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LINEAR QUADRATIC CONTROL FOR NETWORKED CONTROL SYSTEMS WITH INFORMATION LIMITATIONS

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Abstract. *This paper investigates the minimum information rate for linear quadratic (LQ) control of stochastic linear systems, where the sensors and controllers are geographically separated and connected via noiseless, bandwidth-limited digital communication channels. A control scheme is given to achieve the minimum information rate for LQ control. In particular, we argue the tradeoffs between the control cost and the data rate of the channel, present an explicit formula on the tradeoffs between the LQ cost and the information rate, and derive upper bounds on the corresponding LQ cost under data rate limitations.*

Keywords: Linear quadratic control, Limited information rates, Stabilization, Networked control systems

1. **Introduction.** Many advances in recent emerging applications (industrial automation, sensor networks, vehicle systems, aerospace industry, etc.) have led to increasing activity in understanding and designing networked control systems. The problem arises when the sensors, plant and controller are geographically separated and connected via bandwidth-limited digital channels. However, the limitations in the communication links (such as data rate limitations, packet dropout, time delay) often affect control performances significantly [1,2].

Issues of the type discussed are motivated by several pieces of work in the recent literature. The line of work of control under constrained communication channels was initiated by Bansal and Basar [3]. Wong and Brockett [4] further considered the issues of coding, communication protocols, and delays explicitly. A high-water mark in the study of quantized feedback using data rate limited feedback channels is known as the data rate theorem that states the larger the magnitude of the unstable poles is, the larger the required data rate through the feedback loop is. The intuitively appealing result was proved in [5-7], indicating that it quantifies a fundamental relationship between unstable physical systems and the rate at which information must be processed in order to stably control them. When the feedback channel capacity is near the data rate limit, control designs typically exhibit chaotic instabilities. This result was generalized to different notions of stabilization and system models, and was also extended to multi-dimensional systems [8-10]. Control under communication constraints inevitably suffers signal transmission delay, data packet dropout and measurement quantization which might be potential sources of instability and poor performance of control systems [11-13].

This paper is concerned with LQ control problem for networked control systems over bandwidth-limited communication channels. Our purpose is to present a control scheme to achieve some given control objectives under information limitations. Different from the literature, no assumption that system matrix contains only marginally stable and unstable eigenvalues is made. It turns out in our results that the stable part still plays a key role in achieving the LQ cost. Our work here differs in that an explicit formula for the tradeoffs between the LQ cost and the data rate of the channel is proposed and a full knowledge optimal cost is presented in our results.

The remainder of this paper is organized as follows: Section 2 introduces problem formulation; Section 3 presents the control scheme under information limitations; Section 4 deals with LQ control problem for networked control systems with limited information rates; conclusions are stated in Section 5.

2. **Problem Formulation.**

Consider a stochastic linear system described by

$$\dot{X} = AX + BU + FW$$

where $X \in \mathbb{R}^n$ is the state process, $U \in \mathbb{R}^m$ is the control input, and $W \in \mathbb{R}^p$ is the process disturbance. A, B, and F are known constant matrices with appropriate dimensions. The corresponding discrete-time system is

$$X(t+1) = \Upsilon X(t) + \Theta U(t) + W(t) \tag{1}$$

where Υ and Θ are known constant matrices with appropriate dimensions.

Different from the literature, no assumption that system matrix Υ contains only marginally stable and unstable eigenvalues is made. It turns out in our results that the stable part still plays a key role in achieving the LQ cost. It is assumed that

A_0**:** Suppose that the pair (Υ, Θ) is controllable, and the plant states are measurable;

A_1**:** The initial condition $X(0)$ and disturbance $W(0), \cdots, W(t)$ are mutually independent random variables with zero mean, satisfying $E\|X(0)\|^2 < \Phi_0 < \infty$ and $E\|W(t)\|^2 < \Phi_W < \infty$, respectively;

A_2**:** The sensors and controllers are geographically separated and connected by errorless digital channels without time delay. Then, the encoder and decoder have access to the previous control actions.

A_3**:** Let H be the unitary matrix that diagonalizes $\Upsilon = H'\Lambda H$ where $\Lambda = \text{diag}[\lambda_1(\Upsilon), \cdots, \lambda_n(\Upsilon)]$ and $\lambda_i(\Upsilon)$ denotes the ith eigenvalue of Υ $(i = 1, \cdots, n)$.

