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ADAPTIVE OUTPUT-FEEDBACK CONTROL FOR MIMO NONLINEAR SYSTEMS BASED ON FUZZY APPROXIMATION

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Abstract. *An adaptive fuzzy output-feedback control approach for a class of multi-input and multi-output (MIMO) nonlinear systems is proposed by using the backstepping technology, which employs fuzzy logic systems (FLSs) to approximate the unknown nonlinearity. A state observer is first designed to estimate the unmeasured state variables. The robustness of the proposed controller is established by using the Lyapunov stability theory. Theoretical analysis shows that all signals in the closed-loop system can be guaranteed semi-global uniform ultimate boundedness (SGUUB). At last simulation results are presented to validate the effectiveness of the analysis.*

Keywords: Adaptive control, Backstepping, Fuzzy output-feedback control

1. **Introduction.** The adaptive backstepping approach has been the cornerstone of nonlinear systems control for the last two decades. Lots of significant research results have been reported. However, earlier works have a main limitation which usually require some prior information. As an alternative, further research has been carried out on approximation-based adaptive neural or fuzzy control for nonlinear systems. Recently, the adaptive backstepping control based on fuzzy or neural approximation has been developed greatly. By combining Lyapunov stability theory and NNs/FLSs, the adaptive backstepping design has been extensively applied to more broader fields. The works in [1, 2] proposed adaptive neural or fuzzy control schemes for the single-input and single-output (SISO) nonlinear systems. The problems of stabilization or tracing control for MIMO nonlinear systems based on adaptive neural/fuzzy control technique are discussed in [3, 4]. These results are further extended to stochastic nonlinear systems [5, 6] and discrete-time systems [7]. Additionally, observer-based adaptive fuzzy/neural output feedback control approaches are proposed for SISO/MIMO systems by designing state observers in [8, 9, 10]. By the proposed output-feedback controllers, the assumption that the states are available is removed. A key question in these observer-based control schemes is the analysis of the observation error dynamic systems, i.e., how to deal with the term e^TPe is critical, where e is the observation error and P is a positive definite matrix. Most scholars' processing methods are based on nonlinear matrix inequalities, which usually include $||P||$ and are difficult to solve. Just like in [8], in order to achieve the stability of error dynamics, two positive definite matrixes Q and P are needed and the inequalities $q_0 = \lambda_{\min}(Q) - (1+2||P||^2b) - 1$ and $PA_0 + A_0^TP + Q < 0$ must hold, where A_0 is defined as in (3). However, such matrixes Q and P usually are hard to find because of nonlinearity.

Motivated by the aforementioned works and considering that most systems are multivariable in nature, an adaptive fuzzy output-feedback control is discussed for a class of nonlinear MIMO systems. A state observer is designed to estimate the unmeasured states. An adaptive fuzzy output-feedback controller is systematically developed by combining adaptive backstepping technique and FLSs. All signals in the closed-loop system are guaranteed SGUUB. The main contribution of this note is that the stability of the observation error system depends on a set of linear matrix inequalities (LMIs), rather than nonlinear matrix inequalities. This greatly reduces the algorithm design's difficulty, raises the counting yield and makes the simulation more easier to implement. The control theory proposed in this paper should find more applications in a wide variety of problems.

The rest of this paper is organized as follows. Problem statement and preliminaries are proposed in Section 2. The backstepping design procedure and main results are presented in Sections 3 and 4. Simulation results are given in Section 5. At last, concluding remarks are proposed in Section 6.

2. **Problem Statement and Preliminaries.** Consider a class of uncertain MIMO nonlinear systems in strict-feedback form with N subsystems. The ith ($i = 1, 2, \dots, N$) subsystem is in the following form:

$$\begin{cases} \dot{x}_{i,j} = f_{i,j}\left(\bar{x}_{i,j}\right) + x_{i,j+1}, \quad 1 \le j \le n_i - 1, \\ \dot{x}_{i,n_i} = f_{i,n_i}(x) + u_i, \\ y_i = x_{i,1} \end{cases} \tag{1}$$

