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# A NEW ACCELERATION PARTICLE SWARM OPTIMIZATION FOR THE SEQUENTIAL CAPTURE OF MULTIPLE LOCAL OPTIMA OF MULTIMODAL FUNCTIONS

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Abstract. *A new acceleration particle swarm optimization algorithm is developed in this study for the sequential capture of multiple local optima of multimodal functions. The proposed algorithm does not stop but instead continues to search when a new local optimum is found. Particles are attracted by personal and global bests and repelled by the local optima obtained with the proposed algorithm. Particle acceleration depends on resultant forces. A reselection operation is implemented for personal and global bests to prevent particles from flying back to the local optimum. A hybrid mutation strategy is also adopted to increase the diversity of the particle swarm. The computational results on two complex multimodal functions prove that the proposed algorithm is a feasible and effective approach for multimodal function optimization problems.*

**Keywords:** Acceleration particle swarm optimization, Multimodal function, Multiple local optima, Sequential capture

1. **Introduction.** Particle swarm optimization (PSO) was proposed by Kennedy and Eberhart in 1995 and has since become one of the most popular evolution algorithms (EAs) based on swarm intelligence [1]. As a result of its numerous advantages, including simple formula, quick convergence, minimal parameter requirements, and so on, PSO has been regarded as a popular and powerful tool for solving engineering optimization problems and has been improved with various methods for various applications [2-4]. This study focuses on the issue of solving multimodal functions with PSO. A multimodal function refers to a function with multiple local optima, including global optima. In solving these problems, the goal is to identify not only all the global optima but also as many local optima as possible so as to provide comprehensive information or wide selection of choices for decision makers.

Although PSO shows satisfactory performance in numerous optimization problems, it also easily falls into the local optima, especially when used to solve multimodal function optimization problems. To overcome this problem, some researchers focus on helping trapped particles escape from the local optima and gain more local optima, including global optima. For example, a modified PSO with an adaptive mutation strategy was designed for multimodal function optimization problems [5]. Three types of mutation operators can be adopted to maintain swarm diversity. In [6], a parameter adaptive harmony search algorithm was developed to overcome premature convergence and obtain a global optimal solution. In [7], PSO was integrated with a local search technique to locate multiple global and local optimal solutions. In [8], a modified PSO with multiple subpopulations was developed. In the present work, we propose a new acceleration PSO (APSO) inspired by Newton's law of motion. The computational results show that the proposed algorithm can sequentially capture all the local optima including global optima of multimodal functions in a single run. The rest of this paper is organized as follows. Section 2 introduces APSO in detail. Experimental results on two test functions and discussion are presented in Section 3. At last, conclusions and further research aspects are given in Section 4.

2. **Acceleration Particle Swarm Optimization.** First, Newton's law of motion is given.

$$a = \frac{v_2 - v_1}{t} \tag{1}$$

$$v_2 = v_1 + a \cdot t \tag{2}$$

$$x_2 = x_1 + v_1 \cdot t + 0.5a \cdot t^2 \tag{3}$$

During the time step $t$, an object with initial velocity $v_1$ moves from position $x_1$ to position $x_2$. The velocity modifies $v_2$ from $v_1$ after time step $t$. The acceleration is defined by Equation (1). The new velocity $v_2$ is obtained by Equation (2). So the new position of the object is obtained by Equation (3). According to Newton's law of motion, it can be considered that the particles fly with force in the solution space. Particles are attracted by personal and global bests and repelled by the local optima. The acceleration of particle depends on resultant forces. Figure 1 shows the force diagram for a particle.

