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PERFORMANCE LIMITATION OF NETWORKED CONTROL SYSTEMS WITH PACKET DROPOUTS AND QUANTIZATION

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Abstract. *The tracking performance limitation of networked control systems with packet dropouts and quantization constraints is investigated in this paper. The quantization and the packet dropouts of communication channel in the feedback channel are considered. The tracking performance limitation is obtained by using the Youla parameterization technique. It is shown that the tracking performance limitation depends on the non-minimum phase zeros position, unstable poles position of a given plant, quantization interval and packet dropouts probability. The result also shows how the quantization interval and packet dropouts probability may fundamentally constrain the tracking capability of networked control systems. A typical example is given to illustrate the theoretical results.*
Keywords: Packet dropouts probability, Quantization interval, Unstable poles position, Non-minimum phase zeros position

1. **Introduction.** In recent years, networked control systems (NCSs) have been found successfully applied into various fields [1]. While NCSs bring convenience to the life, they have also risen to new challenges due to packet dropouts, delay and quantization, and so on. The predictive controller design of networked systems with communication delay and data loss was studied in paper [2]. The stability problem of NCSs with packet dropouts and time delay was studied in paper [3].

The technologies about stabilization analysis of the NCSs are now fairly mature. From the application point of view, we should study the performance quality of NCSs. Now, more and more scholars are paying their attentions to studying the performance quality of NCSs. For example, the paper [4] studied discrete-time single-input linear time-invariant performance limitation with single-to-noise ratio constraints. The tracking performance limitation for two-channel disturbance rejection under control energy constraint was studied in paper [5]. The tracking performance limitation with communication constraint was studied in paper [6]. The tracking performance limitation of a linear time-invariant system with a quantized control signal was studied in paper [7].

These research results show that the tracking performance limitation of NCSs is determined by plant internal structure and networked parameters, such as non-minimum phase zeros position, and unstable poles position. At present, the study about the tracking performance limitation with quantization interval and packet dropouts is quite few, and it is difficult for us to study the tracking performance limitation with multi-network parameters. However, in NCSs, quantization interval and packet dropouts may exist concurrently. In this paper, we study the problem of the tracking performance limitation of NCSs with packet dropouts and quantization interval. The tracking performance expression is obtained by using the Youla parameterization technique. The result shows that the tracking performance limitation is determined by the non-minimum phase zeros position, unstable poles position, quantization interval and packet dropouts probability. This obtained result will provide a guidance on design of NCSs.

This paper is organized as follows. The problem statement is introduced in Section 2. In Section 3, we study tracking performance limitation of NCSs with packet dropouts and quantization interval as in Figure 1. An example is given to illustrate the theoretical results in Section 4. The paper conclusions are presented in Section 5.

2. **Problem Statement.** The symbol used throughout this paper is described as follows. For any vector z, we devote its conjugate transpose by $\bar{z}$. Let the open right-half and open left-half plans be denoted by $C_+ := \{s : \mathrm{Re}(s) > 0\}$ and $C_- := \{s : \mathrm{Re}(s) < 0\}$ [8], respectively. H_2 and $H_2^{\perp}$ are subspaces containing functions that are analytic in C_+ and C_-, respectively. Finally, $\mathcal{RH}_\infty$ denotes the set of all stable, proper, rational transfer functions.

We consider NCSs with packet dropouts and quantization, as depicted in Figure 1. The stochastic reference signal r is a Brownian motion process, and the parameter d_r denotes whether or not a packet is dropped:

$$d_r = \begin{cases} 1, & \text{if the system output is successfully transmitted to the controller,} \\ 0, & \text{if the system output is not successfully transmitted to the controller} \end{cases}$$

with a probability distribution given by $P\{d_r = 0\} = q$, $P\{d_r = 1\} = 1 - q$, where q is the packet dropouts probability.

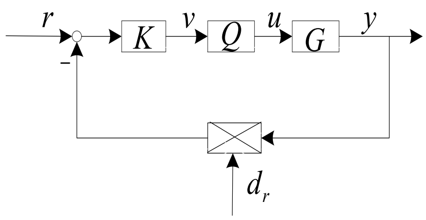

FIGURE 1. Networked control systems with packet dropouts

Figure 1 represented the model of networked control systems with quantization and data dropouts constraints, where data dropouts exist in the feedback path and quantization exist in the forward path. In Figure 1, G denotes the plant model. The signals r, y, v and u represent the reference input, the system output, the communication channel input and the communication channel output, respectively. The uniform quantizer is used in this paper, and $u = v + n$, where n denotes quantization error and obeys uniform distribution in $\left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$, and Δ represented the quantitative interval. Then, we assume the mean and variance of reference input r are zero and θ^2, respectively.

