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MULTI-ATTRIBUTE DECISION MAKING METHOD BASED ON HESITANT FUZZY LINGUISTIC VIKOR

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Abstract. *This paper proposes a new decision making method to the multi-attribute decision making problem, in which attribute values are hesitant fuzzy linguistic numbers and attribute weights information is incomplete or completely unknown. At first, the comprehensive weight optimization model is constructed on the basis of considering the subjective and objective constraints at the same time. Then, the alternatives are ranked with the VIKOR and the distance formula of hesitant fuzzy linguistic. Finally, a numerical example is presented to demonstrate the feasibility and effectiveness of the proposed method.*
Keywords: Multi-attribute decision making, Hesitant fuzzy linguistic (HFL), Maximum deviation, VIKOR

1. **Introduction.** Because of the complexity of decision making environment and the ambiguity of human thinking, decision makers (DMs) have difficulties in expressing evaluation information with exact numerical values. DMs often prefer to express their assessments by linguistic variable [1]. The multi-attribute decision making (MADM) problem with linguistic evaluation information is called linguistic MADM problem which has caused many scholars' attention recently. The existing linguistic MADM methods firstly transform initial linguistic decision making matrix into probability information decision making matrix (PIDMM), then rank alternatives by the sum values of the PIDMM's row elements or by the ideal point method. However, the transforming process may cause information loss. Ranking the alternatives by the sum values of the PIDMM's row elements may lead to decision making deviation because of not considering the benefits or cost characteristic of attributes. In addition, the ideal point method could not always reflect the actual proximity degree of alternatives and the ideal alternative. Therefore, new decision making method should be proposed to deal with linguistic MADM problems reasonably.

The proper information express model is a key factor to solve linguistic MADM problems reasonably. In linguistic MADM, DMs usually hesitate among several linguistic terms and their hesitant degrees are always different. If the DMs' hesitant degree cannot be described properly, information loss may appear. By far, three methods have been proposed to express linguistic evaluation information. (1) Fuzzy number [2-4]: This method can express the linguistic evaluation information intuitively and be calculated easily, but it may cause information loss in the information transformation process. (2) Ordered linguistic term set [5-7]: This method is easy to calculate, too. However, it cannot express DMs' hesitant degree reasonably, which also may cause information loss. (3) Hesitant fuzzy linguistic [8-13]: HFL can express DMs' hesitation degree properly and reduce information loss because of gathering the advantages of the linguistic term set and hesitant fuzzy set.

The sorting method is another key factor to solve linguistic MADM problems reasonably. TOPSIS and VIKOR are frequently used in solving MADM problems. VIKOR presents the multi-attribute ranking index according to the concrete measure of closeness to the ideal alternative and the determined compromise alternative provides the group utility maximization and the individual regret minimization [14]. However, TOPSIS and its extensions only consider the distances from the ideal alternative to each alternative, without considering the relative importance of the distances. So the final results derived by TOPSIS and its extensions are not always the closest to the ideal alternative [15]. In recent years, VIKOR has been widely studied and used to deal with the MADM problem. The purpose of this paper is to extend VIKOR into HFL environment.

Section 2 presents some basic concepts of HFL. Section 3 proposes a new MADM method. In Section 4, a numerical example is presented to illustrate the proposed method. The conclusions and future research directions are presented in Section 5.

2. Preliminaries.

Definition 2.1. *Let $S = \{s_i | i = 0, 1, 2, \ldots, g\}$ be a linguistic term set (LTS) with odd cardinality. The term s_i denotes a linguistic term. The set S should satisfy the following characteristics: 1) $s_i \geq s_j$, if $i \geq j$; 2) Negation operator: $Neg(s_i) = s_j$ such that $i+j = g$.*

Definition 2.2. *Let X be a fixed set, a hesitant fuzzy set (HFS) on X is a function when used to X it returns a subset of $[0, 1]$. An HFS can be denoted as $S = \{< x, h(x) > | x \in X\}$, in which $h(x)$ denotes the membership degree of element $x \in X$ to S, $0 \leq h(x) \leq 1$.*

