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A LEVEL CUT-BASED FUZZY EQUILIBRIUM VALUE MODEL AND ITS APPLICATION IN FUZZY PROGRAMMING

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Abstract. *The algebraic operations of fuzzy number are the bottleneck of fuzzy decision-making problems. How to establish easy and operable computation rules has important theories and application value. In this paper, for the deficiencies of fuzzy equilibrium value based on membership effect, we propose fuzzy effect equilibrium value based on level cut, and give the concrete computation result for trapezoidal and triangular fuzzy numbers with nonlinear membership effect function. These discussions provide an effective method for solving the algebraic operations of fuzzy equilibrium value based on membership effect. We also applied it to a nonlinear fuzzy programming problem. All the results indicate our discussions largely simplify the computations, and it is very useful to a great number of fuzzy decision-making problems.*

Keywords: Membership effect function, Effect equilibrium value, Trapezoidal fuzzy number, Level cut, Fuzzy programming

1. **Introduction.** Linear or nonlinear programming has important applications in many areas of engineering and management. In conventional approach, parameters of programming models must be well defined and precise. However, in real world environment, the problems are very complex, many experts and decision-makers frequently do not precisely know the values of the parameters, and they only give some estimate values. Therefore, it may be more appropriate to consider the parameters as fuzzy data. The fuzzy programming problems with fuzzy parameters would be viewed as a more realistic version than the conventional one. The theory of fuzzy mathematical programming was first proposed by Tanaka et al. [1] based on the fuzzy decision framework of Bellman and Zadeh [2]. Since then, fuzzy programming has been investigated and extended by many researchers from the various point of views, and it has been widely used in many real world applications [3-6]. Some researchers also have developed many approaches in order to better fit the real world problems within the framework of fuzzy programming. Ezzati et al. [7] proposed a method for comparing triangular fuzzy numbers and using it, further proposed an algorithm to find the fuzzy optimal solution of fully fuzzy linear programming problems with equality constraints. Ebrahimnejad and Tavana [8] proposed a method for solving fuzzy linear programming problems in which the coefficients of the objective function and the values of the right-hand-side are represented by symmetric trapezoidal fuzzy numbers while the elements of the coefficient matrix are represented by real numbers. Based on the nearest interval approximation operator, Luhandjula and Rangoaga [9] presented a method for dealing with a multiobjective programming problem with fuzzy-valued objective functions. Li and Wan [10] developed a fuzzy linear programming technique for solving multiattribute decision making problems with multiple types of attribute values

and incomplete weight information. Li et al. [11] analyzed the characteristics of fuzzy decision systematically, and studied the fuzzy programming theory and method based on comprehensive effect. There are still many researches about fuzzy programming. In spite of linear or nonlinear fuzzy programming, the essence of this kind of problem is to find an appropriate ranking method of fuzzy numbers.

Regarding the above reviews, the ranking of fuzzy number is the key to solving fuzzy decision-making, fuzzy programming problems, etc. However, there does not exist total order relation among fuzzy numbers. Therefore, establishing suitable fuzzy number ranking method under different decision background is very valuable in the academic and applications fields. Until now, there are lots of discussions about fuzzy number ranking methods. Jain [12] firstly discussed the comparisons methods of fuzzy variables. Baas and Kwakernaak [13] analyzed the essence of fuzzy multi-attribute decision problems, and further designed a fuzzy relations-based fuzzy information ranking method. Bortolan and Degani [14] put forward the construction strategy of fuzzy number ranking method according to the structure characteristics. Liou and Wang [15] established an integral-based fuzzy number ranking method combined with the representation theorem of fuzzy number. Based on the interval decomposition theory of fuzzy number, Liu et al. [16] took the convex combination of the interval end as the average value, and level importance function as the importance measure of level cuts, and established a concentralized quantitative method of fuzzy number using integral. Asady and Zendehnam [17] proposed a ranking method based on "distance minimization". For the disadvantages of distance minimization, Abbasbandy and Hajjari [18] proposed a ranking method of triangular fuzzy number. Li et al. [19] discussed the ranking criteria of fuzzy number based on numerical characteristic, and further gave the corresponding constructing strategy. Thereafter, Li and Yang [20] established membership effect-based effect equilibrium value model according to the balance theory of particle system, discussed the properties, and further gave the concrete computation method for several special fuzzy numbers.