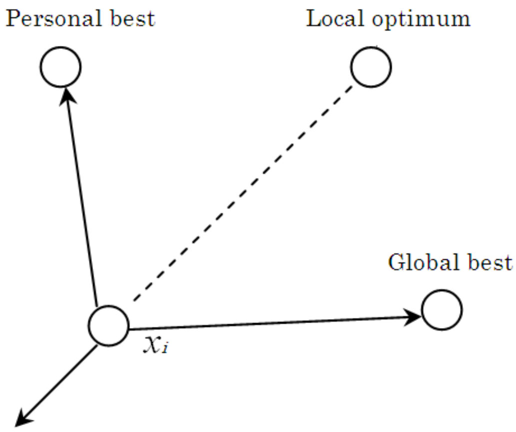



FIGURE 1. Force diagram for a particle

The new acceleration particle swarm optimization (APSO) is presented as follows:

$$a_{id}^t = c_1 \cdot r_1 \left(p_{id}^t - x_{id}^t\right) + c_2 \cdot r_2 \left(p_{gd}^t - x_{id}^t\right) + \sum_{j=1}^{k} c_3 \cdot r_3 \frac{R}{\left(x_{id}^t - p_j\right)^b + c}$$
$$\left(\text{if } c_3 \cdot \frac{R}{\left(x_{id}^t - p_j\right)^b + c} < e, \quad c_3 \cdot \frac{R}{\left(x_{id}^t - p_j\right)^b + c} = 0\right) \tag{4}$$

$$v_{id}^{t+1} = w \cdot v_{id}^t + a_{id}^t \tag{5}$$

$$x_{id}^{t+1} = x_{id}^t + v_{id}^{t+1} + 0.5a_{id}^t \tag{6}$$

Equation (4) is the formula for acceleration update. The first and second parts of the right side of the equation represent the accelerations caused by the attractive forces of the personal and global bests, respectively; the third part represents all the accelerations caused by the repulsive force of the local optima. Equation (5) is the formula for velocity update, and Equation (6) is the formula for position update, where $x_{id}^t$ and $x_{id}^{t+1}$ are the current and next positions, respectively; $v_{id}^t$ and $v_{id}^{t+1}$ are the current and next velocities, respectively; $w$ is the inertia weight; $a_{id}^t$ is the current acceleration; $p_{id}^t$ and $p_{gd}^t$ are the personal and global bests, respectively; $c_1$, $c_2$, and $c_3$ are the positive constants assigned by the designer; $r_1$, $r_2$, and $r_3$ are three uniform random numbers selected from the interval $[0, 1]$; $p_j$ is one of the $k$ local optima obtained currently. The repulsive force should gradually decrease as the particles move away from the local optimum to avoid affecting the motion of the particles in the future. The constants $c_3$, $R$, $b$, and $c$ can be changed to adjust the size and scope of the repulsive force. Furthermore, as shown in Equation (4), when acceleration caused by repulsive force is less than $e$ (positive constant), then it is set to 0.

2.1. **Premature convergence criterion.** The proposed APSO algorithm can help trapped particles jump from the local optimum to avoid premature convergence. We use a diversity valve to estimate possible premature convergence and confirm a local optimum.

**Definition 2.1.** *Diversity valve*

$$Div = \frac{\sum_{i=1}^{N} \sqrt{\sum_{d=1}^{n} \left(x_{id}^t - \overline{p_d}\right)^2}}{NL} \tag{7}$$

Equation (7) is the formula for diversity valve, where $Div$ is the average distance between particles, $N$ is the population size, $L$ is the maximum distance of the decision space, $x_{id}^t$ is the current particle of iteration $t$ and dimension $d$, and $\overline{p_d}$ is the average position in dimension $d$ of all particles. The premature convergence criterion can be described as follows: if the global best is not updated for $T$ iterations and $Div$ is less than $\varepsilon$ ($\varepsilon \in (0, 1)$), then the current global best is considered a new local optimum.

2.2. **Reselection operation.** Once a new local optimum is found, the current particles are reinitiated because they are useless in future search. Although the particles are repelled by the local optimum, they are also attracted by personal and global bests that are probably near the local optimum. The direction of attractive and repulsive forces may be opposite, and the particles would vibrate in a certain space. Thus, the personal and global bests of particles should be reselected under certain conditions.

We use two two-dimensional arrays named $ft(i, t)$ and $aft(i, t)$ to save the fitness and additional fitness valves of all particles in all iterations, where $i$ and $t$ represent the number of particles and iterations respectively. We also employ a three-dimensional array named $x(i, j, t)$ to save the positions of all particles in all iterations, where $j$ represents the number of dimensions. The initial values of all elements of $aft(i, t)$ are set to 0. Furthermore, a collection $G$ is used to save all local optima recently found. Some definitions are provided below. We take the minimization problem as an example.

