H}_1}(1-p_d^*)$, where p_d^* is the minimum detection probability. Therefore, we can formulate the optimization problem as

$$P1: \quad \arg\max_{\rho_1,\rho_2} R(\rho_1,\rho_2)$$
$$s.t. \begin{cases} 0<\rho_1<1 \\ 0<\rho_2<1 \\ 0<\rho_1+\rho_2<1 \\ P_d \geq p_d^*. \end{cases} \tag{7}$$

3.2. **Problem analysis.** In the problem (7), h^2 follows exponential distribution and χ follows Gamma distribution. This makes the problem difficult to solve. Enlightened by [7], we transform the objective function in (7) to be

$$R^\dagger(\rho_1, \rho_2) = P_{\mathcal{H}_0}(1-P_f)(1-\rho_1-\rho_2)\log_2\left(1+\frac{\mathbb{E}[h^2E_t]}{\sigma_\omega^2(1-\rho_1-\rho_2)T}\right). \tag{8}$$

By taking the expectation of h^2E_t, unlike the problem (7) relevant with random variables h^2 and χ, $R^\dagger(\rho_1,\rho_2)$ turns into relating with $\mathbb{E}[h^2]$ and $\mathbb{E}[\chi]$. Hence, the problem (7) is simplified to be

$$P2: \quad \arg\max_{\rho_1,\rho_2} R^\dagger(\rho_1,\rho_2)$$
$$s.t. \begin{cases} 0<\rho_1<1 \\ 0<\rho_2<1 \\ 0<\rho_1+\rho_2<1 \\ P_d \geq p_d^*. \end{cases} \tag{9}$$

Figure 2 shows the results of the problems (7) and (9) for different ρ_2. It can be observed that both the problems have an identical optimal solution of ρ_1. In Figure 3, the optimal solutions of ρ_2 to the two problems for different ρ_1 are presented. It is shown that problems (7) and (9) have the same optimal solution of ρ_2. These numerical results demonstrate that problems (7) and (9) have the identical optimal solution $(\rho_1^{opt}, \rho_2^{opt})$, although $R^\dagger(\rho_1^{opt}, \rho_2^{opt}) > R(\rho_1^{opt}, \rho_2^{opt})$.

Usually, exhaustive search is required to find the optimal solution to the problem (9). Hence, its computational complexity is still high. Figure 4 shows the capacity $R^\dagger$ with respect to (ρ_1, ρ_2). It shows that the problem (9) is concave. We propose to iteratively solve the problem by using the greedy iteration algorithm given in Table 1.

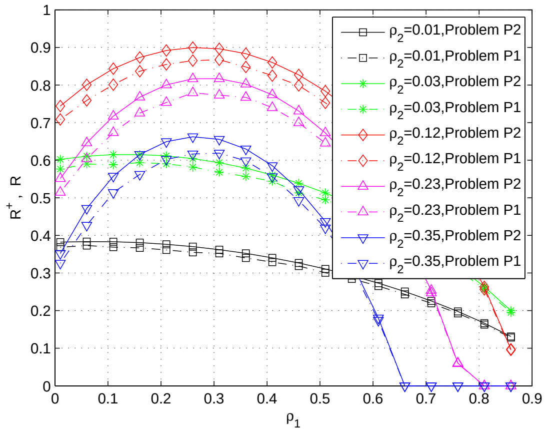

FIGURE 2. R and $R^{\dagger}$ versus ρ_1

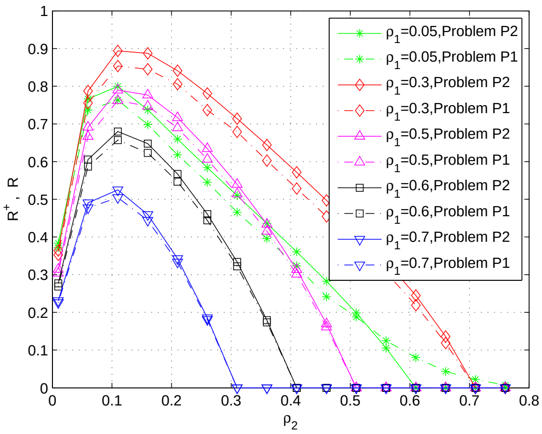

FIGURE 3. R and $R^{\dagger}$ versus ρ_2

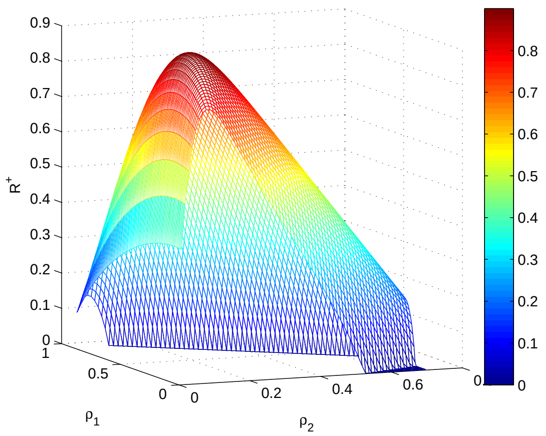

FIGURE 4. $R^{\dagger}$ versus (ρ_1, ρ_2)