where $\bar{x}_{i,j} = [x_{i,1}, x_{i,2}, \dots, x_{i,j}]^T \in R^j$, ($i = 1, \dots, N$; $j = 1, \dots, n_i$) is the state vector for the first j differential equation of the ith subsystem. And $x = \left[x_1^T, \dots, x_N^T\right]^T$ with $x_i = [x_{i,1}, \dots, x_{i,n_i}]^T \in R^{n_i}$ are the whole states. $u_i \in R$ and $y_i \in R$ denote the control input and output variables of the ith nonlinear subsystem, respectively. $f_{i,j}\left(\bar{x}_{i,j}\right)$s are the unknown smooth nonlinear functions. In this paper, it is assumed that the variable $y_i = x_{i,1}$ is measured directly only.

Our control objective is to design an observer-based adaptive fuzzy controller to guarantee all the signals in the resulting closed-loop system are SGUUB and the observer errors are as small as possible. To facilitate the control design, the following assumption will be used in the subsequent developments.

Assumption 2.1. *For the system function $f_{i,j}(\cdot)$ there exist known constants $\underline{a}_{pq}$, $\overline{a}_{pq}$ such that*

$$\underline{a}_{pq} \le \frac{\partial f_{i,j}}{\partial x_{m,n}} \le \overline{a}_{pq}, \quad 1 \le i, m \le N, \quad 1 \le j \le n_i, \quad 1 \le n \le n_m,$$

where n_i and n_m stand for the number of state variables in the ith and jth subsystems, respectively. $p = \sum_{k=0}^{i-1} n_k + j$ and $p = \sum_{k=0}^{m-1} n_k + n$ with $n_0 = 0$.

Remark 2.1. *Since $f_{i,j}(x) = \left[\frac{\partial f_{i,j}}{\partial x_{1,1}}, \dots, \frac{\partial f_{i,j}}{\partial x_{N,n_N}}\right] x$. By Assumption 2.1, there exist constants $h_{i,j} > 0$ such that $|f_{i,j}(x)| \le h_{i,j}||x||$. This implies that the monotonically increasing function $\rho_{i,j}(w) = h_{i,j}w$ is the bounding function of $f_{i,j}(\cdot)$ with $w \in R$.*

3. **Fuzzy State Observer Design.** Because the state vectors $\bar{x}_{i,j}$ cannot be measured directly, a state observer should be established as:

$$\begin{cases} \dot{\hat{x}}_{i,j} = \hat{x}_{i,j+1} + l_{i,j}\left(y_i - \hat{x}_{i,1}\right), \quad 1 \le i \le N, \ 1 \le j \le n_i - 1, \\ \dot{\hat{x}}_{i,n_i} = u_i + l_{i,n_i}\left(y_i - \hat{x}_{i,1}\right) \end{cases} \tag{2}$$

where $\hat{x}_{i,j}$ is the estimation of $x_{i,j}$, and $l_{i,1}, \dots, l_{i,n_i}$ are optional negative constants such that the polynomial $p(s) = s^{n_i} + l_{i,1}s^{n_i-1} + \dots + l_{i,n_i-1}s + l_{i,n_i}$ is Hurwitz. Define the estimation error as $e_{i,j} = x_{i,j} - \hat{x}_{i,j}$. From (1) and (2) the observer error equation is given as:

$$\dot{e} = A_0 e + F(x) \tag{3}$$

where $F(x) = \left[F_1(x)^T, \ldots, F_N(x)^T\right]^T$, with $F_i(x) = \left[f_{i,1}(x_{i,1}), \ldots, f_{i,n_i}(x)\right]^T$. And $e = \left[e_1^T, \ldots, e_N^T\right]^T$, with $e_i = \left[e_{i,1}, \ldots, e_{i,n_i}\right]^T$. $A_0 = diag[A_1, \ldots, A_N]$, with

$$A_i = \begin{bmatrix} L^{n_i-1} & I_{n_i-1} \\ -l_{i,n_i} & 0 \end{bmatrix}, \quad L^{n_i-1} = \left[-l_{i,1}, \ldots, -l_{i,n_i-1}\right]^T, \quad 1 \leq i \leq N.$$