In this paper, the main purpose is to present a control policy to stabilize system (1) in the mean square sense

$$\limsup_{t\to\infty} E\|X(t)\|^2 < \infty \tag{2}$$

using the finite information rate provided by the digital feedback link. The LQ control problem under information rate limitations will be discussed. We also design the control scheme to satisfy some given control objectives, and establish a relation between the LQ cost and the information rate of the channel.

3. **The Control Scheme under Information Limitations.**

In this section, we give the quantization, coding, and control schemes for system (1). Here, we quantize, encode the plant states by an adaptive differential coding strategy, and transmit the information of the plant states over a noiseless, bandwidth-limited digital channel.

Let $X_h(t)$ denote the decoder's estimate of $X(t)$ on the basis of the channel output. Define

$$X_b(t) := HX(t), \quad X_s(t) := HX_h(t).$$

Then, system (1) may be rewritten as

$$X_b(t+1) = \Lambda X_b(t) + H\Theta U(t) + HW(t).$$

Here, we implement a quantized state feedback control law of the form

$$U(t) = KX_h(t) \tag{3}$$

where K is the controller gain.

Let $X_g(t) := (\Upsilon + \Theta K)X_h(t-1)$ denote the prediction value of $X(t)$ at time t. Define $X_v(t) := HX_g(t)$ and $Z(t) := X_b(t) - X_v(t)$, where $Z(t)$ denotes the prediction error. By Assumption A_2, we know that the encoder and decoder have access to previous control actions. Thus, we set $X_g(t) := (\Upsilon + \Theta K)X_h(t-1)$ such that the encoder and decoder can obtain the same prediction value too. Then, we quantize, code $Z(t)$, and then transmit the information of $Z(t)$. Let $Z_h(t)$ denote the estimate of $Z(t)$. Thus, we have $Z(t) = Z_h(t) + V(t)$ and $X_s(t) = X_v(t) + Z_h(t)$, where $V(t) := [v_1(t)\ v_2(t) \cdots v_n(t)]'$ denotes the quantization error with zero mean. Thus, the decoder estimate is defined as

$$X_h(t) = H'X_s(t) = X_g(t) + H'Z_h(t).$$

It means that, the decoder can obtain the estimate $X_h(t)$ of $X(t)$ on the basis of the channel output $Z_h(t)$ such that we may design a closed-loop controller to stabilize the unstable plant.

4. **LQ Control under Information Limitations.** In this section, we deal with LQ control problem for linear time-invariant systems over limited capacity communication channels, and quantify the LQ cost by

$$J_1 = \limsup_{t\to\infty} EX'(t)QX(t)$$

where $Q \in \mathbb{R}^{n\times n}$ is symmetric positive definite. Here we are concerned with how small the state can be made as $t \to \infty$. Here, let the parameter ε denote the regulated variable on the LQ cost. Then, we have the following result.

Theorem 4.1. *Consider system (1) under Assumptions A_0-A_3. Assume that all eigenvalues of $\Upsilon + \Theta K$ lie inside the unit circle. For any $\varepsilon \in (0,1)$, there exists a control policy of the form (3) such that the closed-loop system (1) is stabilizable in the mean square sense (2) with the LQ cost obtained by*

$$\begin{aligned} J_1 &= \limsup_{t\to\infty} EX'(t)QX(t) \\ &< \frac{\Phi_w}{(1-\varepsilon)(1-\|\Upsilon+\Theta K\|^2)}\left[\varepsilon\|\Upsilon+\Theta K\|^2\cdot\left\|Q^{\frac{1}{2}}\Upsilon^{-1}\right\|^2\right. \\ &\quad \left. +\left\|Q^{\frac{1}{2}}\right\|^2 - 2\varepsilon\left\|(\Upsilon+\Theta K)'Q\Upsilon^{-1}\right\|\right] \end{aligned}$$

if the information rate of the channel satisfies the following inequality:

$$R > \tfrac{1}{2}\Sigma_{i\in\Xi}\log_2\tfrac{\lambda_i^2(\Upsilon)}{\varepsilon} \quad \textit{(bits/sample)}$$

where $\Xi := \{i \in \{1,2,\cdots,n\} : \lambda_i^2(\Upsilon) > \varepsilon\}$.