**Definition 2.2.** *Dangerous fly.*

If a particle's fitness and the distance from current position to local optimum decrease or increase together, the particle of current iteration is considered to have experienced *Dangerous fly*. If a particle experienced continuous *Dangerous fly* before it eventually fell into the local optimum, that means the particle and the local optimum are in the same trough. So the particle is not qualified as a personal or global best.

**Definition 2.3.** *Checking operation.*

Check all particles to confirm whether current particle of current iteration has experienced *Dangerous fly*. The *checking operation* will not be executed until at least one local optimum is found.

**Definition 2.4.** *Reselection operation.*

The *reselection operation* is executed after a new local optimum is found and after the particle swarm is reinitiated. The steps of the *reselection operation* are specified as follows.

**Step 1**. When at least one local optimum is found, $ft(i,j)$, $aft(i,j)$, and $x(i,d,j)$ are started to record corresponding values and the *checking operation* is executed.

**Step 2**. Whenever the repulsive force is greater than the attractive force (that means the personal and global bests are near the local optimum), set the additional fitness valves of $aft(i,j)$ corresponding to the last continuous *Dangerous fly* to 1,000 (or any positive constant that is adequately large). So the corresponding position of this particle is no longer selected as a personal or global best.

**Step 3**. Find the best valve of each row of array $(ft(i,j)+aft(i,j))$ and regard the corresponding position as the new personal best of each particle. Find the personal best with the best fitness valve as the new global best.

2.3. **Mutation strategy.** A hybrid mutation strategy that consists of uniform and Gauss mutations is adopted in the proposed APSO algorithm to increase population diversity. Uniform mutation is defined by Equation (8) and Equation (9), where $x_i^t$ and $v_i^t$ are the current position and velocity, respectively; $x_{\max}$ and $x_{\min}$ are the upper and lower bounds of the position, respectively; and $v_{\max}$ and $v_{\min}$ are the upper and lower bounds of velocity, respectively. Gauss mutation is defined by Equation (10) with a variance of 0.1. The proposed mutation strategy is described as follows. (1) Each particle is assigned a probability of $1/N$ to execute uniform mutation in each iteration. (2) The particles with the highest $N/4$ fitness valves are assigned a probability of $4/N$ to execute Gauss mutation in each iteration.

$$x_i^t = x_{\min} + rand \times (x_{\max} - x_{\min}) \tag{8}$$

$$v_i^t = rand \times (v_{\max} - v_{\min}) \tag{9}$$

$$x_i^t = x_i^t(1 + 0.1 \times Gaussian(0.1)) \tag{10}$$

3. **Computational Experiments.** Consider modified Himmelblau function:

$$f(x,y) = \left(x^2+y-11\right)^2 + \left(x+y^2-7\right) + 0.1\left[(x-3)^2+(y-2)^2\right] \tag{11}$$