Definition 2.3. *Let X be a fixed set, and a hesitant fuzzy linguistic set (HFLS) on X can be defined as the following term $A = \{< x, s_{\theta(x)}, h(x) > | x \in X\}$, where $s_{\theta(x)}$ denotes the element of X, $\theta(x)$ denotes the foot of the linguistic evaluation value, and $h(x)$ denotes the confidence level of $s_{\theta(x)}$.*

Definition 2.4. *Let $a_1 =< s_{\theta_1}, h_1 >$ and $a_2 =< s_{\theta_2}, h_2 >$ be two HFL numbers, and the Hausdorff distance between them can be defined as the following:*

$$d(a_1, a_2) = \max\left\{d^*(a_1, a_2), d^*(a_2, a_1)\right\} \tag{1}$$

$$d^*(a_1, a_2) = \max_{s_i \in h_1} \min_{t_j \in h_2} \left|s_i \cdot I(s_{\theta_1})/g - t_j \cdot I(s_{\theta_2})/g\right| \tag{2}$$

Here, $I(s_{\theta_1}) = \theta_1$ and $I(s_{\theta_2}) = \theta_2$ denote the linguistic evaluation values' foots, and g denotes the granularity of the LTS. It is easy to prove that the definition satisfies:

1) $0 \leq d(a_1, a_2) \leq 1$; 2) $d(a_1, a_2) = 0$, only if $a_1 = a_2$; 3) $d(a_1, a_2) = d(a_2, a_1)$.

3. Hesitant Fuzzy Linguistic VIKOR Methods.

3.1. Problem description.

Let $A = \{a_1, a_2, \ldots, a_m\}$ be the alternative set, $C = \{c_1, c_2, \ldots, c_n\}$ be the attribute set, w_j be the weight of attribute c_j, $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$, $D' = (v'_{ij})_{m\times n}$ be the initial HFL decision matrix, and $D = (v_{ij})_{m\times n}$ be the extended decision making matrix.

3.2. **Decision making method.**

Step 1. Construct the initial HFL decision making matrix D'. If HFL evaluation values are different in membership numbers, D' should be extended to D according to the DMs' risk preference.

Step 2. Establish the combined weight optimization model. The evaluation value deviation of alternatives under attribute C_j can be defined as $\sum_{i=1}^{m}\sum_{k=1}^{m} d(v_{ij}, v_{kj})$.

1) The information of attribute weights is incomplete. Usually, the objective weights information can be expressed as: $a_i \le w_i$; $w_i \ge b_i$; $r_i \le w_i \le r_i + \varepsilon_i$ and DMs' subjective preference can be expressed as: $w_i \ge w_j$; $w_i - w_j \ge \alpha$; $w_i \ge \beta w_j$. Here, a_i, b_i, r_i, ε_i, α and β are all non-negative constants. These expression forms can be unified as $w \in W$.

$$\max\ Z(w) = \sum\nolimits_{j=1}^{n} w_j \left(\sum\nolimits_{i=1}^{m}\sum\nolimits_{k=1}^{m} d(v_{ij}, v_{kj})\right) \tag{3}$$
$$\text{s.t.}\ \ w \in W,\ \sum\nolimits_{j=1}^{n} w_j = 1,\ 0 \le w_j \le 1,\ j = 1, 2, \ldots, n,\ i = 1, 2, \ldots, m$$

2) The information of attribute weights is completely unknown.

$$\max\ Z(w) = \sum\nolimits_{j=1}^{n} w_j \left(\sum\nolimits_{i=1}^{m}\sum\nolimits_{k=1}^{m} d(v_{ij}, v_{kj})\right) \tag{4}$$
$$\text{s.t.}\ \ \sum\nolimits_{j=1}^{n} w_j^2 = 1,\ w_j \ge 0,\ j = 1, 2, \ldots, n$$

Step 3. Determine the positive ideal solution and the negative ideal solution.

$$v_j^+ = \left\{\max_i(v_{ij}), C_j \in I' \,|\, \min_i(v_{ij}), C_j \in I''\right\} \tag{5}$$

$$v_j^- = \left\{\min_i(v_{ij}), C_j \in I' \,|\, \max_i(v_{ij}), C_j \in I''\right\} \tag{6}$$

Here, I' denotes the benefit attribute, I'' denotes the cost attribute, $j = 1, 2, \ldots, n$.