Proof: Consider the closed-loop system (1). We have

$$X(t+1) = \Upsilon X(t) + \Theta KX_h(t) + W(t)$$

which can also be rewritten as

$$X(t+1) = \Upsilon(X(t) - X_h(t)) + (\Upsilon + \Theta K)X_h(t) + W(t). \tag{4}$$

Notice that

$$\begin{aligned} X(t) &= H'X_b(t) = H'(X_v(t) + Z(t)), \\ X_h(t) &= H'X_s(t) = H'(X_v(t) + Z_h(t)). \end{aligned}$$

Then,

$$X(t) - X_h(t) = H'(Z(t) - Z_h(t)) = H'V(t). \tag{5}$$

Furthermore, it holds that

$$X_g(t) = (\Upsilon + \Theta K)X_h(t-1).$$

Substitute the equation above and (5) into (4), and obtain

$$X(t+1) = \Upsilon H' V(t) + X_g(t+1) + W(t)$$

which is equivalent to

$$X_b(t+1) = \Lambda V(t) + X_v(t+1) + HW(t).$$

Since we have

$$Z(t+1) = X_b(t+1) - X_v(t+1),$$

it follows that

$$Z(t+1) = \Lambda V(t) + HW(t). \tag{6}$$

Namely,

$$Z(t) = \Lambda V(t-1) + HW(t-1).$$

Furthermore, it follows from (5) that

$$X_v(t) = H(\Upsilon + \Theta K)X_h(t-1) = H(\Upsilon + \Theta K)(X(t-1) - H'V(t-1)).$$

Then, we have

$$\begin{aligned}&E[X_v'(t)HQH'Z(t)]\\ &= E\left[(X(t-1) - H'V(t-1))'(\Upsilon + \Theta K)'H'HQH'(\Lambda V(t-1) + HW(t-1))\right].\end{aligned}$$

Here, $X(t-1)$, $V(t-1)$, and $W(t-1)$ are mutually independent random variables. Then, we have

$$E[X'(t-1)V(t-1)] = 0, \quad E[X'(t-1)W(t-1)] = 0, \quad E[V'(t-1)W(t-1)] = 0.$$

Thus, it follows that

$$E[X_v'(t)HQH'Z(t)] = -E[V'(t-1)H(\Upsilon + \Theta K)'Q\Upsilon H'V(t-1)].$$

Since $X(t) = X_g(t) + H'Z(t)$ holds, we have

$$\begin{aligned}E[X'(t)QX(t)] &= E[(X_g(t) + H'Z(t))'Q(X_g(t) + H'Z(t))]\\ &= E[X_g'(t)QX_g(t)] + E[Z'(t)HQH'Z(t)] + 2E[X_v'(t)HQH'Z(t)]\\ &= E[X_g'(t)QX_g(t)] + E[Z'(t)HQH'Z(t)]\\ &\quad - 2E[V'(t-1)H(\Upsilon + \Theta K)'Q\Upsilon H'V(t-1)].\end{aligned}$$

Notice that

$$\begin{aligned}&\limsup_{t\to\infty} E[V'(t)H(\Upsilon + \Theta K)'Q\Upsilon H'V(t)]\\ &= \limsup_{t\to\infty} E[V'(t-1)H(\Upsilon + \Theta K)'Q\Upsilon H'V(t-1)].\end{aligned}$$

Thus,

$$\begin{aligned}&\limsup_{t\to\infty} E[X'(t)QX(t)]\\ &= \limsup_{t\to\infty} E[X_g'(t)QX_g(t)] + \limsup_{t\to\infty} E[Z'(t)HQH'Z(t)]\\ &\quad - 2\limsup_{t\to\infty} E[V'(t)H(\Upsilon + \Theta K)'Q\Upsilon H'V(t)].\end{aligned} \tag{7}$$

It follows from (6) that

$$E[Z'(t+1)HQH'Z(t+1)] = \mathrm{tr}\left(HQH'\Lambda^2\Sigma_{V(t)}\right) + E\left\|Q^{\frac{1}{2}}W(t)\right\|^2$$

where we define $\Sigma_{V(t)} := E[V(t)V'(t)]$. Here, the parameter ε denotes the regulated variable on the the LQ cost. If we present the quantization scheme to make the following condition hold:

$$\varepsilon\sigma^2(z_i(t)) = \lambda_i^2(\Upsilon)\sigma^2(v_i(t)) \quad (i = 1, 2, \cdots, n) \tag{8}$$

where we define $\sigma^2(z_i(t)) := E\left[z_i^2(t)\right]$ and $\sigma^2(v_i(t)) := E\left[v_i^2(t)\right]$, it holds that

$$\varepsilon E[Z'(t)HQH'Z(t)] = \varepsilon \mathrm{tr}\left(HQH'\Sigma_{Z(t)}\right) = \mathrm{tr}\left(HQH'\Lambda^2\Sigma_{V(t)}\right)$$

where we define $\Sigma_{Z(t)} := E[Z(t)Z'(t)]$. This implies

$$E\left[Z'(t+1)HQH'Z(t+1)\right] = \varepsilon E\left[Z'(t)HQH'Z(t)\right] + E\left\|Q^{\frac{1}{2}}W(t)\right\|^2.$$