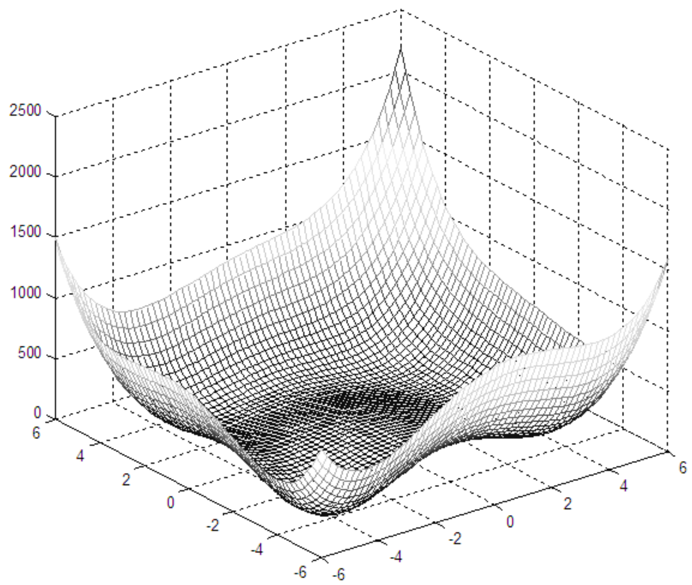



Figure 2. Modified Himmelblau function

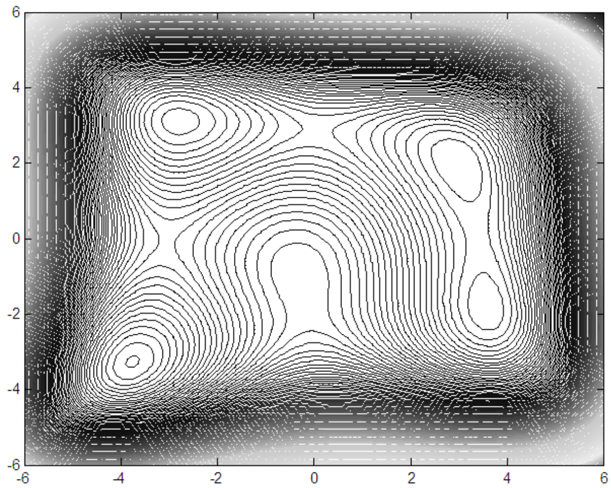



Figure 3. Contour of modified Himmelblau function

Figure 2 shows the 3-D surface of modified Himmelblau function with the defined space $-6 \leq x \leq 6$ and $-6 \leq y \leq 6$, and its corresponding contour is shown in Figure 3. It can easily be found from Figure 3 that this optimized function has four minimum points including one global minimum that occurred at $f(3, 2) = 0$, and the three local minimum points at $f(-3.7634, -3.2660) \approx 7.3673$, $f(3.5814, -1.8208) \approx 1.5043$, and $f(-2.7870, 3.1282) \approx 3.4871$, respectively. Furthermore, the parameters used in APSO are given by the population size $N = 40$, inertia weight $w = [0.9, 0.4]$, positive constants $c_1 = 0.5$, $c_2 = 1.5$, $c_3 = 1$, $R = 2$, $c = 0.5$, $b = 3$, $\varepsilon = 0.05$, $T = 5$, $e = 0.25$, and the maximum iteration number $t_{\max}$ is 100.

Figure 4 demonstrated the convergence behavior of particles by the number of iterations, where circular makers represent particle positions and square makers represent positions of local optima including global optimum. It can be seen in Figure 4 that the local optima including global optimum are found sequentially.