Consider the following Lyapunov candidate $V_e = e^T Pe$ for (3). Its time derivative can be written as:

$$\dot{V}_e = e^T\left(PA_0 + A_0^T P\right)e + 2e^T P\left(F(x) - F(\hat{x})\right) + 2e^T PF(\hat{x}) \tag{4}$$

Additionally, with the fact $P > 0$, the following inequality holds

$$2e^T P(F(x) - F(\hat{x})) = 2e^T P\frac{\partial F}{\partial x}e \leq e^T\left[P\frac{\partial F}{\partial x} + \left(\frac{\partial F}{\partial x}\right)^T P\right]e \tag{5}$$

where $\frac{\partial F}{\partial x} = \left[\frac{\partial f_{i,j}}{\partial x_{m,n}}\right]$ is a Jacobian matrix of g rows and g columns, with $g = \sum_{i=1}^N n_i$. According to Assumption 2.1, every nonzero element in the Jacobian matrix has its own upper and lower bounds. Namely, there exists a function $0 \leq \mu_{pq}(t) \leq 1$ such that $\frac{\partial f_{i,j}}{\partial x_{m,n}} = \mu_{pq}\underline{a}_{pq} + (1 - \mu_{pq})\bar{a}_{pq}$. Thus, $\frac{\partial F}{\partial x}$ can be reformulated as the following form:

$$\frac{\partial F}{\partial x} = \sum_{p=1}^{g}\sum_{q=1}^{g}\left[\mu_{pq}\underline{F}_{pq} + (1 - \mu_{pq})\overline{F}_{pq}\right], \quad 0 < \alpha_{pq} < 1 \tag{6}$$

where $\underline{F}_{pq}$ and $\overline{F}_{pq}$ are constant matrixes and they have only one nonzero element $\underline{a}_{pq}$ and $\bar{a}_{pq}$ at their pth row and qth column, respectively. Because of Jacobian matrix $\frac{\partial F}{\partial x}$ being time-varying, it is difficult for us to complete the stability analysis and controller design. In order to overcome this difficulty, a group of linear matrix inequalities (LMIs) is applied to subsequent procedures. Furthermore, form Remark 1 and Lemma 2 in [11], one can get:

$$2e^T PF(\hat{x}) \leq \varepsilon_0 e^T e + c\left(\sum_{i=1}^{N}\sum_{j=1}^{n_i}|z_{i,j}|^2\phi_{i,j}^2\left(\hat{\theta}_{i,j}\right)\right) \tag{7}$$

with $c = gc_0$ and $c_0 = \varepsilon_0^{-1}\|P^2\|\sum_{j=1}^{n_i-1}h_{i,j}^2$.

Consequently, substituting (5) and (7) into (4) gets

$$\dot{V}_e \leq e^T\left(PA_0 + A_0^T P + P\frac{\partial F}{\partial x} + \left(\frac{\partial F}{\partial x}\right)^T P + \varepsilon_0 I\right)e + c\left(\sum_{i=1}^{N}\sum_{j=1}^{n_i}z_{i,j}^2\phi_{i,j}^2\left(\hat{\theta}_{i,j}\right)\right) \tag{8}$$

Remark 3.1. *The last term at the right side of (8) can be counteracted in the subsequent procedures. And the performance of error dynamics can compensate for the effect of e in the other terms. This treatment method is different from the traditional way in* [8, 9].

4. **Adaptive Fuzzy Control Design and Stability Analysis.** According to the ith subsystems described by (1), the adaptive fuzzy backstepping output-feedback control design is based on the change of coordinates:

$$z_{i,j} = \hat{x}_{i,j} - \alpha_{i,j-1}, \quad i = 1, \ldots, N;\ j = 1, \ldots, n_i \tag{9}$$

where $\alpha_{i,0} = 0$, $\alpha_{i,j}$ is the virtual control signal, and when $j = n_i$, α_{i,n_i} is the actual control input signal $u_i(t)$. The control signal can be constructed as:

$$\alpha_{i,j} = -\frac{1}{2a_{i,j}^2}z_{i,j}\hat{\theta}_{i,j}S_{i,j}(Z_{i,j})^T S_{i,j}(Z_{i,j}) - \frac{1}{2}z_{i,j} - k_{i,j}z_{i,j} \tag{10}$$