Step 4. Compute S_i, R_i and Q_i and rank alternatives by S_i, R_i and Q_i in ascending.

$$\delta_{ij} = d\left(v_j^+, v_{ij}\right) \big/ d\left(v_j^+, v_j^-\right) \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \tag{7}$$

$$S_i = \sum\nolimits_{j=1}^{n} w_j \cdot \delta_{ij} \tag{8}$$

$$R_i = \max_j w_j \cdot \delta_{ij} \tag{9}$$

$$Q_i = u \cdot (S_i - S^*) \big/ (S^- - S^*) + (1 - u) \cdot (R_i - R^*) \big/ (R^- - R^*) \tag{10}$$

Here, $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$, u and $1 - u$ denote the weights of group utility and individual regret respectively, $0 \le u \le 1$.

Step 5. Confirm the compromise solution. Supposing a_1 and a_2 are the first and second position respectively in the ranking list by Q_i. Alternative a_1 will be the compromise solution if it meets: Con1 Acceptable advantage: $Q(a_2) - Q(a_1) \ge 1/m - 1$; Con2 Acceptable stability in decision making: a_1 must also be the best ranking by S_i, as well as by R_i. If Con2 is not satisfied, both a_1 and a_2 are compromise solutions. If Con1 is not satisfied, a_1, a_2, $\ldots$, a_m are compromise solutions, $Q(a_m) - Q(a_1) < 1/m - 1$.

4. Application of the Proposed Method.

4.1. **Implementation.** A government desires to evaluate four cities' emergency response capability. The cities are denoted as a_1, a_2, a_3, a_4. Attributes are warning capability c_1, resource support capability c_2, rescue capability c_3 and recovery capability c_4. These attributes are all beneficial. Attribute weights are denoted as w_j $(j = 1, 2, \ldots, 4)$, $0 \le w_j \le 1$, $\sum_{j=1}^{4} w_j = 1$. The LTS is $S = \{s_0 = vp, s_1 = p, s_2 = mp, s_3 = m, s_4 = mg, s_5 = g, s_6 = vg\}$.

Step 1. Construct the initial HFL decision making matrix D'. Suppose DMs are risk preference. Based on this, matrix D' is extended to D. The results are shown in Table 1.

TABLE 1. Extended hesitant fuzzy linguistic decision making matrix

D	a_1	$< s_3, \{0.5, 0.6, 0.6\} >$	$< s_4, \{0.5, 0.6, 0.6\} >$	$< s_2, \{0.5, 0.6, 0.7\} >$	$< s_3, \{0.4, 0.6, 0.7\} >$
	a_2	$< s_4, \{0.5, 0.6, 0.7\} >$	$< s_3, \{0.6, 0.7, 0.7\} >$	$< s_4, \{0.6, 0.6, 0.6\} >$	$< s_4, \{0.6, 0.6, 0.6\} >$
	a_3	$< s_4, \{0.6, 0.7, 0.7\} >$	$< s_3, \{0.5, 0.6, 0.6\} >$	$< s_3, \{0.5, 0.6, 0.7\} >$	$< s_5, \{0.5, 0.6, 0.7\} >$
	a_4	$< s_4, \{0.6, 0.7, 0.7\} >$	$< s_3, \{0.6, 0.6, 0.6\} >$	$< s_5, \{0.5, 0.6, 0.6\} >$	$< s_4, \{0.4, 0.6, 0.7\} >$

Step 2. Construct the optimization model to determine the attribute weights.