It follows from Figure 1 that

$$v = Kr - Kd_r y, \quad u = v + n, \quad y = Gu \tag{1}$$

According to (1), we can obtain the tracking error of the NCS is

$$e = r - y = \left(1 - \frac{KG}{1 + KGd_r}\right) r - \frac{G}{1 + KGd_r} n$$

According to [8], we can obtain

$$S_e(j\omega) = \left(1 - \frac{K(s)G(s)}{1 + d_r K(s)G(s)}\right) S_{re}(j\omega) + \frac{G(s)}{1 + d_r K(s)G(s)} S_{ne}(j\omega)$$

Then we have

$$\sigma_e^2 = \left\|1 - \frac{K(s)G(s)}{1+(1-q)K(s)G(s)}\right\|_2^2 \theta^2 + \frac{\Delta^2}{12}\left\|\frac{G(s)}{1+(1-q)K(s)G(s)}\right\|_2^2$$

Denote $J := \sigma_e^2$, and the performance limitation is measured by the possible minimal tracking error achievable by all possible linear stabilizing controllers (denoted by $\mathcal{K}$), determined as

$$J^* = \inf_{K\in\mathcal{K}}\left\|1 - \frac{K(s)G(s)}{1+(1-q)K(s)G(s)}\right\|_2^2 \theta^2 + \frac{\Delta^2}{12}\inf_{K\in\mathcal{K}}\left\|\frac{G(s)}{1+(1-q)K(s)G(s)}\right\|_2^2 \quad (2)$$

For the rational transfer function $(1-q)G$, consider a coprime factorization of $(1-q)G$ as

$$(1-q)G = \frac{N}{M} \quad (3)$$

where $M, N \in \mathcal{RH}_\infty$ and satisfy the Bezout identity

$$MX - NY = 1 \quad (4)$$

where $X, Y \in \mathcal{RH}_\infty$. Then a stabilizing compensator K can be characterized by the Youla parameterization [9]

$$\mathcal{K} := \left\{K : K = -\frac{(Y-MQ)}{X-NQ}, Q \in \mathcal{RH}_\infty\right\} \quad (5)$$

It is well known that a non-minimum phase transfer function can be factorized as a minimum phase part and an all-pass factor [10]

$$N = (1-q)L_z N_n, \quad M = B_P M_m \quad (6)$$

where L_z and B_p are the all-pass factors, and N_n and M_m are the minimum phase Parts. L_z includes all the right-half plane zeros of the plant $z_i \in C_+$, $i = 1, 2, \ldots, n$, B_p includes all the right-half plane poles of the plant $p_j \in C_+$, $j = 1, 2, \ldots, m$. We consider the coprime factorization of $L_z(s)$ and $B_p(s)$ respectively as

$$L_z(s) = \prod_{i=1}^{n} \frac{s - z_i}{s + \bar{z}_i} \quad B_p(s) = \prod_{j=1}^{m} \frac{s - p_j}{s + \bar{p}_j} \quad (7)$$

3. **Performance Limitation of Networked Control Systems.** According to (2), (3), (4) and (5), we can obtain

$$J^* \geq \inf_{Q\in\mathcal{RH}_\infty}\left\|1 + \frac{1}{1-q}N(Y-MQ)\right\|_2^2 \theta^2 + \frac{\Delta^2}{12}\inf_{Q\in\mathcal{RH}_\infty}\left\|-\frac{1}{1-q}N(X-NQ)\right\|_2^2 \quad (8)$$

It is clear that in order to obtain the minimum J^*, Q must be appropriately selected.

Theorem 3.1. *If the plant is factorized as in (3) and (6), then the tracking performance limitation is given by*

$$J^* \geq J_1^*\theta^2 + \frac{\Delta^2}{12}J_2^* \quad (9)$$

where

$$J_1^* = \sum_{i=1}^{n} 2\mathrm{Re}(z_i) + \sum_{i,j\in N}^{n} \frac{4\mathrm{Re}(p_j)\mathrm{Re}(p_i)}{(\bar{p}_j + p_i)\,\bar{b}_j b_i}\left[\left(1 - \frac{1}{1-q}L_z^{-1}(p_j)\right)^H \left(1 - \frac{1}{1-q}L_z^{-1}(p_j)\right)\right]$$

$$J_2^* = \sum_{i,j\in N}^{n} \frac{4\mathrm{Re}(z_j)\mathrm{Re}(z_i)}{(\bar{z}_i + z_j)\bar{a}_i a_j}\left[(N_n(z_i)X(z_i))^H (N_n(z_i)X(z_i))\right]$$

$$b_j = \prod_{\substack{i\in N\\ i\neq j}} \frac{p_j - p_i}{p_j + \bar{p}_i} \qquad a_i = \prod_{\substack{j\in N\\ j\neq i}} \frac{z_i - z_j}{z_i + \bar{z}_j}$$