where $k_{i,j}$ and $a_{i,j}$ are positive design parameters. $S_{i,j}(Z_{i,j})$ is the basis function vector with $Z_{i,j} = \left[\hat{x}_{i,1}, \ldots, \hat{x}_{i,j}, \hat{\theta}_{i,1}, \ldots, \hat{\theta}_{i,j}\right]^T$. $\hat{\theta}_{i,j}$, which will be defined later, is the estimation of an unknown constant $\theta_{i,j}$, and the evaluated error is $\tilde{\theta}_{i,j} = \theta_{i,j} - \hat{\theta}_{i,j}$. Its adaptive law is given as follows:

$$\dot{\hat{\theta}}_{i,j} = \frac{r_{i,j}}{2a_{i,j}^2} z_{i,j}^2 S_{i,j}^T(Z_{i,j}) S_{i,j}(Z_i) - \sigma_{i,j}\hat{\theta}_{i,j} \tag{11}$$

where $r_{i,j}$ and $\sigma_{i,j}$ are positive design parameters.

Remark 4.1. *It is apparent that (11) means that for any initial condition $\hat{\theta}_{i,j}(t_0) \geq 0$, the solution $\hat{\theta}_{i,j}(t) \geq 0$ holds for $t \geq t_0$. Thus, throughout this paper, it is assumed that $\hat{\theta}_{i,j}(t) \geq 0$.*

Consider the Lyapunov function candidate for the ith subsystem:

$$V_i = V_{zi} + V_{\theta i} = \frac{1}{2}\sum_{j=1}^{n_i} z_{i,j}^2 + \sum_{j=1}^{n_i} \frac{1}{2r_{i,j}}\tilde{\theta}_{i,j}^2$$

Next, the time derivative of $V_{zi} = \frac{1}{2}\sum_{j=1}^{n_i} z_{i,j}^2$ is calculated as:

$$\begin{aligned}\dot{V}_{zi} = \sum_{j=1}^{n_i} z_{i,j}\dot{z}_{i,j} &= \sum_{j=1}^{n_i-1} z_{i,j}\left(\alpha_{i,j} - \sum_{k=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{x}_{i,k}}\hat{x}_{i,k+1} - \sum_{k=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_{i,k}}\dot{\hat{\theta}}_{i,k}\right) \\ &\quad + \sum_{j=1}^{n_i-1} z_{i,j}\left(l_{i,j}e_{i,1} - \sum_{k=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{x}_{i,k}} l_{i,k}e_{i,1}\right) + \sum_{j=1}^{n_i-1} z_{i,j}z_{i,j+1} \\ &\quad + z_{i,n_i}\left(u_i - \dot{\alpha}_{n_i-1} + l_{i,n_i}e_{i,1}\right)\end{aligned} \tag{12}$$

The completion of squares is used to handle the term $z_{i,j}\left(l_{i,j}e_{i,1} - \sum_{k=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{x}_{i,k}} l_{i,k}e_{i,1}\right)$ in (12) as follows:

$$z_{i,j}\left(l_{i,j}e_{i,1} - \sum_{k=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{x}_{i,k}} l_{i,k}e_{i,1}\right) \leq \frac{1}{2\beta_{i,j}} z_{i,j}^2\left(l_{i,j} - \sum_{k=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{x}_{i,k}} l_{i,k}\right)^2 + \frac{1}{2}\beta_{i,j}e_{i,1}^2 \tag{13}$$

