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PARAMETERS OPTIMIZATION AND INTERVAL CONCEPT LATTICE UPDATE WITH CHANGE OF PARAMETERS

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Abstract. *Interval concept lattice has been proposed recently as the new definition of concept lattice on the parameters range. It can flexibly reflect the uncertainty information, and provide a solution for the mining problem of uncertain rules. Interval parameters α and β determine the interval concepts and structures, and then affect mining interval association rules. Generally interval parameters are determined artificially, that has certain subjectivity, so the interval parameters optimization is of practical significance. Firstly the related theories of interval concept lattice, the building algorithm of lattice structure and the mining algorithm of interval association rules are introduced, and then interval update algorithm of concept lattice structure is designed based on the change of parameters. Through the analysis to the lattice structure update degree, we build the interval parameter optimization model of concept lattice and use the method of parameter approaching to obtain the optimal parameters. Finally the instance is given to demonstrate the effectiveness of the model.*

Keywords: Interval concept lattice, Interval association rules, Update degree, Interval parameter optimization

1. **Introduction.** Interval concept lattice [1] is an extension of the classical [2] and rough [3] concept lattice, and the concept extension is the sets of objects meeting a certain percentage of intension, namely α-upper and β-lower extension must meet the given precision. [4] and [5] give the incremental generation algorithm and dynamic compression of interval concept lattice, and use the values of parameters confirmed subjectively to build lattice and then mine interval association rules. Usually we could not obtain the best lattice structure or association rules based on the subjective-certain parameters, and it has some limits on the reflection of truth. In practical life, users always concern the objects sets of attributes meeting a certain percentage of intension, thus mining more targeted association rules. Therefore, it is necessary to provide a feasible method of parameters optimization. At present there are a lot of algorithms of parameters optimization, including Genetic Algorithm, Neural Network Algorithm, Simulated Annealing Algorithm and so on, which have been applied to the problem of parameters optimization in complex system. However, the implementation of these algorithms largely depends on the setting of various parameters and they tend to ignore the characteristics of the problem themselves. Interval concept lattice parameters optimization problems should not only consider the terminal users' demanding for the concept lattice structure and further related interval association rules, but also try to find certain influence rules of changing parameters in lattice structure.

Based on the above analysis, in combination with interval concept lattice structure features, firstly put forward the lattice structure updating algorithm based on the change of parameters; secondly the parameter optimization algorithm of interval concept lattice is presented; finally through the example the optimal parameters of interval concept lattice are found. The research of different parameters affecting the interval concept and association rules and building the optimization model of interval parameters lays the foundation of mining uncertain rules and formulating uncertain decisions.

2. Interval Concept Lattice Theory.

Definition 2.1. *The formal context* (U, A, R) *is given.* $L(U, A, R)$ *is a classic concept lattice based on it.* (M, N, Y) *is a rough concept lattice based on* RL. *Assume the interval* $[\alpha, \beta]$, $0 \le \alpha < \beta \le 1$,

$$\alpha\text{-upper extension } M^{\alpha} : M^{\alpha} = \{x | x \in M, |f(x) \cap Y|/|Y| \ge \alpha, 0 \le \alpha \le 1\}$$
$$\beta\text{-lower extension } M^{\beta} : M^{\beta} = \{x | x \in M, |f(x) \cap Y|/|Y| \ge \beta, 0 \le \alpha \le \beta \le 1\}$$

Among them, Y is the intension of the concept. $|Y|$ is the number of elements contained by set Y, namely cardinal number. M^{α} expresses the objects covered by at least $\alpha \times |Y|$ attributes from Y.