Proof: From (8), we denote

$$J_1{}^* = \inf_{Q\in\mathcal{RH}_\infty} \left\|1 + \frac{1}{1-q}N(Y-MQ)\right\|_2^2, \quad J_2^* = \inf_{Q\in\mathcal{RH}_\infty} \left\|-\frac{1}{1-q}N(X-NQ)\right\|_2^2 \quad (10)$$

According to (6) and (10), because L_z and B_p are the all-pass factors, it follows that

$$J_1{}^* = \inf_{Q\in\mathcal{RH}_\infty} \left\|(L_z{}^{-1} - 1) + (1 + N_n(Y - MQ))\right\|_2^2$$

As $(L_z{}^{-1} - 1) \in \mathrm{H}_2^\perp$, $(1 + N_n(Y - MQ)) \in \mathrm{H}_2$, thus

$$J_1^* = \|L_z^{-1} - 1\|_2^2 + \inf_{Q\in\mathcal{RH}_\infty} \left\|B_p^{-1}(1 + N_nY) - N_nM_mQ\right\|_2^2$$

It follows the same arguments as in [11] that

$$\left\|L_z^{-1} - 1\right\|_2^2 = \sum_{i=1}^{n_s} 2\mathrm{Re}(z_i) \quad (11)$$

Denote $J_{11}^* = \inf_{Q\in\mathcal{RH}_\infty} \left\|B_p^{-1}(1 + N_nY) - N_nM_mQ\right\|_2^2$, and then based on a partial fraction procedure, we can obtain $B_p^{-1}(N_nY + 1) = \sum_{j=1}^{m} \frac{s+\bar{p}_j}{s-p_j}\frac{N_n(p_j)Y(p_j)+1}{b_j} + R_1$, where $R_1 \in \mathcal{RH}_\infty$, $b_j = \prod_{\substack{i\in N\\ i\neq j}} \frac{p_j-p_i}{p_j+\bar{p}_i}$.

Therefore,

$$J_{11}^* = \inf_{Q\in\mathcal{RH}_\infty} \left\|\sum_{j=1}^{m} \left(\frac{s+\bar{p}_j}{s-p_j} - 1\right) \frac{N_n(p_j)Y(p_j)+1}{b_j} + R_1 + \sum_{j=1}^{m} \frac{N_n(p_j)Y(p_j)+1}{b_j} - N_nM_mQ\right\|_2^2.$$

As $\left(\frac{s+\bar{p}_j}{s-p_j} - 1\right) \in \mathrm{H}_2^\perp$, and $\left(R_1 + \sum_{j=1}^{m} \frac{N_n(p_j)Y(p_j)+1}{b_j} - N_nM_mQ\right) \in \mathrm{H}_2$, it follows that

$$J_{11}^* = \left\|\sum_{j=1}^{m} \frac{2\mathrm{Re}(p_j)}{s-p_j}\frac{N_n(p_j)Y(p_j)+1}{b_j}\right\|_2^2 + \inf_{Q\in\mathcal{RH}_\infty} \left\|R_1 + \sum_{j=1}^{m} \frac{N_n(p_j)Y(p_j)+1}{b_j} - N_nM_mQ\right\|_2^2$$

Because N_n and M_m are outer functions and minimum phases, we can obtain that $\inf_{Q\in\mathcal{RH}_\infty} \left\|R_1 + \sum_{j=1}^{m} \frac{N_n(p_j)Y(p_j)+1}{b_j} - N_nM_mQ\right\|_2^2 = 0$, hence

$$J_{11}^* = \sum_{i,j\in N}^{n} \frac{4\mathrm{Re}(p_j)\mathrm{Re}(p_i)}{(\bar{p}_j + p_i)\,\bar{b}_jb_i} \left[(N_n(p_j)Y(p_j) + 1)^H\,(N_n(p_j)Y(p_j) + 1)\right] \quad (12)$$