By this way, the effect of the first term at the right side of Equation (13) can be counteracted directly by α_i. The performance of error can compensate for the effect of the second one. Now, substituting inequality (13) into (12) and defining functions $\bar{f}_{i,j}(z_{i,j})$ as

$$\begin{aligned}\bar{f}_{i,j}(Z_{i,j}) &= -\sum_{k=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{x}_{i,k}}\hat{x}_{i,k+1} - \sum_{k=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_{i,k}}\dot{\hat{\theta}}_{i,k} + \frac{1}{2\beta_{i,j}} z_{i,j}\left(\sum_{k=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \hat{x}_{i,k}} l_{i,k} - l_{i,j}\right)^2 \\ &\quad + z_{i,j-1} + c z_{i,j}\phi_{i,j}^2\left(\hat{\theta}_{i,j}\right), \quad 1 \leq j \leq n_i - 1 \\ \bar{f}_{i,n_i}(Z_{i,n_i}) &= -\dot{\alpha}_{i,n_i-1} + l_{i,n_i}e_{i,1} + z_{i,n_i-1} + c z_{i,n_i}\phi_{i,n_i}^2\left(\hat{\theta}_{i,n_i}\right)\end{aligned}$$

then, Equation (12) can be rewritten as:

$$\dot{V}_{zi} \leq \sum_{j=1}^{n_i} z_{i,j}\left(\alpha_{i,j} + \bar{f}_{i,j}(z_{i,j})\right) + \sum_{j=1}^{n_i-1}\frac{1}{2\beta_{i,j}}e_{i,1}^2 - c\sum_{j=1}^{n_i} z_{i,j}^2\phi_{i,j}^2\left(\hat{\theta}_{i,j}\right) \tag{14}$$

Because these functions $\bar{f}_{i,j}(z_{i,j})$s are unknown, they cannot be directly used to design controllers. We should use FLSs to approximate them. In light of the theory in [12], for any given $\varepsilon_{i,j} > 0$, there exists an FLS $W_{i,j}^T S_{i,j}(Z_{i,j})$ such that

$$\bar{f}_{i,j}(Z_{i,j}) = W_{i,j}^T S_{i,j}(Z_{i,j}) + \delta_{i,j}(Z_{i,j})$$

where $\delta_{i,j} \le \varepsilon_{i,j}$ denotes the approximation error.

Applying completion of squares again gets

$$z_{i,j}\bar{f}_{i,j} \le \frac{1}{2a_{i,j}^2} z_{i,j}^2 \theta_{i,j} S_{i,j}^T S_{i,j} + \frac{1}{2}a_{i,j}^2 + \frac{1}{2}z_{i,j}^2 + \frac{1}{2}\varepsilon_{i,j}^2,\ 1 \le i \le N,\ 1 \le j \le n_i \tag{15}$$

where the unknown constant $\theta_{i,j} = \|W_{i,j}\|^2$. Substituting (10) and (15) into (14) obtains:

$$\begin{aligned} \dot{V}_{zi} \le & -\sum_{j=1}^{n_i} k_{i,j} z_{i,j}^2 + \sum_{j=1}^{n_i} \frac{1}{2a_{i,j}^2} z_{i,j}^2 \tilde{\theta}_{i,j} S_{i,j}^T S_{i,j} + \sum_{j=1}^{n_i} \frac{1}{2}\left(a_{i,j}^2 + \varepsilon_{i,j}^2\right) \\ & + \sum_{j=1}^{n_i-1} \frac{1}{2}\beta_{i,j} e_{i,1}^2 - c\sum_{j=1}^{n_i} z_{i,j}^2 \phi_{i,j}^2\left(\hat{\theta}_{i,j}\right) \end{aligned} \tag{16}$$

Consider the whole Lyapunov function candidate as $V = V_e + \sum_{i=1}^{N}(V_{zi} + V_{\theta i})$. Noticing $\dot{V}_{\theta i} = \sum_{j=1}^{n_i} \frac{-1}{r_{i,j}} \tilde{\theta}_{i,j}\dot{\hat{\theta}}_{i,j}$ and taking (8) and (16) into account, one has

$$\begin{aligned} \dot{V} \le & e^T\left[PA_0 + A_0^T P + P\frac{\partial F}{\partial x} + \left(\frac{\partial F}{\partial x}\right)^T P + (\varepsilon_0 I + \beta)\right] e - \sum_{i=1}^{N}\sum_{j=1}^{n_i} k_{i,j} z_{i,j}^2 \\ & + \sum_{i=1}^{N}\sum_{j=1}^{n_i} \frac{\tilde{\theta}_{i,j}}{r_{i,j}}\left(\frac{r_{i,j}}{2a_{i,j}^2} z_{i,j}^2 S_{i,j}^T S_{i,j} - \dot{\hat{\theta}}_{i,j}\right) + \sum_{i=1}^{N}\sum_{j=1}^{n_i} \frac{1}{2}\left(a_{i,j}^2 + \varepsilon_{i,j}^2\right) \end{aligned} \tag{17}$$

where $\beta = diag\left[\sum_{i=1}^{N}\sum_{j=1}^{n_i-1} \frac{1}{2}\beta_{i,j}, 0, \ldots, 0\right]$.