Definition 2.2. *The formal context* (U, A, R), *the ternary ordered pairs* $(M^{\alpha}, M^{\beta}, Y)$ *are called interval concept, and among them,* Y *is the intension and concept description;* M^{α} *is the* α*-upper extension;* M^{β} *is the* β*-lower extension.*

Definition 2.3. *The ordered pairs* $(M^{\alpha}, M^{\beta}, Y)$ *is any node in interval concept lattice, and* $\tau^{\alpha}_{\beta}(Y) = |M^{\beta}|/|M^{\alpha}|$ *expresses the approximation precision of specified relation* R *which is about attribute set* Y *in the specified parameters* α *and* β *covering the objects.*

2.1. The construction of interval concept lattice.

Interval concept lattice construction is the prerequisites of data analysis and mining association rules. The resulting of Hasse diagram [6,7] can vividly reflect the generalization and specialization of relationship between concepts. [4] provided a kind of algorithm of interval concept lattice construction based on the power sets of the attribute sets.

2.2. The mining algorithm of interval association rules.

We can respectively mine α-upper interval association rules and β-lower interval association rules in interval concept lattice, here for example of α-upper interval association rules.

Algorithm: The mining algorithm of upper interval association rules

Input: Concept lattice $L^{\beta}_{\alpha}(U, A, R)$, minimum support and confidence threshold: θ and φ, interval parameter α.

Output: α-upper interval association rules

Process: Step1, Breadth-first traverse interval concept lattice, and obtain α-upper frequent nodes set α-Fcset. For any concept node of interval concept $C1 = (M^{\alpha}, M^{1}, Y)$, if $|M^{\alpha}| \ge |U|\theta$, α-Fcset $= \alpha$-Fcset $\cup \{C\}$, finally obtain upper frequent nodes set after traversing all the interval concepts.

Step2, Generate all α-upper candidate binary groups. Assuming that set of nodes of candidate binary groups formed with any node $C1 = (M^{\alpha}_1, M^{1}_1, Y_1)$ in α-upper frequent nodes set α-Fcset is PAIRS($C1$). For other node $C2 = (M^{\alpha}_2, M^{1}_2, Y_2)$ in α-Fcset, if $C2 > C1$ and $|M^{\alpha}_1|/|M^{\alpha}_2| \ge \varphi$, PAIRS($C1$) = PAIRS($C1$) $\cup \{C2\}$. Repeat above steps until finding all PAIRS in α-Fcset.

Step3, Eliminate superfluous candidate binary groups. Arrange the nodes of α-Fcset in the descending order of the intension cardinal numbers. If $C1 > C2$, PAIRS($C1$) = PAIRS($C1$) $-$ PAIRS($C2$). After eliminating superfluous of α-Fcset, obtain α-upper candidate binary groups, and then based on it, we can count α-upper frequent nodes set α-Fiset.

Step4, Generate α-upper interval association rules: α-Rluesset.

Based on this algorithm we can extract more refined uncertain association rules, and improve the reliability of the rules.

3. **The Interval Parameters Optimization Model.** In the process of parameters optimization about interval concept lattice, an important link is to adjust parameters. As parameters change, interval concept lattice structure will be changed. How to only adjust some parts of the interval concept and parent-child relationships on the basis of existing lattice structure is one of the key problems of parameters optimization.

3.1. **The update of interval concept lattice structure based on the change of parameters.** Updating interval concept lattice is not reconstructing it. Its updating mainly is divided into two sorts. One is about the change of formal context data; the other is about the change of interval parameters. Reference [8] has already carried on the detailed discussion to the first sort, and we only discuss the second sort here. According to the characteristics of the changing concept nodes, we will give definitions of the follows.

Assume interval concept lattice $L_{\alpha_0}^{\beta_0}(U, A, R)$. Interval parameters are changed from $[\alpha_0, \beta_0]$ into $[\alpha_1, \beta_1]$.

Definition 3.1. *If* $\exists G_1 = (M^{\alpha_1}, M^{\beta_1}, Y) \in L_{\alpha_1}^{\beta_1}(U, A, R)$ *and* $G_1 = (M^{\alpha_1}, M^{\beta_1}, Y) = (M^{\alpha_0}, M^{\beta_0}, Y) = G_0 \in L_{\alpha_0}^{\beta_0}(U, A, R)$*, call* G_0 *invariable concept.*