Simultaneously, according to (4) and $M(p_j) = 0$, we can obtain

$$N_n(p_j)Y(p_j) = -(1-q)^{-1}L_z^{-1}(p_j)$$

Hence

$$N_n(p_j)Y(p_j) + 1 = 1 - (1-q)^{-1}L_z^{-1}(p_j) \quad (13)$$

According to (10)-(13), we can obtain

$$\begin{aligned} J_1^* &= \sum_{i=1}^{n} 2\mathrm{Re}(z_i) + \sum_{i,j\in N}^{n} \frac{4\mathrm{Re}(p_j)\mathrm{Re}(p_i)}{(\bar{p}_j+p_i)\bar{b}_jb_i}\omega^H\omega \\ b_j &= \prod_{\substack{i\in N\\ i\neq j}} \frac{p_j-p_i}{\bar{p}_i+p_j}, \quad \omega = 1 - (1-q)^{-1}L_z^{-1}(p_j) \end{aligned} \quad (14)$$

According to (6) and (10), because L_z is the all-pass factors, it follows that

$$J_2^* = \inf_{Q\in\mathcal{RH}_\infty} \left\| \frac{N_n X}{L_z} - (1-q)N_n N_n Q \right\|_2^2$$

Based on a partial fraction procedure, we can obtain $\frac{N_n X}{L_z} = \sum_{i=1}^{n} \frac{s+\bar{z}_i}{s-z_i} \frac{N_n(z_i)X(z_i)}{a_i} + R_2$, where $R_2 \in \mathcal{RH}_\infty$, $a_i = \prod_{\substack{j\in N\\ j\neq i}} \frac{z_i - z_j}{z_i+\bar{z}_j}$.

Therefore,

$$J_2^* = \inf_{Q\in\mathcal{RH}_\infty} \left\| \sum_{i=1}^{n} \left(\frac{s+\bar{z}_i}{s-z_i} - 1\right) \frac{N_n(z_i)X(z_i)}{a_i} + R_2 + \sum_{i=1}^{n} \frac{N_n(z_i)X(z_i)}{a_i} - (1-q)N_n N_n Q \right\|_2^2.$$

Because $\left(\frac{s+\bar{z}_i}{s-z_i} - 1\right) \in \mathrm{H}_2^\perp$, and $\left(R_2 + \sum_{i=1}^{n} \frac{N_n(z_i)X(z_i)}{a_i} - (1-q)N_n N_n Q\right) \in \mathrm{H}_2$, it follows that

$$J_2^* = \left\| \sum_{i=1}^{n} \left(\frac{s+\bar{z}_i}{s-z_i} - 1\right) \frac{N_n(z_i)X(z_i)}{a_i} \right\|_2^2 + \inf_{Q\in\mathcal{RH}_\infty} \left\| R_2 + \sum_{i=1}^{n} \frac{N_n(z_i)X(z_i)}{a_i} - (1-q)N_n N_n Q \right\|_2^2$$

We choose an appropriate Q, which makes $\inf_{Q\in\mathcal{RH}_\infty} \left\| R_2 + \sum_{i=1}^{n} \frac{N_n(z_i)X(z_i)}{a_i} - (1-q)N_n N_n Q \right\|_2^2 = 0$.

By a simple computation, we can obtain

$$J_2^* = \sum_{i,j\in N}^{n} \frac{4\mathrm{Re}(z_i)\mathrm{Re}(z_j)}{(z_i+\bar{z}_j)\bar{a}_i a_j} \left[(N_n(z_i)X(z_i))^H (N_n(z_i)X(z_i))\right] \tag{15}$$

where $a_i = \prod_{\substack{j\in N\\ j\neq i}} \frac{z_i - z_j}{z_i+\bar{z}_j}$.

According to (8), (10), (14) and (15), the proof is completed.

Theorem demonstrates the tracking performance limitation of SISO NCSs generally depends on the non-minimum phase zeros position, unstable poles position of a given plant, quantization interval and packet dropouts probability. It is obvious that this effect will in general degrade the tracking performance limitation because of quantitative interval and packet dropouts probability constraint in NCSs.

4. **Numerical Example.**

Consider an unstable plant model described by $G(s) = \frac{s-0.1}{(s+0.1)(s-k)}$, the non-minimum phase zeros is located at $z_1 = 0.1$, and it has an unstable pole for any $k > 0$ at $p_1 = k$. Assume the packet dropouts probability $q = 0.5$, where $\theta = 1$, when the quantization interval is $\Delta_1 = 0$, $\Delta_2 = 1$, $\Delta_3 = 2$. From Theorem 3.1, the tracking performance limitation of the system with different quantization intervals is shown in Figure 2. From Figure 2, we can see that the tracking performance limitation is connected with the quantization interval, moreover, the tracking performance limitation will degrade when the quantization interval increases. From Figure 2, it can also be seen that the tracking performance limitation is degraded when the non-minimum phase zeros position move closer to the unstable pole position.