Now, we can summarize our main results in the following theorem.

Theorem 4.1. *Consider the nonlinear MIMO system (1) under Assumption 2.1. If there exists a definitive positive matrix P, it can make the following inequality true*

$$PA_0 + A_0^T P + \varepsilon_0 I + \beta + PF_{pq} + F_{pq}^T P < 0, \quad 1 \le p,\ q \le g \tag{18}$$

where F_{pq} is a constant matrix whose element at the pth row and the qth column is $\bar{a}_{pq}$ or $\underline{a}_{pq}$ and others are zero. Then the controller (10), the fuzzy state observer (2) and the adaptive law (11) can guarantee all the signals in closed-loop system are SGUUB and the error signals $e_{i,j}$, $z_{i,j}$, and $\tilde{\theta}_{i,j}$ eventually converge to a small enough neighborhood around the origin.

Proof: According to inequality (7) and Lemma 3 in [13], the following inequity

$$PA_0 + A_0^T P + P\frac{\partial F}{\partial x} + \left(\frac{\partial F}{\partial x}\right)^T P + \varepsilon_0 I + \beta < 0, \tag{19}$$

is equivalent to a set of linear matrix inequalities as (18) described. Next, using $\tilde{\theta}\hat{\theta} \le -\frac{1}{2}\tilde{\theta}^2 + \frac{1}{2}\theta^2$ and (11), we can obtain:

$$\begin{aligned} \dot{V} \le & e^T\left[PA_0 + A_0^T P + P\frac{\partial F}{\partial x} + \left(\frac{\partial F}{\partial x}\right)^T P + (\varepsilon_0 I + \beta)\right] e - \sum_{i=1}^{N}\sum_{j=1}^{n_i} k_{i,j} z_{i,j}^2 \\ & - \sum_{i=1}^{N}\sum_{j=1}^{n_i} \frac{\sigma_{i,j}}{2r_{i,j}} \tilde{\theta}_{i,j^2} + \sum_{i=1}^{N}\sum_{j=1}^{n_i} \frac{\sigma_{i,j}}{2r_{i,j}} \theta_{i,j}^2 + \sum_{i=1}^{N}\sum_{j=1}^{n_i} \frac{1}{2}\left(a_{i,j}^2 + \varepsilon_{i,j}^2\right) \end{aligned} \tag{20}$$

According to inequalities (19) and (18), there exists a constant $\alpha > 0$ such that

$$PA_0 + A_0^T P + PF_{pq} + F_{pq}^T P + \varepsilon_0 I + \beta < -\alpha I$$

which is equivalent to

$$PA_0 + A_0^T P + P\frac{\partial F}{\partial x} + \left(\frac{\partial F}{\partial x}\right)^T P + \varepsilon_0 I + \beta < -\alpha e^T e \leq -\frac{\alpha}{\lambda_M(P)} e^T P e$$

where $\lambda_M(P)$ is the maximal eigenvalue of matrix P.

Now let

$$a_0 = \min\left\{\frac{\alpha}{\lambda_M(P)}, 2k_{i,j}, \sigma_{i,j},\ 1 \leq i \leq N,\ 1 \leq j \leq n_i\right\}$$

$$b_0 = \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left[\frac{1}{2}\left(a_{i,j}^2 + \varepsilon_{i,j}^2\right) + \frac{\sigma_{i,j}}{2r_{i,j}}\theta_{i,j}^2\right]$$

and then (20) can be rewritten as

$$\dot{V} \leq -a_0 V + b_0 \tag{21}$$

Therefore, all the signals in the closed-loop system are SGUUB. The proof is completed.