Definition 3.2. *If* $\exists G_1 = (M^{\alpha_1}, M^{\beta_1}, Y) \in L_{\alpha_1}^{\beta_1}(U, A, R)$*,* $M^{\alpha_1} \supseteq M^{\alpha_0}$ *and* $M^{\beta_1} \supseteq M^{\beta_0}$*,* $(M^{\alpha_0}, M^{\beta_0}, Y) = G_0 \in L_{\alpha_0}^{\beta_0}(U, A, R)$*, call* G_1 *the extension concept of* G_0*.*

Property 3.1. *If* $\alpha' < \alpha$ *and* $\beta' < \beta$*,* $M^{\alpha'} \supseteq M^{\alpha}$ *and* $M^{\beta'} \supseteq M^{\beta}$*.*

Proof: $M^{\alpha'} = M^{\alpha} \cup \{x'\}$ and $\{x' | \alpha \leq |f(x') \cap Y|/|Y| \leq \alpha'\}$, because $M^{\alpha} = \{x | x \in M, |f(x) \cap Y|/|Y| \geq \alpha > \alpha'\}$ and $M^{\beta} = \{x | x \in M, |f(x) \cap Y|/|Y| \geq \beta > \beta'\}$. Similarly, $M^{\beta'} = M^{\beta} \cup \{x''\}$ and $\{x'' | \beta' \leq |f(x'') \cap Y|/|Y| \leq \beta\}$, obviously $M^{\alpha'} \supseteq M^{\alpha}$ and $M^{\beta'} \supseteq M^{\beta}$.

Definition 3.3. *If* $\exists G_1 = (M^{\alpha_1}, M^{\beta_1}, Y) \in L_{\alpha_1}^{\beta_1}(U, A, R)$*,* $M^{\alpha_1} \subseteq M^{\alpha_0}$*,* $M^{\beta_1} \subseteq M^{\beta_0}$ *and* $(M^{\alpha_0}, M^{\beta_0}, Y) = G_0 \in L_{\alpha_0}^{\beta_0}(U, A, R)$*, call* G_1 *the reduction concept of* G_0*.*

Property 3.2. *If* $\alpha' > \alpha$ *and* $\beta' > \beta$*,* $M^{\alpha'} \subseteq M^{\alpha}$ *and* $M^{\beta'} \subseteq M^{\beta}$*.*

Proof: $M^{\alpha'} = M^{\alpha} - \{x'\}$ and $\{x' | \alpha \leq |f(x') \cap Y|/|Y| \leq \alpha'\}$, because $M^{\alpha} = \{x | x \in M, \alpha' > |f(x) \cap Y|/|Y| \geq \alpha\}$ and $M^{\beta} = \{x | x \in M, \beta' > |f(x) \cap Y|/|Y| \geq \beta\}$. Similarly, $M^{\beta'} = M^{\beta} - \{x''\}$ and $\{x'' | \beta \leq |f(x'') \cap Y|/|Y| \leq \beta'\}$, obviously $M^{\alpha'} \subseteq M^{\alpha}$ and $M^{\beta'} \subseteq M^{\beta}$.

Definition 3.4. *If* $\exists G_1 = (M^{\alpha_1}, M^{\beta_1}, Y) \in L_{\alpha_1}^{\beta_1}(U, A, R)$ *and* $M^{\alpha_1} = \emptyset$*, call* $G_1 = (M^{\alpha_1}, M^{\beta_1}, Y)$ *deletion concept.*

Definition 3.5. *Assume* G' *is the concept in the updated interval concept lattice and* $M^{\alpha}_{G'_{father}} = M^{\alpha}_{G'_{children}}$*,* $M^{\beta}_{G'_{father}} = M^{\beta}_{G'_{children}}$*. Call* $G'_{children}$ *redundancy concept.*

We should update the interval concept lattice with the adjustment of the parameters.Parameters changing into $[\alpha_1, \beta_1]$ from $[\alpha_0, \beta_0]$, there are four kinds of circumstances: (1) $\alpha_1 > \alpha_0$; (2) $\alpha_1 < \alpha_0$; (3) $\beta_1 > \beta_0$; (4) $\beta_1 < \beta_0$; First two changes need to update the upper extension of the interval concept $M^{\alpha_0} \to M^{\alpha_1}$; the other changes need to update the lower extension of the interval concept $M^{\beta_0} \to M^{\beta_1}$. So the following four functions are given, respectively updating the interval concept.