From the above references reviews, the existing fuzzy number ranking methods have different forms and features, and they also have different application scope. They all play an important role for fuzzy decision-making and fuzzy programming. Although some authors have already given the algebraic operations of fuzzy numbers, the computation processes were often comparatively complicated. Therefore, in this paper, we have the following contributions: 1) for the deficiencies of nonadditivity of the membership effect-based effect equilibrium value model, we firstly establish level cut-based effect equilibrium value method; 2) we secondly give the general computation formulas for the trapezoidal fuzzy number and triangular fuzzy number, by combining with different membership effect functions; 3) we verify the effectiveness and operability of our model through a concrete nonlinear fuzzy programming problem.

This paper is organized as follows. Section 2 reviews the concept of fuzzy numbers and its arithmetic operations, and also gives the fuzzy equilibrium value. Section 3 proposes fuzzy equilibrium value model based on level cut, and discusses the computation method of effectiveness equilibrium value. Section 4 further establishes an effectiveness equilibrium value-based solution method for fuzzy programming and shows the experimental results through a concrete programming problem. Conclusions and further study are noted in Section 5.

2. **Preliminaries.**

2.1. **Fuzzy number.**

For the better understanding of this paper, let us briefly review some concepts and results on fuzzy numbers. In this paper, we use R to denote the family of all real numbers, $\mathtt{I}(R)$ the family of all interval numbers, and $\mathbb{F}(R)$ the family of all fuzzy sets. For $A \in \mathbb{F}(R)$, the membership function of A will be denoted $A(x)$, the λ-cut set by $A_\lambda = \{x|A(x) \geq \lambda\}$, the suppose set by supp$A = \{x|A(x) > 0\}$, and the closure of supp$A$ will be denoted by A_0.

Definition 2.1. [21] *Suppose $A \in \mathbb{F}(R)$, if it satisfies the following conditions: 1) $A_1 \neq \emptyset$; 2) $A_\lambda \in \mathcal{I}(R)$ for any $\lambda \in (0,1]$; 3) supp$A$ is bounded, then we call A a fuzzy number, and the family of all fuzzy numbers can be called fuzzy space and denoted by $\tilde{R}$. In particular, 1) if there exist $a, b_1, b_2, c \in R$ satisfying: i) $A(x) = 0$ for any $x < a$ or $x > c$; ii) $A(x) = (x-a)/(b_1-a)$ for any $a \leq x < b_1$; iii) $A(x) = 1$ for any $b_1 \leq x \leq b_2$; iv) $A(x) = (x-c)/(b_2-c)$ for any $b_2 < x \leq c$, then we call A a trapezoidal fuzzy number and simply denoted as $A = (a, [b_1, b_2], c)$; 2) if $b_1 = b_2 = b$, then we call $A = (a, [b,b], c)$ a triangular fuzzy number and is simply denoted as $A = (a, b, c)$.*

Obviously, if we take interval number $[a,b]$ as a fuzzy set whose membership function is $[a,b](x) = 1$ for any $x \in [a,b]$, and $[a,b](x) = 0$ for any $x \notin [a,b]$, and real number a as a fuzzy set whose membership function is $a(a) = 1$, and $a(x) = 0$ for any $x \neq a$, then the interval numbers and real numbers are special fuzzy numbers, which shows that fuzzy numbers are the extension of the interval numbers and real numbers.