5. Simulation Example.

Example 5.1. *Consider the following MIMO systems transformed from* [14] *with* $i = 1, 2$*:*

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} \\ \dot{x}_{i,2} = \dfrac{u_i}{J_i} + f_{i,2}(x) \\ y_i = x_{i,1} \end{cases}$$

where $f_{1,2} = ((m_1 gr/J_1) - (Kr^2/4J_1))\sin(x_{1,1})$, $f_{2,2} = ((m_2 gr/J_2) - (Kr^2/4J_2))\sin(x_{2,1})$, *and* $J_1 = 5$, $J_2 = 6.25$, $m_1 = 2$, $m_2 = 2,5$, $K = 100$, $r = 0.5$, $g = 9.81$. *By our method, choosing* $\varepsilon_0 = 0.01$, $\beta = 0.1I$, *and for given* F_{pq}, *solving LMIs (18) one can get* $l_{1,1} = 2$, $l_{1,2} = 4.5$, $l_{2,1} = 2.5$, $l_{2,2} = 7$ *and obtain* $P = diag[P_1, P_2]$, *where*

$$P_1 = \begin{bmatrix} 16.4 & -5.1 \\ -5.1 & 3.3 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 16.4 & -4.1 \\ -4.1 & 1.9 \end{bmatrix}$$

For $i = 1, 2$ *and* $j = 1, 2$, *the design parameters are set as* $k_{i,j} = 10$, $a_{i,j} = 0.5$, $\sigma_{i,j} = 1$, $r_{i,j} = 7.5$, *and the initial conditions are chosen as,* $x_{1,j}(0) = 0.2$, $x_{2,j}(0) = 0.1$. *The other initial conditions are chosen as zeros. In the simulation, FLSs* $W_{i,j}^T S_{i,j}(\cdot)$*s contain 14 nodes and the widths of the Gaussian function are equal to two.*