(1) Function: $CL1(C, \alpha_0, \alpha_1)$ // C is any node in concept lattice; and $\alpha_1 > \alpha_0$
 $\{Ma = \{\emptyset\}$
  For each x in M^{α_0} of C:
   If $\frac{|f(x) \cap Y|}{|Y|} \geq \alpha_1$ then $Ma = Ma \cup x \quad M^{\alpha_1} = Ma\}$

(2) Function: $CL2(C, \alpha_0, \alpha_1)$ // C is any node in concept lattice; and $\alpha_1 < \alpha_0$
{$Ma = M^{\alpha_0}$
For the upper extension Maf of any father-node CF in C:
{Make $maf1 = Maf - M^{\alpha_0}$
For $\forall x \in maf1$:
If $\frac{|f(x) \cap Y|}{|Y|} \geq \alpha_1$ then $Ma = Ma \cup x$} // Y is the intension set of C
$M^{\alpha_1} = Ma$}

(3) Function: $CL3(C, \beta_0, \beta_1)$ // C is any node in concept lattice; and $\beta_1 > \beta_0$
{$Mb = \{\emptyset\}$
For each x in M^{β_0} of C:
If $\frac{|f(x) \cap Y|}{|Y|} \geq \beta_1$ then $Mb = Mb \cup x$
$M^{\beta_1} = Mb$}

(4) Function: $CL4(C, \beta_0, \beta_1)$ // C is any node in concept lattice; and $\beta_1 < \beta_0$
{$Mb = M^{\beta_0}$
For the upper extension Mbf of any father-node CF in C:
{Make $mbf1 = Mbf - M^{\beta_0}$
For $\forall x \in mbf1$:
If $\frac{|f(x) \cap Y|}{|Y|} \geq \beta_1$ then $Mb = Mb \cup x$} // Y is the intension set of C
$M^{\beta_1} = Mb$}

Based on the four functions, when the parameters change, we use the method of breadth first to visit and judge each node from the root node in the interval concept lattice. According to the four different situations, update and adjust the nodes, delete the redundancy concepts empty concepts from the lattice structure, and adjust the father-son relationship.

Algorithm: Update algorithm of interval concept lattice based on parameters $LCP1$

Input: Formal context M, $L_{\alpha_0}^{\beta_0}(U, A, R)$, interval parameter (α_1, β_1).

Output: $L_{\alpha_1}^{\beta_1}(U, A, R)$

Process: Step1, $C_1 = (M^{\alpha_0}, M^{\beta_0}, Y)$ is the root node of $L_{\alpha_0}^{\beta_0}(U, A, R)$, if $Y = \emptyset$, C_1 does not change; if $Y \neq \emptyset$ and $\alpha_1 > \alpha_0$, call function: $CL1(C, \alpha_0, \alpha_1)$, else call function: $CL2(C, \alpha_0, \alpha_1)$, and then update M^{α_0} to M^{α_1}; as the same, if $\beta_1 > \beta_0$, call function: $CL3(C, \beta_0, \beta_1)$, else call function: $CL4(C, \beta_0, \beta_1)$, and then update M^{β_0} to M^{β_1} and C_1 to $(M^{\alpha_1}, M^{\beta_1}, Y)$.

Step2, Visit each children-nodes C_i in C_1.

Step3, Assume $C_i = (M_i^{\alpha_0}, M_i^{\beta_0}, Y_i)$. If $\alpha_1 > \alpha_0$, call function: $CL1(C, \alpha_0, \alpha_1)$, else call function: $CL2(C, \alpha_0, \alpha_1)$, and then update $M_i^{\alpha_0}$ to $M_i^{\alpha_1}$; if $M_i^{\alpha_0} = \emptyset$, delete node C_i, else continue updating the lower extension: if $\beta_1 > \beta_0$, call function: $CL3(C, \beta_0, \beta_1)$, else call function: $CL4(C, \beta_0, \beta_1)$, then update $M_i^{\beta_0}$ to $M_i^{\beta_1}$, and C_i is totally updated to $(M_i^{\alpha_1}, M_i^{\beta_1}, Y_i)$.