Theorem 2.1. [22] *Let $A, B \in \mathbb{F}(R)$, $k \in R$, $f(x,y)$ be a continuous binary function, $A_\lambda = [\underline{a}(\lambda), \overline{a}(\lambda)]$, $B_\lambda = [\underline{b}(\lambda), \overline{b}(\lambda)]$ be the λ-cut set of A and B, respectively, and then $f(A,B) \in \mathbb{F}(R)$. For any $\lambda \in (0,1]$, we have $(f(A,B))_\lambda = f(A_\lambda, B_\lambda) = \{f(x,y)|x \in A_\lambda, y \in B_\lambda\}$. In particular, we have:*

1) $A + B = B + A$, $A \cdot B = B \cdot A$, $k(A \pm B) = kA \pm kB$;
2) $(A+B)_\lambda = [\underline{a}(\lambda) + \underline{b}(\lambda), \overline{a}(\lambda) + \overline{b}(\lambda)]$, $(A-B)_\lambda = [\underline{a}(\lambda) - \overline{b}(\lambda), \overline{a}(\lambda) - \underline{b}(\lambda)]$;
3) $(A \times B)_\lambda = A_\lambda \times B_\lambda = [\min\{\underline{a}(\lambda)\underline{b}(\lambda), \underline{a}(\lambda)\overline{b}(\lambda), \overline{a}(\lambda)\underline{b}(\lambda), \overline{a}(\lambda)\overline{b}(\lambda)\}, \max\{\underline{a}(\lambda)\underline{b}(\lambda), \underline{a}(\lambda)\overline{b}(\lambda), \overline{a}(\lambda)\underline{b}(\lambda), \overline{a}(\lambda)\overline{b}(\lambda)\}]$, $(A \div B)_\lambda = A_\lambda \div B_\lambda = [\underline{a}(\lambda), \overline{a}(\lambda)] \times [(\overline{b}(\lambda))^{-1}, (\underline{a}(\lambda))^{-1}]$, $0 \notin [\underline{b}(\lambda), \overline{b}(\lambda)]$;
4) If $\underline{a}(\lambda) \geq 0, \overline{b}(\lambda) \geq 0$, then $(A \times B)_\lambda = [\underline{a}(\lambda)\underline{b}(\lambda), \overline{a}(\lambda)\overline{b}(\lambda)]$, $(A \div B)_\lambda = [\underline{a}(\lambda) \div \overline{b}(\lambda), \overline{a}(\lambda) \div \underline{b}(\lambda)]$;
5) If $A = (a_1, b_1, c_1)$, $B = (a_2, b_2, c_2)$, then $A + B = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$, $A - B = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$.

Fuzzy numbers have many good analytical properties. For details, please see [22].

2.2. Fuzzy equilibrium value based on membership effectiveness.

Definition 2.2. [20] *$T(t)$ is a mapping from $[0,1]$ to $[0,1]$ satisfying the following conditions: 1) $T(0) = 0$, $T(1) = 1$; 2) $T(t_1) \leq T(t_2)$ for any $0 \leq t_1 \leq t_2 \leq 1$; 3) it is continuous on $[0,1]$, and then we call it a membership effect function.*

Definition 2.3. [20] *Let A be a fuzzy number, $T(t)$ be a membership effect function, and*

$$E_T(A) = E(T(A)) = \begin{cases} \frac{\int_{-\infty}^{+\infty} xT(A(x))dx}{\int_{-\infty}^{+\infty} T(A(x))dx}, & T(A) \neq a, \\ a, & T(A) = a. \end{cases} \tag{1}$$

Then we call $E_T(A)$ the Effectiveness Equilibrium Value of A on membership effect function T.

Theorem 2.2. *Let $(a, [b_1, b_2], c)$ be a trapezoidal fuzzy number, $a < c$, $T(t)$ be a membership effect function, $K_0 = \int_0^1 T(t)dt$, $K_1 = \int_0^1 tT(t)dt$. Then*

$$E_T(A) = \frac{\frac{1}{2}(b_2^2 - b_1^2) + [a(b_1 - a) + c(c - b_2)]K_0 + [(b_1 - a)^2 - (c - b_2)^2]K_1}{(b_2 - b_1) + (b_1 - b_2 + c - a)K_0}. \tag{2}$$