When the packet dropouts probability $q = 0.5$, and the quantization interval $\Delta = 3$, according to Theorem 3.1, the tracking performance limitation is $J_3{}^* = 0.2 + 2k\left(1 - 2\frac{k+0.1}{k-0.1}\right)^2 + \frac{3}{20}\left(\frac{1}{0.1-k}\right)^2$.

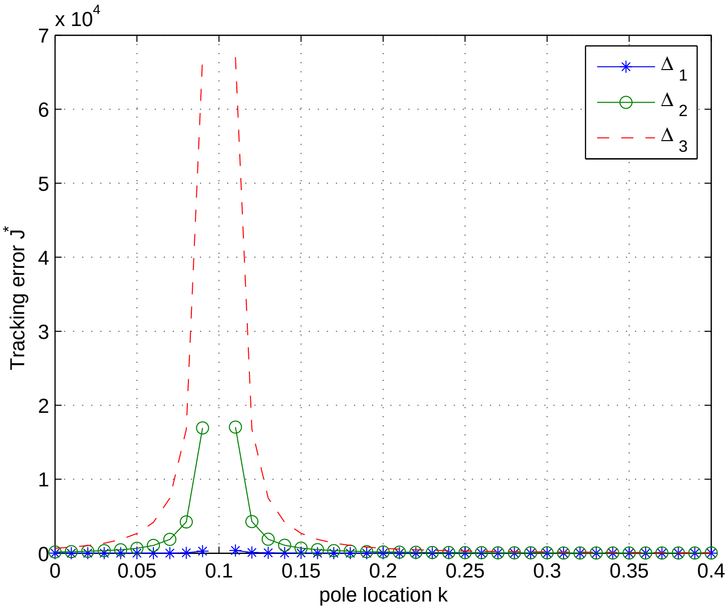

FIGURE 2. Tracking performance limitation with different quantization intervals

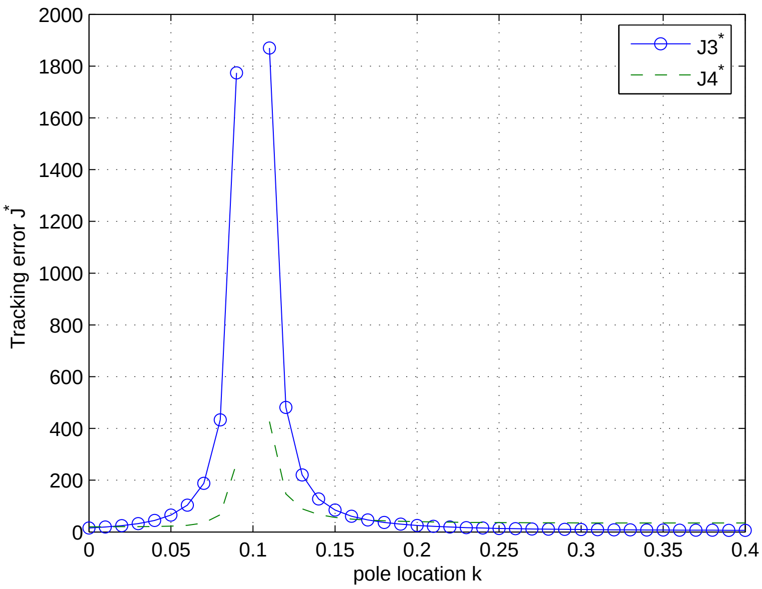

FIGURE 3. Tracking performance limitation of NCSs

The tracking performance limitation of networked control systems with packet dropouts constraint was studied in paper [12]. According to [12], we can obtain $J_4{}^* = 20 + 2k\left(1 - 2\frac{k+0.1}{0.1-k}\right)^2$. Then, the tracking performance limitation in the tow cases is shown in Figure 3.

From Figure 3, we can see that the tracking performance limitation is degraded when the non-minimum phase zeros position move closer to the unstable pole position. Compared with [12], the tracking performance limitation is worse by the quantization interval and packet dropouts. Thus, the quantization error affects the tracking performance limitation.

5. **Conclusions.** The tracking performance limitation of networked control systems with packet dropouts and quantization has been investigated in this paper. The tracking

performance limitation is obtained by using the Youla parameterization technique. It is shown that the tracking performance limitation is dependent on the non-minimum phase zeros position, the unstable pole position of the plant, the quantization interval and the packet dropouts probability. It is also proved how quantization and packet dropouts probability may fundamentally destroy the NCSs tracking capability. Finally, a typical example is given to illustrate the theoretical results.

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