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MULTI-PERIOD PRODUCTION PLANNING USING SHAPLEY VALUE WITH CONSTRAINTS

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Abstract. *Deterioration of products, environmental changes, demand uncertainty, among with other factors require efficient strategies in order to prevent a set of negative revenues to a company. For shaping a loss distribution popular functions managing risk, namely value-at-risk (VaR) and conditional value-at-risk (CVaR) have been used. In this paper, we employ linear programming (LP) techniques under cooperative game theory framework to estimate the risk on a multi-period production planning problem. First, a comparison between the traditional Shapley value and our results is made to analyze the efficiency of our model. Further, a discussion of the strengths and limitations of the proposed LP scheme follows a numerical illustration.*
Keywords: Production planning, Shapley value, Linear programming, Risk, VaR, CVaR

1. **Introduction.** Managers in every level of decision process are responsible for strategical and efficient policies which requires consistent models in order to increase the profits in a business. The concept on risk management is related to distinct perspectives. However, just as stated by [1], it is important before defining risk management to understand the concept of risk. Risk combines both the uncertainty of outcomes and utility or benefit of outcomes. Outcomes are summarized by the profit and loss statement ($P\&L$), and the uncertainty in profits is described by the distribution or density function which maps the many possible realizations for the ($P\&L$), with profits sometimes high and sometimes low [1]. In their tutorial, [2] considers risk management as a procedure for shaping a loss distribution, while [3] defines risk measure as a mapping from the set of random variables representing the risk exposure to a real number. Though several innovations have been proposed for measuring risks, value-at-risk (VaR), conditional value-at-risk (CVaR) also known as tail conditional expectation and shortfall expectation (SE) are on the top of approaches accepted by practitioners [2, 3, 5, 9]. Despite its heavy application in engineering and in financial sector, VaR is inferior to CVaR in optimization applications. However, a close correspondence between both is found when using the same confidence level [4].

In this study, we aim to extend previous work [8] solving problems related to financial engineering, precisely n periods production planning problem. For this purpose, we combined CVaR with cooperative game theory and linear programming (LP); players are interpreted as periods due to the correlation between game theory and production planning models. The game consists of finding Shapley values [7], understood as risk allocated to each period, but avoiding the conventional formulation, and as research contribution, we use an alternative equivalent Shapley value which is demonstrated through a numerical example.

The remainder of the paper is organized as follows. Section 2 outlines VaR and CVaR as the well-known approaches in estimating risk measures. In Section 3 a proposal for multi-period production planning is presented, starting with a summary on coalitional game theory, analyzing the correlation between cooperative games and production planning and as well, and present an optimization model to calculate the Shapley values. Section 4 provides a numerical example and Section 5 concludes the paper with pointers to future work.

2. An Overview on VaR and CVaR Risk Measure Approaches.

Basic aspects about VaR and CVaR are described in this section due to the objectives of this paper. Thus, it cannot be perceived as a review regarding these two measure approaches. For more details on the above subject, we refer readers to [2, 4] and the references therein.

Risk measure can be defined as a procedure for shaping a loss distribution, for instance an investor's risk profile. Value-at-risk (VaR), which is a percentile of a loss distribution and conditional value-at-risk (CVaR) are among the few risk measure approaches mostly accepted by practitioners. There is a close correspondence between CVaR and VaR, and with the same confidence level, VaR is a lower bound for CVaR. In terms of optimization applications CVaR is superior to VaR [2]. Consider X a loss random variable with the cumulative distribution function $F_x(z) = P\{X \geq z\}$.

Definition 2.1 (**VaR**). *The VaR of X with confidence level $\alpha \in]0,1[$ is*

$$VaR_\alpha(X) = \min\{z | F_x(z) \geq \alpha\} \tag{1}$$

i.e., $VaR_\alpha(X)$ is a lower α-percentile of X and is proportional to the standard deviation if X is normally distributed, that is, $X \sim N(\mu, \sigma^2)$, then,

$$F_\alpha(X) = F_x^{-1}(\alpha) = \mu + k(\alpha)\sigma, \tag{2}$$

where

$$k(\alpha) = \sqrt{2} erf^{-1}(2\alpha - 1) \tag{3}$$

and

$$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z \mathrm{e}^{-x^2} dt. \tag{4}$$

Other characteristics of $\mathrm{VaR}_\alpha(X)$ include [2]:

- For discrete distributions, $\mathrm{VaR}_\alpha(X)$ is a nonconvex and discontinuous function of the confidence level α.
- $\mathrm{VaR}_\alpha(X)$ is non-subadditive.
- VaR has many extrema for discrete, therefore, it is difficult to control or optimize for nonnormal distributions.