Corollary 2.1. *Let $(a, [b_1, b_2], c)$ be a trapezoidal fuzzy number, and $a < c$, for $T(t) = t$, we have*

$$E_T((a, [b_1, b_2], c)) = \frac{b_2^2 - b_1^2 - ab_1 - a^2 + c^2 + cb_2}{3(b_2 - b_1 + c - a)}. \tag{3}$$

From Corollary 2.1, we know that if $A = (1, [3, 4], 5)$, $B = (2, [4, 6], 7)$, for $T(t) = t$, we have $A + B = (3, [7, 10], 12)$, $E_T(A) = 16/5$, $E_T(B) = 33/7$, $E_T(A + B) = 95/12$, $E_T(A + B) \neq E_T(A) + E_T(B)$.

3. **Fuzzy Equilibrium Value Model Based on Level Cut.** Based on the above discussions, we know that Effectiveness Equilibrium Value of fuzzy numbers does not meet the allocation rate on algebraic operation. The membership functions of fuzzy number on algebraic operation are often difficult to determine, and algebraic operation is an essential component of fuzzy decision-making (fuzzy programming). Therefore, establishing an operable computation method of Effectiveness Equilibrium Value is a key to solving fuzzy decision-making problems. According to the decomposition theorem and representation theorem of fuzzy number, combined with the properties on algebraic operation, we will give a level cut-based computation method of Effectiveness Equilibrium Value in the following.

Theorem 3.1. *Let $A \in \mathbb{F}(R)$, if taking membership effect function as a tool for dealing with fuzzy preference, then $T(A)(x) = T(A(x))$, and suppose*

$$T(A)(x) = \begin{cases} 0, & x \notin [\underline{a}(0), \overline{a}(0)] \\ L(x), & x \in [\underline{a}(0), \underline{a}(1)] \\ 1, & x \in [\underline{a}(1), \overline{a}(1)] \\ R(x), & x \in [\overline{a}(1), \overline{a}(0)]. \end{cases} \tag{4}$$

If A is not a real number, then $T(A)$ is not a real number, and it satisfies: 1) $L(x)$ is strictly monotone increasing in $[\underline{a}(0), \underline{a}(1)]$; 2) $R(x)$ is strictly monotone decreasing in $[\overline{a}(1), \overline{a}(0)]$, then we have

$$E_T(A) = \frac{\int_0^1 \lambda L^{-1}(T^{-1}(\lambda))(L^{-1}(T^{-1}(\lambda)))' d\lambda - \int_0^1 \lambda R^{-1}(T^{-1}(\lambda))(R^{-1}(T^{-1}(\lambda)))' d\lambda + \frac{1}{2}[\underline{a}(1) + \overline{a}(1)]\delta(\overline{a}(1) - \underline{a}(1))}{\int_0^1 [R^{-1}(T^{-1}(\lambda)) - L^{-1}(T^{-1}(\lambda))] d\lambda},$$

where 1) $f'(x)$ is the derivative of $f(x)$ on x; 2) we suppose $f'(x) = 0$, if $f(x)$ is non derivative at point x; 3) $\delta(x) = x$ for $x > 0$, and $\delta(x) = 0$ for $x \leq 0$.

Proof: Using the properties of integrals, we know that

$$\int_{-\infty}^{+\infty} T(A(x))dx = \int_0^1 [R^{-1}(T^{-1}(\lambda)) - L^{-1}(T^{-1}(\lambda))] d\lambda.$$

In the following, we will prove that

$$\begin{aligned} \int_{-\infty}^{+\infty} xT(A(x))dx = & \int_0^1 \lambda L^{-1}(T^{-1}(\lambda))(L^{-1}(T^{-1}(\lambda)))' d\lambda - \int_0^1 \lambda R^{-1}(T^{-1}(\lambda))(R^{-1}(T^{-1}(\lambda)))' d\lambda \\ & + \frac{1}{2}[\underline{a}(1) + \overline{a}(1)]\delta(\overline{a}(1) - \underline{a}(1)). \end{aligned}$$