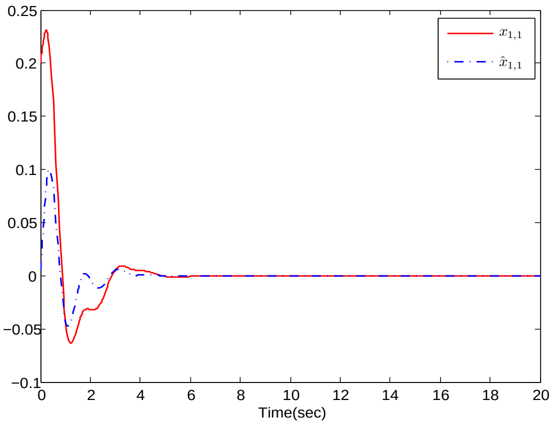

FIGURE 1. $x_{1,1}$ and $\hat{x}_{1,1}$

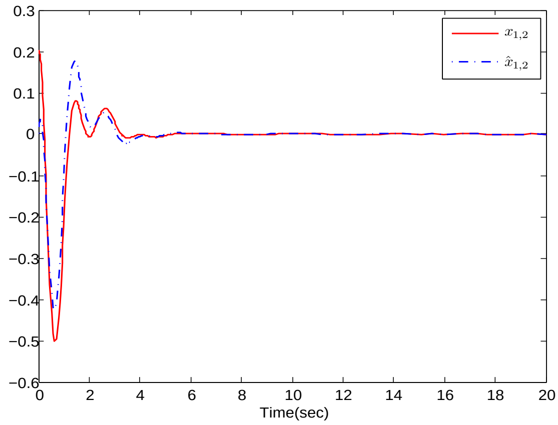

FIGURE 2. $x_{1,2}$ and $\hat{x}_{1,2}$

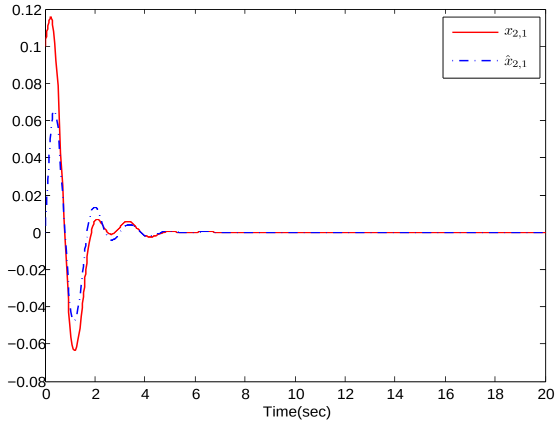

FIGURE 3. $x_{2,1}$ and $\hat{x}_{2,1}$

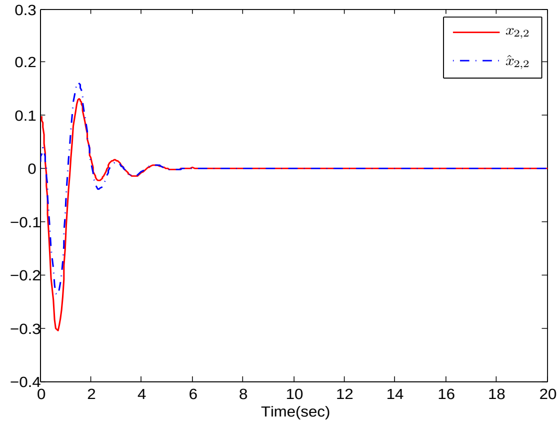

FIGURE 4. $x_{2,2}$ and $\hat{x}_{2,2}$

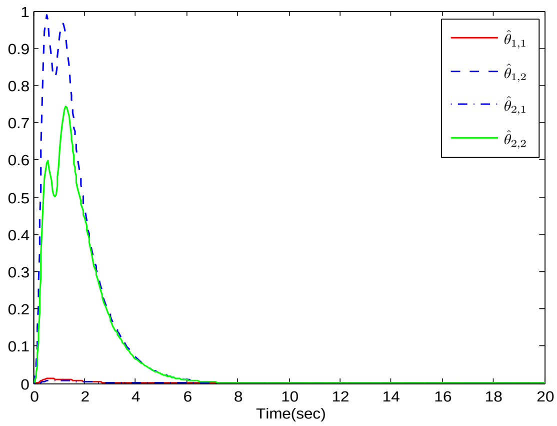

FIGURE 5. $\hat{\theta}_{i,j}$ $(i, j = 1, 2)$

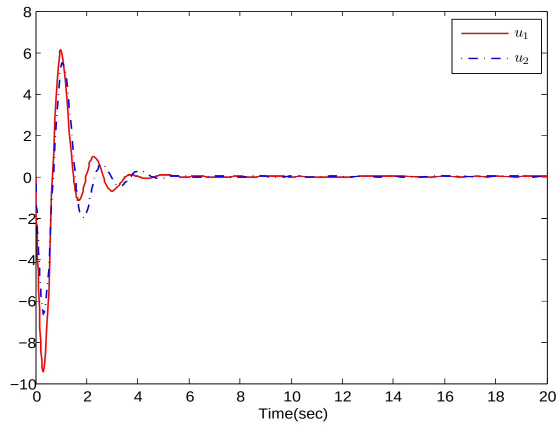

FIGURE 6. u_1 and u_2

The simulation results are shown in Figures 1-6. From the simulation results, it can be clearly shown that even though the nonlinear systems contain uncertain nonlinearities, the proposed adaptive fuzzy output-feedback controllers can guarantee the stability of the control systems and the boundedness of all the signals in the system.

6. **Conclusion.** An adaptive fuzzy control strategy has been proposed for a class of uncertain MIMO nonlinear system in strict-feedback form. FLSs are used to approximate the structure uncertainties and an observer is designed for state estimations. Based on the backstepping method and the Lyapunov stability theory, the designed controller guarantees all the closed-loop signals are SGUUB. The errors can be arbitrarily small by choosing suitable design parameters. The main efficiency of the proposed technique is due to a group of linear matrix inequalities. Compared with the existing method, solving linear matrix inequalities greatly reduces the computing difficulty, and makes our method more suitable for practical applications. Finally, computer simulations reveal the efficiency of the proposed method and also verify the theoretical results. Moreover, it is significant and challenging to apply the proposed method to nonlinear systems with time delays. This is an area that we will look at and that we will explore.

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