Definition 2.2 (**CVaR**). *$CVaR_\alpha(X)$ equals the conditional expectation of X subject to $X \geq VaR_\alpha(X)$ for random variables with continuous distribution. Formally,*

$$CVaR_\alpha(X) = \{E[X] \quad s.t. \quad X \geq VaR_\alpha(X)\}. \tag{5}$$

The CVaR of X with $\alpha \in]0,1[$ is the mean of the generalized α-tail distribution:

$$CVaR_\alpha(X) = \int_{-\infty}^{\infty} \mathrm{z} \; dF_x^\alpha(z) \tag{6}$$

where

$$F_x^\alpha(z) = \begin{cases} 0, & for\ z < VaR_\alpha(X) \\ \frac{F_x(z)-\alpha}{1-\alpha}, & for\ z \geq VaR_\alpha(X) \end{cases}. \tag{7}$$

$\text{CVaR}_\alpha(X)$ is not equal to an average of outcomes greater than $\text{VaR}_\alpha(X)$.

3. A Proposal for Multi-period Production Planning Problem.

This section begins by introducing basic concepts concerning cooperative game theory for being one of the frameworks on which our proposed scheme is built. Its last part returns to CVaR already described in Section 2 and ends with the computational issues regarding our methodology.

3.1. Coalitional games with transferable utility (TU).

A *coalitional game* on a finite set of players is a pair (Ω, v) where $\Omega = \{1, 2, \ldots, n\}$ is the set of *players* and $v : 2^N \to \mathbb{R}$ is a real-valued (also called characteristic function) mapping with $v(\emptyset) = 0$. Any nonempty subset of Ω [6] (including Ω itself and all the one-element subsets) is called a *coalition*. The characteristic function $v(S)$, the worth of coalition S, represents the total amount of transferable utility that members of S could earn without any help from the players outside of S, i.e., the maximum sum utility payoffs that the members of coalition S can guarantee themselves against the best offensive threat by the complementary coalition $N \backslash S$.

Definition 3.1 (**Superadditivity**). (Ω, v) *is said to be superadditive if*

$$v(S \cup T) \geq v(S) + v(T), \ \ if\ S \cap T = \emptyset, \quad \forall\ S, T \subset \Omega. \tag{8}$$

Shapley value. Motivated by the need of a theory that would predict a unique expected payoff allocation for every given coalitional game, the concept of Shapley value, as a solution concept in cooperative game theory, was proposed by [7]. It considers the relative importance of each player to the game in deciding the payoff to be allocated to the players and is formally represented as

$$\pi_i = \sum_{\mathcal{H} \subset \Omega - \{i\}} \frac{|\mathcal{H}|!(|\Omega|! - |\mathcal{H}|! - 1)!}{|\Omega|!} \{v(\mathcal{H} \cup \{i\}) - v(\mathcal{H})\} \tag{9}$$

where, Ω denotes the set of players and $\mathcal{H}$ represents the coalition under study. [7] demonstrated his theory through the following properties:

- Efficiency (Group rationality): players precisely distribute among themselves the resources available to the grand coalition: $\sum_{i=1}^{n} \pi_i = v(\Omega)$.
- Individual fairness (Individual rationality): every player gets at least as much as he would have received without cooperation: $\pi_i \geq v(\{i\})$, $\forall i \in \{1, 2, \ldots, n\}$.
- Symmetry: if two players i and j are equivalent, then $\pi_i = \pi_j$.
- Additivity: if v_1 and v_2 are two games, then $\pi_i(v_1 + v_2) = \pi_i(v_1) + \pi_i(v_2)$.
- Null player: $v(\emptyset) = 0$.

Usually, finding the characteristic function $v(S)$ is a complex work, particularly in n-person games, since the amount of coalitions players in the game have to build increases exponentially according to the number of players, i.e., $2^n - 1$. Because of its importance in terms of applications, this function has been an important topic for research and, as result a lot of algorithms to support the computation of $v(S)$ have been suggested. In this study, we employ CVaR as the characteristic function and its computation is described next.