Thus,

$$\limsup_{t\to\infty} E[Z'(t)HQH'Z(t)] < \frac{1}{1-\varepsilon}\left\|Q^{\frac{1}{2}}\right\|^2 \Phi_W. \tag{9}$$

It means that we quantize only $z_i(t)$ corresponding to the ith eigenvalue of Υ subject to the condition: $\lambda_i^2(\Upsilon) > \varepsilon$. Then, it follows that the condition (8) can hold if the information rate satisfies the following condition:

$$R > \frac{1}{2}\Sigma_{i\in\Xi}\log_2\frac{\sigma^2(z_i(t))}{\sigma^2(v_i(t))} = \frac{1}{2}\Sigma_{i\in\Xi}\log_2\frac{\lambda_i^2(\Upsilon)}{\varepsilon} \quad \text{(bits/sample)}$$

where $\Xi := \{i \in \{1, 2, \cdots, n\} : \lambda_i^2(\Upsilon) > \varepsilon\}$.

It also follows from the condition (8) that

$$\frac{\varepsilon}{\lambda_i^2(\Upsilon)}\left[H(\Upsilon+\Theta K)'Q\Upsilon H'\right]_{i,i}\sigma^2(z_i(t)) = \left[H(\Upsilon+\Theta K)'Q\Upsilon H'\right]_{i,i}\sigma^2(v_i(t)) \ (i = 1, 2, \cdots, n)$$

where $[\cdot]_{ij}$ denotes an element of a matrix $(i, j = 1, \cdots, n)$. Thus,

$$\varepsilon\mathrm{tr}\left[H(\Upsilon+\Theta K)'Q\Upsilon^{-1}H'\Sigma_{Z(t)}\right] = \mathrm{tr}\left[H(\Upsilon+\Theta K)'Q\Upsilon H'\Sigma_{V(t)}\right].$$

It means that

$$E\left[V'(t)H(\Upsilon+\Theta K)'Q\Upsilon H'V(t)\right] = \varepsilon E\left[Z'(t)H(\Upsilon+\Theta K)'Q\Upsilon^{-1}H'Z(t)\right].$$

Furthermore, it follows from (6) that

$$\begin{aligned} &E[Z'(t+1)H(\Upsilon+\Theta K)'Q\Upsilon^{-1}H'Z(t+1)] \\ &= \mathrm{tr}[H(\Upsilon+\Theta K)'Q\Upsilon^{-1}H'\Lambda^2\Sigma_{V(t)}] + E[W'(t)(\Upsilon+\Theta K)'Q\Upsilon^{-1}W(t)]. \end{aligned}$$

If the condition (8) holds, we have

$$\begin{aligned} \varepsilon E[Z'(t)H(\Upsilon+\Theta K)'Q\Upsilon^{-1}H'Z(t)] &= \varepsilon\mathrm{tr}\left[H(\Upsilon+\Theta K)'Q\Upsilon^{-1}H'\Sigma_{Z(t)}\right] \\ &= \mathrm{tr}\left[H(\Upsilon+\Theta K)'Q\Upsilon^{-1}H'\Lambda^2\Sigma_{V(t)}\right]. \end{aligned}$$

Then,

$$\begin{aligned} &E[Z'(t+1)H(\Upsilon+\Theta K)'Q\Upsilon^{-1}H'Z(t+1)] \\ &= \varepsilon E\left[Z'(t)H(\Upsilon+\Theta K)'Q\Upsilon^{-1}H'Z(t)\right] + E[W'(t)(\Upsilon+\Theta K)'Q\Upsilon^{-1}W(t)]. \end{aligned}$$

Thus, it follows that

$$\begin{aligned} &\limsup_{t\to\infty} E[V'(t)H(\Upsilon+\Theta K)'Q\Upsilon H'V(t)] \\ &= \limsup_{t\to\infty} \varepsilon E[Z'(t)H(\Upsilon+\Theta K)'Q\Upsilon^{-1}H'Z(t)] \\ &< \frac{\varepsilon}{1-\varepsilon}\|(\Upsilon+\Theta K)'Q\Upsilon^{-1}\|\Phi_W. \end{aligned} \tag{10}$$