TABLE 1. Greedy iteration algorithm to solve the problem in (9)

Greedy Iteration Algorithm
Initialization:
$\rho_1 = 0 \sim 1,\ \rho_2 = 0 \sim 1,\ \rho_1^* = 0,\ \rho_2^* = 0,\ \epsilon = 10^{-6}$;
Iterative:
1: **while** $\left\| \max\limits_{\rho_1} R^{\dagger}(\rho_1, \rho_2^*) - \max\limits_{\rho_2} R^{\dagger}(\rho_1^*, \rho_2) \right\| > \epsilon$ **do**
2: $\rho_1^* = \arg \max\limits_{\rho_1} R^{\dagger}(\rho_1, \rho_2^*)$;
3: $\rho_2^* = \arg \max\limits_{\rho_2} R^{\dagger}(\rho_1^*, \rho_2)$;
4: **end while**
5: $\rho_1^{opt} = \rho_1^*,\ \rho_2^{opt} = \rho_2^*$.

4. **Simulation Results.** In this section, several numerical results are presented to validate the proposed algorithm. Let $P_{\mathcal{H}_0} = 0.7$ and $P_{\mathcal{H}_1} = 1 - P_{\mathcal{H}_0} = 0.3$. In LTE-A systems, the time-slot is 1ms and its system bandwidth can be 5MHz. Hence, we assume the length of a frame to be $T = 1$ms, the sampling frequency $f_s = 10$MHz. According to the IEEE 802.22 standard, the minimum detection probability is selected as $p_d^* = 0.9$. In addition, PU's SNR $\gamma = -10$dB, the sensing power $p_s = 1$, and the scale and shape parameters of Gamma function $a = 5$, $b = 1$.