By the monotonicity of $L^{-1}(T^{-1}(\lambda))$ and $R^{-1}(T^{-1}(\lambda))$ on $[0, 1]$, we know that $L^{-1}(T^{-1}(\lambda))$ and $R^{-1}(T^{-1}(\lambda))$ are almost everywhere differentiable on $[0, 1]$. Suppose $T(A)(x)$ be continuous on $(-\infty, +\infty)$ (when $T(A)(x)$ has discontinuous point on $(-\infty, +\infty)$, we can prove the conclusions similarly combined with the properties of integra).

$$\begin{aligned} \int_{-\infty}^{+\infty} xT(A(x))dx = & \int_{-\infty}^{\underline{a}(0)} xT(A(x))dx + \int_{\underline{a}(0)}^{\underline{a}(1)} xT(A(x))dx + \int_{\underline{a}(1)}^{\overline{a}(1)} xT(A(x))dx \\ & + \int_{\overline{a}(1)}^{\overline{a}(0)} xT(A(x))dx + \int_{\underline{a}(0)}^{+\infty} xT(A(x))dx \\ = & \int_{\underline{a}(0)}^{\underline{a}(1)} xT(A(x))dx + \int_{\underline{a}(1)}^{\overline{a}(1)} xT(A(x))dx + \int_{\overline{a}(1)}^{\overline{a}(0)} xT(A(x))dx \\ = & \int_{\underline{a}(0)}^{\underline{a}(1)} xT(A(x))dx + \int_{\overline{a}(1)}^{\overline{a}(0)} xT(A(x))dx + \frac{1}{2}[\underline{a}(1) + \overline{a}(1)]\delta(\overline{a}(1) - \underline{a}(1)). \end{aligned}$$

We have, 1) when $L(x)$ is strictly monotone increasing in $[\underline{a}(0), \underline{a}(1)]$,

$\int_{\underline{a}(0)}^{\underline{a}(1)} xT(A(x))dx = \int_0^1 \lambda L^{-1}(T^{-1}(\lambda))dL^{-1}(T^{-1}(\lambda)) = \int_0^1 \lambda L^{-1}(T^{-1}(\lambda))[L^{-1}(T^{-1}(\lambda))]'d\lambda$;

2) when $R(x)$ is strictly monotone decreasing in $[\overline{a}(0), \overline{a}(1)]$,

$\int_{\overline{a}(1)}^{\overline{a}(0)} xT(A(x))dx = \int_0^1 \lambda R^{-1}(T^{-1}(\lambda))dR^{-1}(R^{-1}(\lambda)) = \int_0^1 \lambda R^{-1}(T^{-1}(\lambda))[R^{-1}(T^{-1}(\lambda))]'d\lambda$.

That is, when conditions 1) and 2) hold true, we have

$$\int_{-\infty}^{+\infty} xT(A(x))dx = \int_0^1 \lambda L^{-1}(T^{-1}(\lambda))(L^{-1}(T^{-1}(\lambda)))'d\lambda - \int_0^1 \lambda R^{-1}(T^{-1}(\lambda))(R^{-1}(T^{-1}(\lambda)))'d\lambda + \frac{1}{2}[\underline{a}(1) + \overline{a}(1)]\delta(\overline{a}(1) - \underline{a}(1)).$$

Especially, if $L(x) = 0$ for any $x \in (-\infty, \underline{a}(1))$, we have

$$\int_{\overline{a}(1)}^{\overline{a}(0)} xT(A(x))dx = -\int_0^1 \lambda L^{-1}(T^{-1}(\lambda))[L^{-1}(T^{-1}(\lambda))]'d\lambda = 0.$$

If $R(x) = 0$ for any $x \in (\overline{a}(1), +\infty)$, we have

$$\int_{\overline{a}(1)}^{\overline{a}(0)} xT(A(x))dx = -\int_0^1 \lambda R^{-1}(T^{-1}(\lambda))[R^{-1}(T^{-1}(\lambda))]'d\lambda = 0.$$

It is easy to know that, not only trapezoidal fuzzy number and triangular fuzzy number satisfy the conditions of Theorem 3.1, but also their algebraic operation still satisfies the conditions of Theorem 3.1. Therefore, Theorem 3.1 provides us a method of computing Effectiveness Equilibrium Value on algebraic operation of trapezoidal fuzzy number and triangular fuzzy number. Especially, we have the following conclusions.