Step4, For each father-node $C_i' = C_i \rightarrow$ Parent in C_i, and $C_i' = (M_i'^{\alpha_1}, M_i'^{\beta_1}, Y_i')$, if $M_i'^{\alpha_1} = M_i^{\alpha_1}$ and $M_i'^{\beta_1} = M_i^{\beta_1}$, $C_i \rightarrow$ Parent $= C_i' \rightarrow$ Parent, namely delete C_i'.

Step5, For each children-node in C_i, and $C_i = C_i \rightarrow$ Children, turn to Step3 until visiting the final node in $L_{\alpha_0}^{\beta_0}(U, A, R)$.

Step6, Output the new concept lattice structure: $L_{\alpha_1}^{\beta_1}(U, A, R)$.

3.2. **The parameter optimization algorithm of interval concept lattice.** For any given formal context and parameters, using the method of document [4] can quickly build interval concept lattice structure, and extract association rules. However, the parameters are specified artificially and the extracted rules often have low utilization and accuracy. Therefore, this section is to construct an interval parameters optimization model.

Definition 3.6. *Assume interval concept set is L_0 in $L_{\alpha_0}^{\beta_0}(U,A,R)$. When interval parameter changed from $[\alpha_0,\beta_0]$ into $[\alpha_1,\beta_1]$, the new interval concept set is L_1 in $L_{\alpha_1}^{\beta_1}(U,A,R)$. The update degree of interval concept lattice can be expressed:*

$$\omega = (|L_1| - |L_1 \cap L_0|)/|L_0|$$

Algorithm: Parameter optimization algorithm of interval concept lattice: LPO

Input: Formal context M, minimum support threshold θ, minimum confidence threshold φ

Output: Suitable interval parameters: α and β, the number of corresponding upper and lower frequent nodes and rules as well as interval association rules.

Process: Step1, According to formal context, obtain the number of attribute: n, and set up the length of step: $\lambda = 1/n$;

Step2, Initialize the interval parameters: $\alpha = 1/n$, $\beta = 1$, and then build $L_\alpha^\beta(U,A,R)$;

Step3, Set up the update degree: $\omega = 0$;

Step4, Count the number of lattice node con0; refer to the algorithm of mining association rules in Section 2.2, and then obtain upper and lower association rules;

Step5, Make $\alpha = \alpha + \lambda$, $\beta = \beta - \lambda$, and update lattice structure by Section 3.1;

Step6, Count the number con1 of nodes and the update degree ω_1;

Step7, If $\omega_1 - \omega$ approaches 0, end and output corresponding α, β, upper and lower frequent nodes and association rules; else con0 = con1, $\omega = \omega_1$, and turn Step5.

3.3. **Model analysis.** We try to find specific function relationship between parameters and the interval concept lattice nodes and association rules, then figure out the best parameters, but the specific function relation is uncertain and hard to get. To establish a model of interval parameters optimization is feasible in dealing with the issue, the calculation of the model is not greater than the number n of condition attribute in a given formal context, and according to the equal step change of parameters, we just need to adjust part of the interval concepts and parent-child relationships based on the original interval concept lattice, and need not rebuild all the concept lattice nodes, which has greatly reduced the time complexity of the model. However, when the formal context is too complex, the efficiency of the model is reduced.

4. **Example Analysis.** For simplicity, we set parameter β value to 1 and explore the effect of α-upper interval association rules and interval concept lattice structure with the change of parameter α. The formal context is shown in Table 1.

The number of attribute A is 8 and object U is 15 according to given formal context (U,A,R) by Table 1. Find the best interval parameter based on the model (Section 3.2).