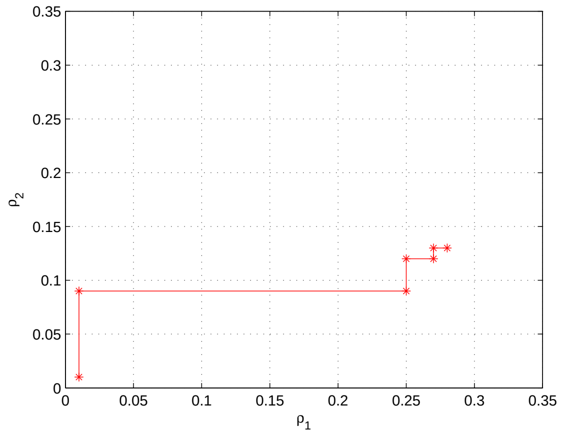

FIGURE 5. Iteration of ρ_1^{opt}, ρ_2^{opt}

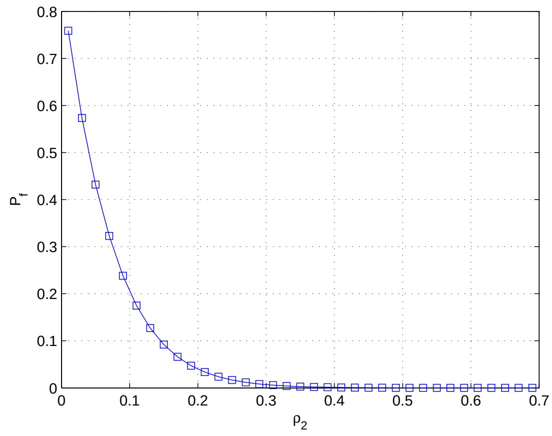

FIGURE 6. P_f versus ρ_2

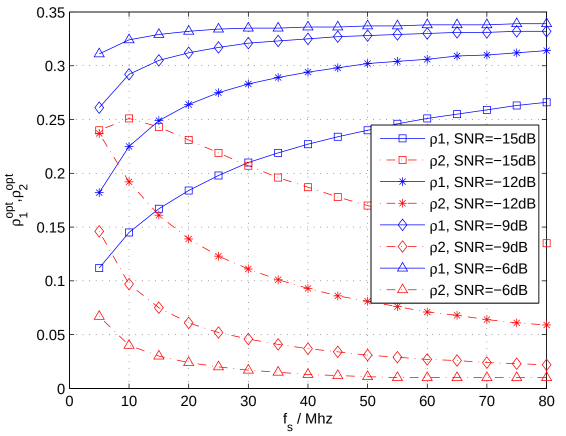

FIGURE 7. ρ_1^{opt}, ρ_2^{opt} versus f_s

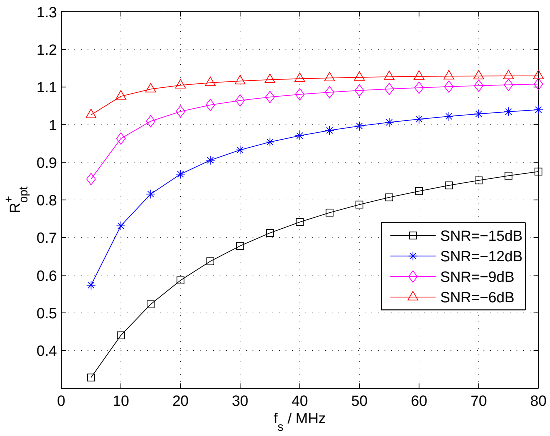

FIGURE 8. $R_{opt}^{\dagger}$ versus f_s

It has been shown in Figure 4 that, by applying the greedy iteration algorithm proposed in Table 1, the optimal ρ_1, ρ_2 can be iteratively found at the point of $(\rho_1 = 0.28, \rho_2 = 0.13)$. In Figure 5, it can be seen that the optimal ρ_1, ρ_2 can be reached after seven iterations, which apparently reduces computational complexity.

Figure 6 shows that P_f decreases dramatically at the very beginning when ρ_2 increases, then P_f goes down moderately, which is for the reason that sharing more time to spectrum sensing gives rise to a more reliable test-statistic. In the scenario above, $f_s = 10$MHz, $\gamma = -10$dB, the corresponding P_f is about 0.13, which is acceptable.

In Figure 7, we further investigate the variation trend of optimal ρ_1^{opt}, ρ_2^{opt} when sample frequency f_s varies with SNR. The curves show that when the sample frequency rises, the optimal sensing ratio goes down, and the optimal save ratio goes up. Meanwhile, if f_s keeps fixed, the higher SNR results in relatively higher harvesting ratio and lower sensing ratio. All that is rational, since that increasing the sample frequency is equivalent to taking more samples so that the test-statistic could be more trustworthy, and that increasing SNR could result in CU more sensitive to the behavior of PU. Under such circumstances, the corresponding $R_{opt}^{\dagger}$ in Figure 8, shows that increasing sample frequency or SNR gives rise to boosting $R_{opt}^{\dagger}$.

5. **Conclusions.** In this paper, we have analyzed the sensing and energy harvesting intervals of cognitive radio with energy harvesting. An equivalent transformation of the original optimization problem has been proposed to get the optimal sensing and energy

harvesting ratio. After finding that the proposed objective function is concave, we proposed a greedy iteration algorithm for the optimization problem, whose computational complexity is greatly reduced. In the future work, we will investigate the optimization of the frame length of energy harvesting cognitive radio for achieving a better throughput performance.

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REFERENCES

[1] B. Wang and K. J. R. Liu, Advances in cognitive radio networks: A survey, *IEEE Journal of Selected Topics in Signal Processing*, vol.5, no.1, pp.5-23, 2010.

[2] T. Yucek and H. Arslan, A survey of spectrum sensing algorithms for cognitive radio applications, *IEEE Communications Surveys & Tutorials*, vol.11, no.1, pp.116-130, 2009.

[3] Y. C. Liang, Y. Zeng, E. C. Y. Peh and A. T. Hoang, Sensing-throughput tradeoff for cognitive radio networks, *IEEE Transactions on Wireless Communications*, vol.7, no.4, pp.1326-1337, 2008.

[4] M. L. Ku, W. Li, Y. Chen and K. J. R. Liu, Advances in energy harvesting communications: Past, present, and future challenges, *IEEE Communications Surveys & Tutorials*, vol.18, no.2, pp.1384-1412, 2016.

[5] Z. Qu, Q. Song and S. Yin, Optimal spectrum sensing in energy harvesting cognitive radio systems, *China Communications*, vol.11, no.5, pp.83-90, 2014.

[6] S. Yin, Z. Qu and S. Li, Achievable throughput optimization in energy harvesting cognitive radio systems, *IEEE Journal on Selected Areas in Communications*, vol.33, no.3, pp.407-422, 2015.

[7] L. Zhang, M. Xiao, G. Wu, S. Li and Y. C. Liang, Energy-efficient cognitive transmission with imperfect spectrum sensing, *IEEE Journal on Selected Areas in Communications*, vol.34, no.5, pp.1320-1335, 2016.

[8] S. Luo, R. Zhang and T. J. Lim, Optimal save-then-transmit protocol for energy harvesting wireless transmitters, *IEEE Transactions on Wireless Communications*, vol.12, no.3, pp.1196-1207, 2013.

[9] J. Jia, Q. Zhang and X. S. Shen, HC-MAC: A hardware-constrained cognitive MAC for efficient spectrum management, *IEEE Journal on Selected Areas in Communications*, vol.26, no.1, pp.106-117, 2008.