Furthermore, notice that

$$X_g(t+1) = (\Upsilon+\Theta K)X_h(t) = (\Upsilon+\Theta K)(X(t) - H'V(t)).$$

Thus,

$$\begin{aligned} &E[X_g'(t+1)QX_g(t+1)] \\ &= E[X'(t)(\Upsilon+\Theta K)'Q(\Upsilon+\Theta K)X(t)] + E[V'(t)H(\Upsilon+\Theta K)'Q(\Upsilon+\Theta K)H'V(t)] \\ &< \|\Upsilon+\Theta K\|^2(E[X'(t)QX(t)] + E[V'(t)HQH'V(t)]). \end{aligned}$$

It means that

$$\begin{aligned} &\limsup_{t\to\infty} E[X_g'(t)QX_g(t)] \\ &= \limsup_{t\to\infty} EX_g'(t+1)QX_g(t+1) \\ &< \|\Upsilon+\Theta K\|^2 \left(\limsup_{t\to\infty} E[X'(t)QX(t)] + \limsup_{t\to\infty} E[V'(t)HQH'V(t)]\right). \end{aligned} \tag{11}$$

It follows from the condition (8) that

$$\begin{aligned} \varepsilon E[Z'(t)H(\Upsilon^{-1})'Q\Upsilon^{-1}H'Z(t)] &= \varepsilon \mathrm{tr}[H(\Upsilon^{-1})'Q\Upsilon^{-1}H'\Sigma_{Z(t)}] \\ &= \mathrm{tr}[HQH'\Sigma_{V(t)}] = E[V'(t)HQH'V(t)]. \end{aligned}$$

Furthermore, it follows from (6) that

$$\begin{aligned} &E[Z'(t+1)H(\Upsilon^{-1})'Q\Upsilon^{-1}H'Z(t+1)] \\ &= \mathrm{tr}[H(\Upsilon^{-1})'Q\Upsilon^{-1}H'\Lambda^2\Sigma_{V(t)}] + E[W'(t)(\Upsilon^{-1})'Q\Upsilon^{-1}W(t)]. \end{aligned}$$

If the condition (8) holds, we have

$$\begin{aligned} &E[Z'(t+1)H(\Upsilon^{-1})'Q\Upsilon^{-1}H'Z(t+1)] \\ &= \varepsilon E\left[Z'(t)H(\Upsilon^{-1})'Q\Upsilon^{-1}H'Z(t)\right] + E\left[W'(t)(\Upsilon^{-1})'Q\Upsilon^{-1}W(t)\right]. \end{aligned}$$

Thus, it holds that

$$\begin{aligned} &\limsup_{t\to\infty} E[V'(t)HQH'V(t)] \\ &= \limsup_{t\to\infty} \varepsilon E\left[Z'(t)H\left(\Upsilon^{-1}\right)'Q\Upsilon^{-1}H'Z(t)\right] < \frac{\varepsilon}{1-\varepsilon}\left\|Q^{\frac{1}{2}}\Upsilon^{-1}\right\|^2 \Phi_W. \end{aligned} \tag{12}$$

Substitute (9), (10), (11), and (12) into (7), and obtain

$$\begin{aligned} \limsup_{t\to\infty} EX'(t)QX(t) < \frac{\Phi_w}{(1-\varepsilon)(1-\|\Upsilon+\Theta K\|^2)} &\left[\varepsilon\|\Upsilon+\Theta K\|^2 \cdot \left\|Q^{\frac{1}{2}}\Upsilon^{-1}\right\|^2\right. \\ &\left. + \left\|Q^{\frac{1}{2}}\right\|^2 - 2\varepsilon\left\|(\Upsilon+\Theta K)'Q\Upsilon^{-1}\right\|\right]. \end{aligned}$$

Thus, system (1) is stabilizable in the mean square sense (3). □

Notice that the parameter ε plays a key role in the inherent tradeoffs between control and communication costs. An explicit formula on the relation between the LQ cost J_1 and the information rate of the channel is given. A sufficient condition on the information rate for stabilization is given in Theorem 4.1. Under the condition, some LQ costs can be obtained too. In the literature, the information of the plant states corresponding to the eigenvalues of system matrix Υ which lie outside the unit circle is transmitted. A distinction with the existing literature is that we have to transmit some information of the plant states corresponding to the eigenvalues of system matrix Υ which lie inside the unit circle in order to achieve the specified LQ cost.

5. **Conclusions.** This paper addressed the performance control problem for stochastic linear systems with limited information rates. The approach taken here was based on the hypothesis that the sensors, plant and controller were connected by a rate-limited, errorless communication channel. A control scheme was presented to achieve some given LQ control objectives. An explicit formula on the tradeoff between the LQ cost and the information rate of the channel was proposed in our results. The researches on robust control for linear control systems under information limitations will be our future work.

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