Let $D = d_1 + d_2 + \cdots + d_n$ be the cumulative demand and α, the confidence level. The CVaR is defined in Equation (10).

$$v(\mathcal{H}) = CVaR_{\mathcal{H}}(1-\alpha) = \sum_{i\in\mathcal{H}} d_i + \sum_{i\in\mathcal{H}}\sum_{j\in\mathcal{H}} \sigma_{ij} \frac{\varphi(z_{1-\alpha})}{1-\Phi(z_{1-\alpha})} \tag{10}$$

where,

$$z_{1-\alpha} = \frac{VaR_D(1-\alpha) - \sum_{i\in\mathcal{H}} d_i}{\sqrt{\sum_{i\in\mathcal{H}}\sum_{j\in\mathcal{H}} \sigma_{ij}}}. \tag{11}$$

Here, φ is the standard normal density, Φ denotes the cumulative function and σ_{ij} are the element of the variance-covariance matrix Σ shown in (12).

$$\mathbf{\Sigma} = \begin{bmatrix} \omega_1^2 & \omega_1^2 & \cdots & \omega_1^2 \\ \omega_1^2 & \omega_1^2 + \omega_2^2 & \cdots & \omega_1^2 + \omega_1^2 \\ \vdots & \vdots & \ddots & \vdots \\ \omega_1^2 & \omega_1^2 + \omega_2^2 & \cdots & \omega_1^2 + \omega_2^2 + \cdots + \omega_n^2 \end{bmatrix}. \tag{12}$$

Notice that CVaR is subadditive while – CVaR is a characteristic function observing the superadditive property [3].

3.2. **Relation between production planning and game theory.** Game theory can be applied to production planning problems. Table 1 summarizes the parallelism between these two techniques. In our methodology, we use the corresponding second column to perform the computations.

TABLE 1. Comparison between production planning and game theory

	Game theory	**Production planning**
$\boldsymbol{i}$	Player	Period
$\boldsymbol{S}$	Coalition	Set of periods
Characteristic function	v	$-$ CVaR$(1-\alpha)$ [1]
$\boldsymbol{\pi}_i$	Shapley value	individual risk

The estimated demand can be represented as $d = [d_1, d_2, \ldots, d_n]$, and is normally distributed, i.e., $d \sim N(\bar{d}, \Sigma)$, with $\bar{d}$ denoting the expected demand.

For the planning process, we assume that there exists relation between production, the inventory and Shapley value. The higher the expected demand is, the higher the production volume will be as can be observed in the graphical illustration for n periods production planning model presented in Figure 1 where S_0 is the initial inventory and x_i the production volume. The expected demand $\bar{d}_i$ is considered favorable and consequently, uncertain risk can be avoided. In the graph, $\bar{S}_i$ is the average inventory, d_i denotes demand with normal distribution and is proportional to the variance, that is, $\mathrm{N}(\bar{d}_i, \omega_i)$. The inventory quantity S_i at period i [5] is given by

$$S_i = S_0 + \sum_{t=1}^{i} x_t - \sum_{t=1}^{i} d_t \tag{13}$$

d_i is a random variable so that S_i becomes random variable and it is assumed to obey an average $\bar{d}_i$ and standard deviation ω_i, where d_i, d_j $(i \neq j)$ are independent each other and $\omega_i = \sigma \times \bar{d}_i$, where σ is the deviation of order. In this paper, following a previous study [8], Shapley values were computed by using an alternative equivalent formulation

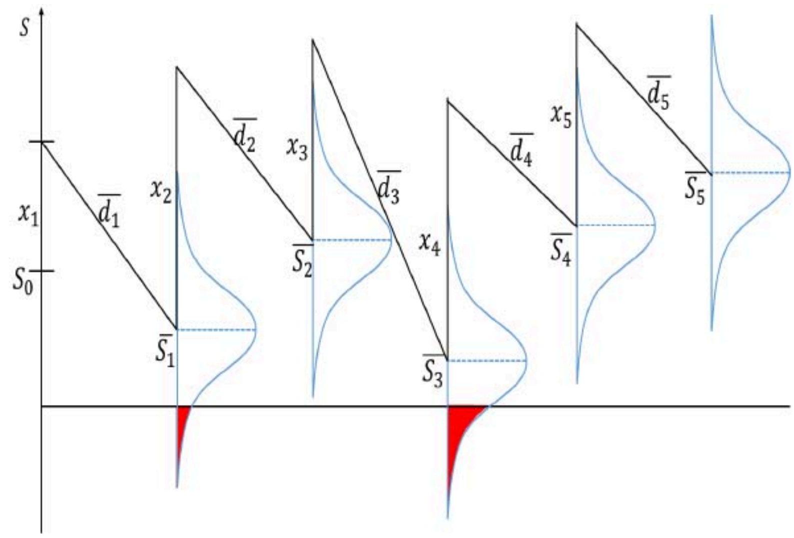

Figure 1. Production planning for multi-period

which is an LP model. The model is described briefly next referring readers to [8] and the references therein for more details.