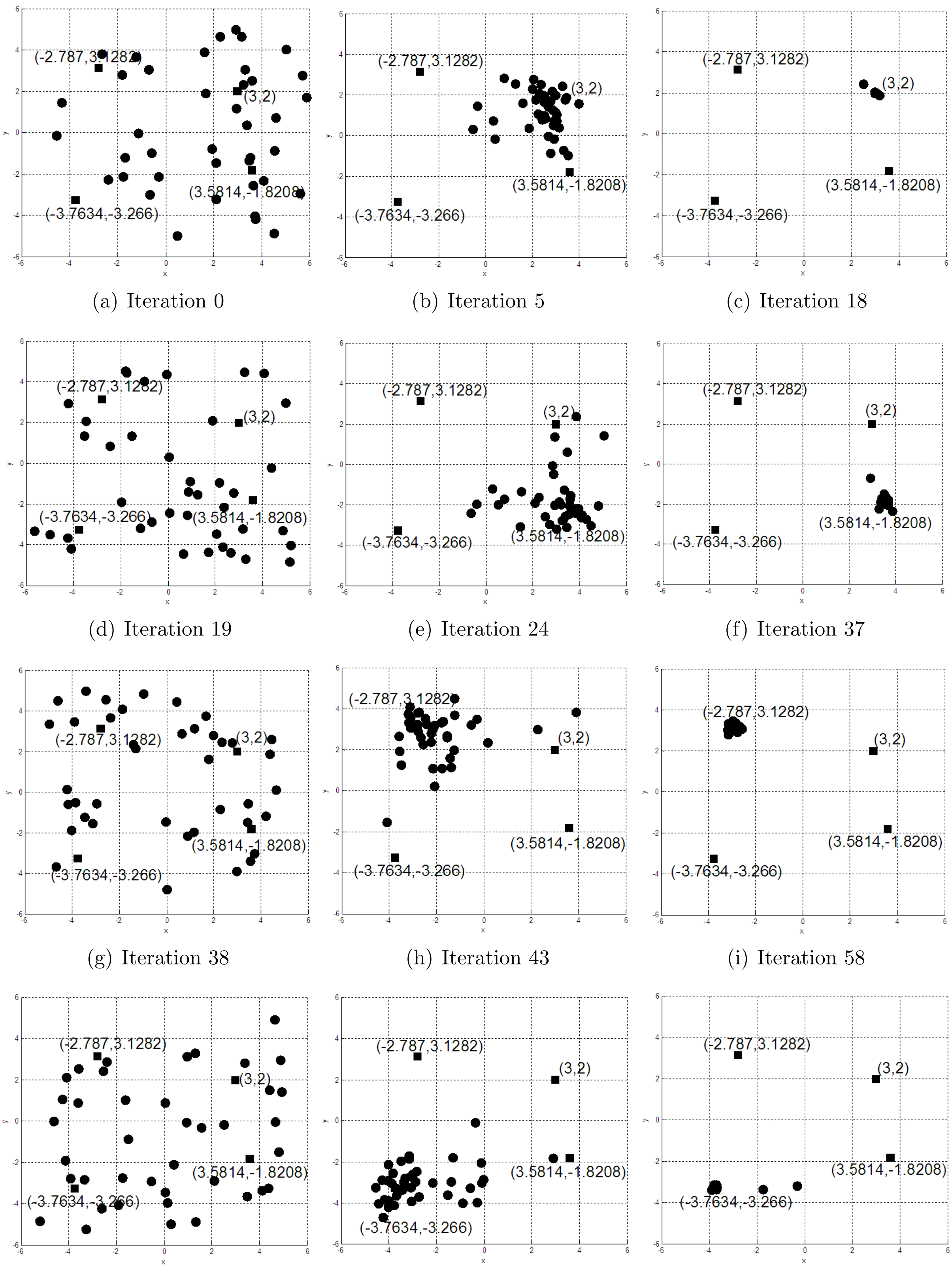
Consider another function named Foxholes:

$$\min f(X) = \frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{2} (x_i - a_{ij})^6} \tag{12}$$

$$a\{1\} = \begin{bmatrix} -32 & -16 & 0 & 16 & 32 \\ -32 & -16 & 0 & 16 & 32 \\ -32 & -16 & 0 & 16 & 32 \\ -32 & -16 & 0 & 16 & 32 \\ -32 & -16 & 0 & 16 & 32 \end{bmatrix}$$

$$a\{2\} = \begin{bmatrix} -32 & -32 & -32 & -32 & -32 \\ -16 & -16 & -16 & -16 & -16 \\ 0 & 0 & 0 & 0 & 0 \\ 16 & 16 & 16 & 16 & 16 \\ 32 & 32 & 32 & 32 & 32 \end{bmatrix}, \quad a_{ij} = a\{i\}(j)$$

The Foxholes function has twenty five local maximum points including one global maximum point. APSO is used to solve this optimization problem with the purpose of finding all local and global maximum points. The parameter settings are the same as the first problem except that the maximum iteration number $t_{\max}$ is 500. Figure 5 showed that all the local maximum points including one global maximum point marked by white dots have been found in a single run and the contour of this function is shown in Figure 6.




(a) Iteration 0 (b) Iteration 5 (c) Iteration 18

(d) Iteration 19 (e) Iteration 24 (f) Iteration 37

(g) Iteration 38 (h) Iteration 43 (i) Iteration 58

(j) Iteration 59 (k) Iteration 64 (l) Iteration 78

FIGURE 4. The convergence behavior of particles by the number of iterations

We make a comparison between APSO and MPSO [8]. As a modified PSO with multiple subpopulations, MPSO can find local optima simultaneously and each subpopulation could find at most one local optimum. Obviously a large number of subpopulations increase the possibility of finding a large number of local optima, but the number of