(1) Initialize the interval parameter $\alpha = 1/8$ and the update degree $\omega = 0$, build the interval concept lattice based on parameters as $\alpha = 1/8$ and $\beta = 1$, and further mine α-upper interval association rules based on given minimum support and confidence

Table 1. Formal context

Objects	a	b	c	d	e	f	g	h	Objects	a	b	c	d	e	f	g	h
1	0	0	1	1	1	0	0	0	9	0	0	1	1	1	0	0	0
2	0	1	0	1	0	1	0	0	10	0	0	0	1	1	0	0	1
3	1	0	0	0	1	0	0	1	11	0	1	1	1	0	0	0	0
4	1	1	1	0	0	0	0	0	12	1	1	0	1	0	0	0	0
5	0	1	0	0	0	0	1	0	13	1	0	0	1	0	0	0	0
6	0	0	0	0	1	1	0	0	14	1	0	1	0	1	0	0	0
7	0	0	0	1	0	0	0	1	15	0	0	1	1	1	0	0	1
8	1	1	0	1	0	0	0	0									

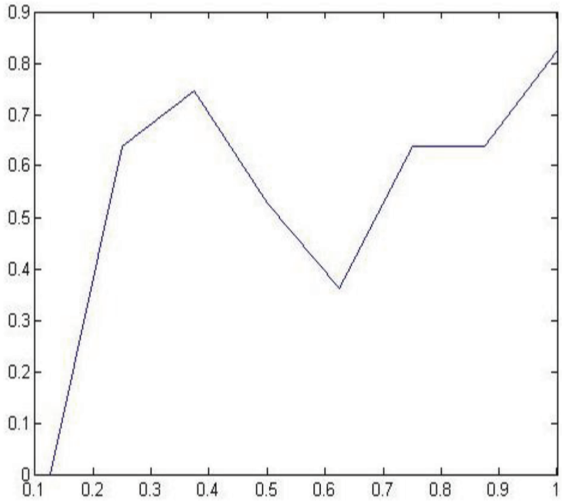

Figure 1. α and the update degree relational graph

threshold: $\theta = 0.6$ and $\varphi = 0.7$, obtain the number of α-upper interval association rules: 16, concept nodes: 110; (2) When interval parameter α increases to 2/8 by the equal step, change the objects set covered by local nodes on the basis of concept lattice of $\alpha = 1/8$, such as the extension of concept nodes whose numbers of attribute sets are not less than 4 is correspondingly reduced, and that less than 4 stays the same. According to Definition 3.6, we can calculate the update degree ω_1; (3) With the increase of interval parameter α by the equal step, calculating the update degree ω of concept lattice. If the difference of update degree between the old and the new concept lattice tends to zero, output α. The diagram of parameter and update is shown in Figure 1.

Easy to see, when α changes from 4/8 into 5/8, at this point update degree reaches the minimum value: 0.363, and output $\alpha = 4/8$, as α increases, the update degree gradually stays the same, which shows that after α changing to a certain value, its effect on the lattice structure is gradually stabilized, and thus to extract the number of association rules as well as degree of confidence will not have great changes.

Finally the model outputs: $\alpha = 4/8$; Interval concept lattice nodes: 178; Frequent nodes: 32; Number of α-upper interval association rules: 27; Association rules: ab $\Rightarrow$ cdeh, ac $\Rightarrow$ bdeh, ad $\Rightarrow$ cf, ad $\Rightarrow$ bceh, ae $\Rightarrow$ dg, ae $\Rightarrow$ bcde and so on. According to the outputs of interval parameter optimization model, setting $\alpha = 0.5$ and $\beta = 1$ in the formal context M means that we can obtain the satisfactory association rules. Meanwhile, because of the early-setting minimum support threshold and minimum confidence threshold, the outputs of the confidence of association rules are not less than the given minimum threshold.

5. **Conclusion.** This article mainly aims at the analysis of interval parameters, which directly affect the change of interval concept lattice structure, and mining association rules. Constructed update algorithm of concept lattice structure based on the change of parameters and interval parameters optimization model help find the best parameter. Preliminarily believing, when the parameter is 0.5, we can get the optimal association rules and lattice structure, which provides an effective method of selecting interval parameters. Developing new solution methods (based on cost function or game theories) as well as more realistic versions of the parameters optimization could be considered in future studies.

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