Suppose a sample β from a data set, where x_i and y_i are the values of the referred sample. One can find the error among the data using the following expression:

$$e_\beta = f_\beta(\boldsymbol{A}, \boldsymbol{M}, \boldsymbol{v}) = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{v} - \boldsymbol{A}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{A}\boldsymbol{z})_\beta \tag{14}$$

where $(\ )_\beta$ denotes the selection of the β^{th} row value. So, the sum of all error functions is given by $E = \sum_\beta |e_\beta|$ using a multiple linear regression model which minimizes the sum of the absolute values of the residuals. Equality (14) can be transformed into an optimization problem in Equation (15).

$$\begin{aligned} \text{Minimize} \quad & \epsilon \\ \text{Subject to} \quad & \boldsymbol{A}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{v} + \boldsymbol{s}^+ - \boldsymbol{s}^- = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{A}\boldsymbol{z} \\ & \sum_{d \in K} z_\beta(K, v) = v(K) \\ & 0 \le s^+ \le \epsilon,\ 0 \le s^- \le \epsilon \end{aligned} \tag{15}$$

where $s^+ = [s_1^+, s_2^+, s_3^+, \ldots, s_n^+]^{\mathrm{T}}$ and $s^- = [s_1^-, s_2^-, s_3^-, \ldots, s_n^-]^{\mathrm{T}}$ denote the set of slack variables; A is a matrix which observes the supperaditivity property; v is a column matrix whose elements are the real values $v(\mathcal{H})$ and $M = (\text{diag}\, M_{\Omega,s})$ obtained through Equation (16) as follows.

$$M_{\Omega,s} = \frac{1}{\Omega - 1}\ \{_{\Omega-2}C_{s-1}\}^{-1}. \tag{16}$$

In Equation (16), $M_{\Omega,s}$ corresponds to a set of weights, Ω as defined previously indicates the set of players and s denotes the number of elements in the coalition being estimated ($s \subset \mathcal{H}$). This can be visualized through the following example.

Example 3.1. *Let (Ω, v) be a 3-person game with $\Omega = \{1, 2, 3\}$ the finite set of players, so*

$$\boldsymbol{z} = \begin{bmatrix} z_1(\Omega, v) \\ z_2(\Omega, v) \\ z_3(\Omega, v) \end{bmatrix}, \qquad \boldsymbol{v} = \begin{bmatrix} v(\{1\}) \\ v(\{2\}) \\ v(\{3\}) \\ v(\{1,2\}) \\ v(\{1,3\}) \\ v(\{2,3\}) \end{bmatrix} \tag{17}$$

and

$$\boldsymbol{A}^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}. \tag{18}$$

Computational issues. The process scheme described in this paper can be applied through the following stepwise procedures:

Step 1: **Define the confidence level** $1 - \alpha$, the **estimated demand** d_i and **initial inventory** S_0.

Step 2: **To determine the characteristic functions**: Measure each player's record. Using Equation (10) the obtained solution is assigned as the characteristic function for the corresponding players/coalitions.

Step 3: **To determine the Shapley value**: Compute the weights $M_{\Omega,s}$ by employing Equation (16). Using the characteristic function values of all coalitions and the weights, Shapley value and error function are calculated by solving (15).

4. **Numerical Example.** Suppose a company desires to analyze its production in 5 periods. How can they estimate the risk distribution for each period?

Hence, it is a 5-person game, that is, $\Omega = \{1, 2, 3, 4, 5\}$. Consider as prerequisites: initial inventory $S_0 = 10$, the estimated demand for each period is given by $d = [10, 20, 24, 6, 12]$, where the variance $\omega = 3$ and the confidence level $\alpha = 0.1$. Using Equation (10) the coalitions' characteristic functions $v(\mathcal{H})$ were computed and are presented in Table 2.