1) The information of attribute weights is incomplete. Assume that the attribute weights' objective allowable ranges are $0.10 \le w_1 \le 0.25$, $0.15 \le w_2 \le 0.30$, $0.10 \le w_3 \le 0.35$ and $0.15 \le w_4 \le 0.30$, and the subjective preference of attribute weights satisfy $w_1 \le w_4 \le w_2 \le w_3$. The optimization model is constructed as the following:

$$MAX\ Z(w) = 0.5167w_1 + 0.6500w_2 + 0.8000w_3 + 0.6167w_4$$
$$\text{s.t.} \begin{cases} 0.1 \le w_1 \le 0.25;\ 0.15 \le w_2 \le 0.3;\ 0.1 \le w_3 \le 0.35; \\ 0.15 \le w_4 \le 0.3;\ w_1 \le w_4 \le w_2 \le w_3;\ \sum_{j=1}^{4} w_j = 1;\ w_j \ge 0 \end{cases}$$

Using Lingo 11.0, attribute weights can be obtained $w = (0.10, 0.30, 0.35, 0.25)$.

2) The information of attribute weights is unknown. $w = (0.20, 0.25, 0.31, 0.24)$.

Step 3. Determine the positive ideal solution and the negative ideal solution.

$$v^+ = \{< s_4, \{0.6, 0.7, 0.7\} >, < s_4, \{0.5, 0.6, 0.6\} >, < s_5, \{0.5, 0.6, 0.6\} >, < s_5, \{0.5, 0.6, 0.7\} >\}$$
$$v^- = \{< s_3, \{0.5, 0.6, 0.6\} >, < s_2, \{0.6, 0.6, 0.6\} >, < s_2, \{0.5, 0.6, 0.7\} >, < s_3, \{0.4, 0.6, 0.7\} >\}$$

Step 4. Compute S_i, R_i, Q_i and rank alternatives. Results are shown in Table 2 and Table 3.

TABLE 2. Ranking results of weight information being incomplete

	S_i	Rank	R_i	Rank	Q_i	Rank
a_1	0.600	4	0.328	4	1.000	4
a_2	0.130	1	0.05	1	0.000	1
a_3	0.344	2	0.219	3	0.532	3
a_4	0.361	3	0.200	2	0.516	2

TABLE 3. Ranking results of weight information being unknown

	S_i	Rank	R_i	Rank	Q_i	Rank
a_1	0.594	4	0.291	4	1.000	4
a_2	0.158	1	0.080	1	0.000	1
a_3	0.298	2	0.194	3	0.431	3
a_4	0.321	3	0.167	2	0.393	2

Step 5. Determine the compromise solution. Table 2 and Table 3 show the same results: $a_2 \succ a_4 \succ a_3 \succ a_1$ ranking by Q_i. Both Con1 and Con2 are met. So a_2 is the compromise solution. The emergency capability of a_2 is the best among the four cities.

4.2. **Discussion.** To illustrate the effectiveness and validity of the proposed method, the F-VIKOR proposed in [15] is used to solve the problem mentioned above. Supposing $w = (0.10, 0.30, 0.35, 0.25)$, the results are shown in Table 4. The results obtained by these two methods are basically the same, and a_2 and a_1 rank the first and the last position respectively, which show the validity of our proposed method. However, our proposed

Table 4. Comparison of two methods' results

	a_1	a_2	a_3	a_4
The proposed method	4	1	3	2
F-VIKOR	4	1	2	3

method obtained $a_4 \succ a_3$, while F-VIKOR obtained $a_3 \succ a_4$. Table 2 shows that the evaluation values of a_3 and a_4 under c_1 are equal, the evaluation values of a_4 under c_2 and c_3 are greater than a_3, and the total weight of c_2 and c_3 reaches 0.65. So $a_4 \succ a_3$ is reasonable.

5. **Conclusion.** For the MADM problems, in which the attribute values are HFL numbers and attribute weights information is incomplete or unknown, a new decision making method is proposed. At first, the combined weights optimization model is constructed to obtain the attribute weights. The goal of the model is to maximize the deviation of attribute values. The constraints of the model are the interval range of attribute weights and the subjective preference of DMs. Then, the alternatives are ranked with the VIKOR method and the distance formula of HFL. Finally, a numerical example is presented to demonstrate the feasibility and validity of the proposed method. In the future, the proposed method should be integrated with other methods or theories, such as Shapley value-based method and regret theory, which will make it more applicable.

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