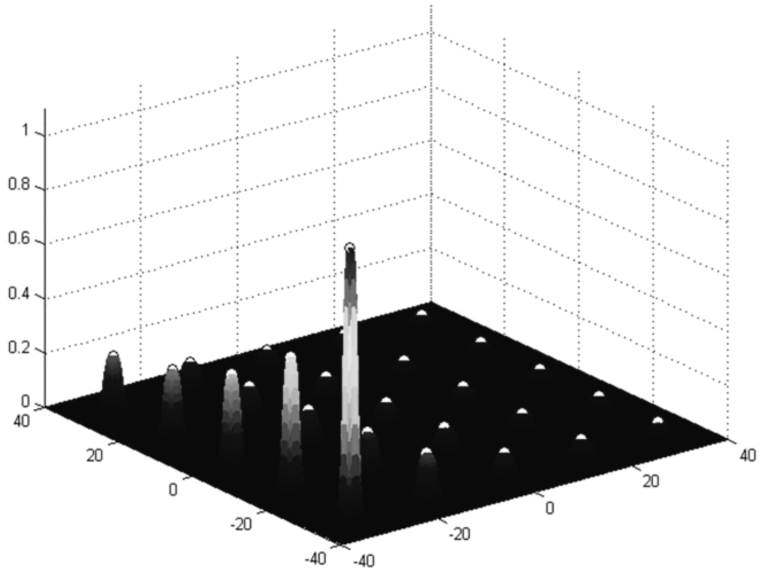



Figure 5. Maximum points of Foxholes function

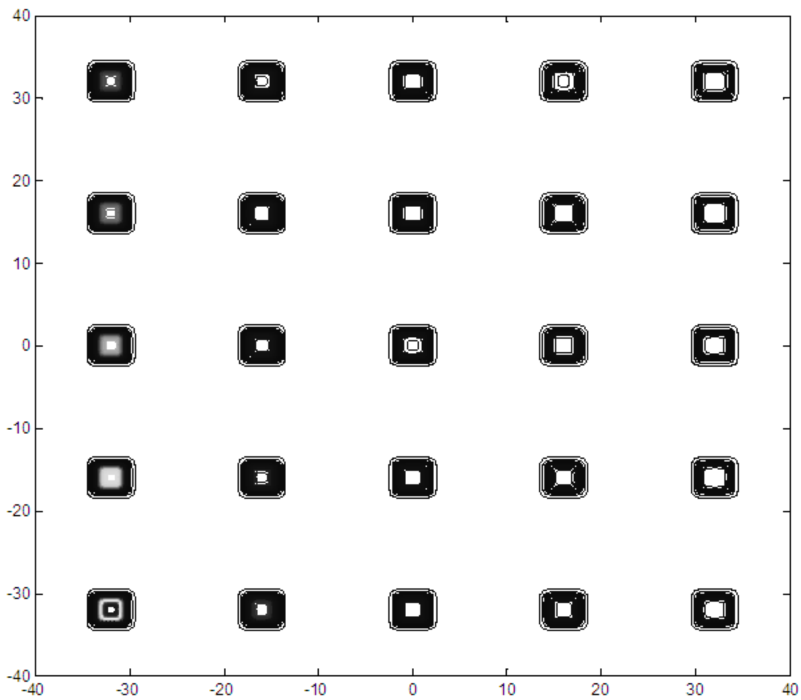



Figure 6. Contour of Foxholes function

subpopulations is uncertain because the number of local optima is unknown in advance. Moreover, different subpopulations may find a same local optimum. So finding all the local optima in a single run cannot be guaranteed. By contrast, the main idea of APSO is to prevent the search in obtained local optima areas, and the major strategies are repulsive force of local optima and reselection operation of personal and global bests. In this way, the algorithm can find all the local optima sequentially in a single run. The experimental results on two test functions have shown the excellent performance of the proposed algorithm.

4. **Conclusions.** A new APSO algorithm is successfully developed in this study for the sequential capture of the multiple local optima of multimodal functions. With this algorithm, the particles are repelled by the local optima and are thus never trapped. The

personal and global bests of particles are reselected under certain conditions to prevent particles from flying back to the local optima. The computational results sufficiently reveal the applicability of the proposed algorithm. This algorithm could also be combined with existing algorithms to improve convergence accuracy. In the future, the new APSO algorithm may be utilized to solve discrete and multi-objective optimization problems.

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