TABLE 2. The coalitions' characteristic functions $v(\mathcal{H})$

Coalitions	$v(\mathcal{H}) = -\mathrm{CVaR}_{\mathcal{H}}(1-\alpha)$	Coalitions	$v(\mathcal{H}) = -\mathrm{CVaR}_{\mathcal{H}}(1-\alpha)$
$v\{\emptyset\}$	0	$v\{123\}$	83.91695542
$v\{1\}$	17.99564266	$v\{124\}$	62.51854666
$v\{2\}$	31.30754629	$v\{125\}$	73.98257064
$v\{3\}$	37.84885933	$v\{134\}$	73.92263887
$v\{4\}$	21.99128532	$v\{135\}$	80.85219835
$v\{5\}$	29.87880051	$v\{145\}$	65.50288835
$v\{12\}$	47.87880051	$v\{234\}$	88.34575512
$v\{13\}$	53.58524469	$v\{235\}$	92.64063772
$v\{14\}$	37.15448205	$v\{245\}$	79.54657798
$v\{15\}$	44.61509258	$v\{345\}$	87.23018516
$v\{23\}$	67.98692798	$v\{1234\}$	103.7939385
$v\{24\}$	51.28444217	$v\{1235\}$	110.5178543
$v\{25\}$	58.51854666	$v\{1245\}$	94.62220772
$v\{34\}$	58.82869959	$v\{1345\}$	101.9327724
$v\{35\}$	65.91695542	$v\{2345\}$	116.229087
$v\{45\}$	50.96687924	$v\{12345\}$	131.297273

In Table 3 the Shapley values are shown. The first row contains the solution set using Equation (9) and second row, those from Program (15). Through this solution one can obtain information regarding the risk in each period. As expected both approaches present the same results, i.e., the Shapley values for each period or player π_1, π_2, π_3, π_4 and π_5 are respectively, 17.58, 15.13, 32.47, 26.38 and 39.73.

One can easily observe that Shapley's efficiency (Group rationality) property is fulfilled, that is, the overall sum of the values gives the same value for the grand coalition $v(\mathcal{H})$ as in Table 2.

Table 3. Shapley values for the 5 periods

Methods	***Periods***					
	$\boldsymbol{\pi_1}$	$\boldsymbol{\pi_2}$	$\boldsymbol{\pi_3}$	$\boldsymbol{\pi_4}$	$\boldsymbol{\pi_5}$	$\boldsymbol{v\{\Omega\}}$
Conventional Shapley value	17.58	15.13	32.47	26.38	39.73	131.30
LP model	17.58	15.13	32.47	26.38	39.73	131.30

In Table 4, we imposed some new constraints into the problem (15) in order to analyze the distribution of the risk among the periods. The problem was analyzed through different cases as defined below.

- Case 1: $\pi_5 \geq 3\pi_1$
- Case 2: $\pi_5 \geq 48$
- Case 3: $\pi_1 \leq 10$
- Case 4: $\pi_5 \geq 3\pi_2$
- Case 5: $\pi_5 \leq 35$
- Case 6: $\pi_2 \leq 6$
- Case 7: $\pi_3 \leq 20$
- Case 8: $\pi_4 \leq 12$
- Case 9: $\pi_5 \leq 24$

Table 4. Constrained Shapley value indicating risk distribution among the 5 periods

Cases	$\boldsymbol{\epsilon'}$	$\boldsymbol{\pi_1}$	$\boldsymbol{\pi_2}$	$\boldsymbol{\pi_3}$	$\boldsymbol{\pi_4}$	$\boldsymbol{\pi_5}$	$\boldsymbol{s_1^+}$	$\boldsymbol{s_1^-}$	$\boldsymbol{s_2^+}$	$\boldsymbol{s_3^+}$	$\boldsymbol{s_3^-}$	$\boldsymbol{s_4^+}$	$\boldsymbol{s_5^+}$
Main	0.614	17.58	15.13	32.47	26.38	39.73	0.614	0	0.614	0.614	0	0.614	0.614
Case 1	2.948	14.02	14.51	31.98	28.71	42.07	0	2.948	0	0.121	0	2.948	2.948
Case 2	8.877	16.97	14.51	26.05	25.76	48.00	0	0	0	0	5.807	0	8.877
Case 3	6.970	10.00	14.51	31.86	28.83	46.09	0	6.970	0	0	0	3.068	6.970
Case 4	1.11	18.077	13.41	32.72	26.86	40.22	1.11	0	0	0	0	0	0
Case 5	3.13	16.97	14.44	35.00	25.76	39.12	0	0	0	0	0	0	0
Case 6	8.51	16.97	6.00	31.86	28.83	47.63	0	0	0	0	0	0	0
Case 7	11.86	16.97	14.51	20.00	37.62	42.19	0	0	0	0	0	0	0
Case 8	13.76	16.97	14.51	34.93	12.00	52.89	0	0	0	0	0	0	0
Case 9	15.12	16.97	14.51	34.93	40.88	24.00	0	0	0	0	0	0	0

The other slack variables are: $s_2^- = \boldsymbol{s_4^-} = \boldsymbol{s_5^-} = \boldsymbol{s_6^+} = \boldsymbol{s_6^-} = 0$ (for all cases).

Now, we desire to estimate how much is the difference of the cost or risk for each period as compared to the main case presented in Tables 3 and 4. The result of this process is displayed in Table 5.

Subtracting the results from Table 4 we quantified the difference of the estimated demand for each period, and this arithmetic is presented in Table 5. Moreover, the planning

Table 5. Differences between the estimated risk in the 5 periods

Cases	$\boldsymbol{\epsilon'}$	**1**	**2**	**3**	**4**	**5**	$\boldsymbol{s_1^+}$	$\boldsymbol{s_1^-}$	$\boldsymbol{s_2^+}$	$\boldsymbol{s_3^+}$	$\boldsymbol{s_3^-}$	$\boldsymbol{s_4^+}$	$\boldsymbol{s_5^+}$
Case 1 - Main	2.33	−3.56	−3.07	−0.49	2.33	2.33	−0.61	2.95	−0.61	−0.49	0	2.33	2.33
Case 2 - Main	8.26	−0.61	−0.61	−6.42	−0.61	8.27	−0.61	0	−0.61	−0.61	5.81	−0.61	8.26
Case 3 - Main	6.36	−7.58	−0.61	−0.61	2.46	6.36	−0.61	6.97	−0.61	−0.61	0	2.45	6.36
Case 4 - Main	0.49	−0.49	−4.18	0.24	0.49	0.49	0.50	0	−0.61	−0.61	0	−0.61	−0.61
Case 5 - Main	2.53	−0.61	−3.14	2.53	−0.61	−0.61	−0.61	0	−0.61	−0.61	0	−0.61	−0.61
Case 6 - Main	7.90	−0.61	−11.58	−0.61	2.46	7.90	−0.61	0	−0.61	−0.61	0	−0.61	−0.61
Case 7 - Main	11.24	−0.61	−3.07	−12.47	11.25	2.46	−0.61	0	−0.61	−0.61	0	−0.61	−0.61
Case 8 - Main	13.15	−0.61	−3.07	−2.46	−14.38	13.15	−0.61	0	−0.61	−0.61	0	−0.61	−0.61
Case 9 - Main	14.50	−6.61	−3.07	−2.46	14.51	−15.73	−0.61	0	−0.61	−0.61	0	−0.61	−0.61

process, in this table, is explained as follows: one can predict a higher demand in the last period while the estimated demand for other periods is expected to decrease considerably since the corresponding Shapley value for the fifth period increases n times of the first period (**case 1**); if the estimated demand is increased, for instance, 4 times in the last period, we should expect a decreasing of the demand in the first and second periods and as well as a significant fall off in the third period (**case 2**); in **case 3**, allocating the lowest demand, 10, to the first period will have a positive impact on the last two periods in terms of demand, i.e., 2.46 and 6.36, respectively. Hence, in general, positive or negative impacts regarding demands and risks depend on the constraints the model is subject to, and for this reason we consider our model to a Shapley constraint approach.

5. **Concluding Remarks.** Game theory is a strong tool to solve problems which require strategic policies. As one solution concept, Shapley value can predict efficient decisions. In this work we combined game theory, risk measure and linear programming techniques to find the Shapley values in order to acquire information regarding risk in a multi-period production planning environment. Our model used CVaR as characteristic function and equivalent results were obtained employing the conventional Shapley value approach. For n periods production planning problems, decision-makers can evaluate theirs strategies according to several cases which imply to analyze the set of constraints in the optimization model.

One of limitations of our model is the lack of an overall satisfaction of Shapley value axioms which leads to a future direction in this work, that is, the consistency of the constraints and as well as a strong response when dealing with data uncertainty.

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