Remark 3.1. *Theorem 3.1 gives a level cut-based method of computing Effectiveness Equilibrium Value. In Theorem 3.1, membership effect function is required to be strictly monotone and has inverse function. In reality, membership effect function is often regarded as a tool for dealing with different fuzzy preference. Therefore, level cut-based Effectiveness Equilibrium Value model is very practical and important.*

The commonly used membership effect functions which satisfy the required conditions include: $T_1(t) = t^\alpha$, $\alpha > 0$, $T_2(t) = 1 - (1 - t)^{\alpha+1}$, $\alpha = 0, 1, 2, \cdots$.

Corollary 3.1. *Let* $A = (a, [b_1, b_2], c)$, $T(t) = t^\alpha$, *then we have: If* $T(t) = t$, *then* $E_T(A) = \frac{a^2+ab_1+b_1^2-b_2^2-b_2c-c^2}{3(a+b_1-b_2-c)}$; *If* $T(t) = t^{1/2}$, *then* $E_T(A) = \frac{8a^2+4ab_1+3b_1^2-3b_2^2-4b_2c-8c^2}{10(2a+b_1-b_2-2c)}$; *If* $T(t) = t^2$, *then* $E_T(A) = \frac{a^2+2ab_1+3b_1^2-3b_2^2-2b_2c-c^2}{4(a+2b_1-2b_2-c)}$; *Especially, if* $b_1 = b_2 = b$, *then the above conclusions can be,* $E_T(A) = \frac{a^2+ab-bc-c^2}{3(a-c)}$ *for* $T(t) = t$; $E_T(A) = \frac{2a^2+ab-bc-2c^2}{5(a-c)}$ *for* $T(t) = t^{1/2}$; $E_T(A) = \frac{a^2+2ab-2bc-c^2}{4(a-c)}$ *for* $T(t) = t^2$.

Corollary 3.2. *Let* $A = (a, [b_1, b_2], c)$, $T(t) = 1 - (1 - t)^{\alpha+1}$, *and then we have: If* $\alpha = 0$, *then* $E_T(A) = \frac{a^2+ab_1+b_1^2-b_2^2-b_2c-c^2}{3(a+b_1-b_2-c)}$; *If* $\alpha = 1$, *then* $E_T(A) = \frac{3a^2+2ab_1+b_1^2-b_2^2-2b_2c-3c^2}{4(2a+b_1-b_2-2c)}$; *If* $\alpha = 5$, *then* $E_T(A) = \frac{21a^2+6ab_1+b_1^2-b_2^2-6b_2c-21c^2}{8(6a+b_1-b_2-6c)}$; *Especially, if* $b_1 = b_2 = b$, *then the above conclusions can be,* $E_T(A) = \frac{a^2+ab-bc-c^2}{3(a-c)}$ *for* $\alpha = 0$; $E_T(A) = \frac{3a^2+2ab-2bc-3c^2}{8(a-c)}$ *for* $\alpha = 1$; $E_T(A) = \frac{7a^2+2ab-2bc-7c^2}{16(a-c)}$ *for* $\alpha = 5$.

From the above corollaries, we can see that the product of fuzzy numbers varies with membership effect function.

4. **An Effectiveness Equilibrium Value-Based Solution Method for Fuzzy Programming Problems.** Based on the analysis of Section 3, level cut-based Effectiveness Equilibrium Value model can well embody the algebraic operation of fuzzy number. Since trapezoidal fuzzy number and triangular fuzzy number are usually used to represent the uncertainty in reality, also their algebraic operations have special properties (see Theorem 3.1). Next, we will further discuss the features of our model through a non-linear programming problem with trapezoidal and triangular fuzzy coefficients.

4.1. **Fuzzy programming problems.** In this paper we will consider the following linear or nonlinear programming problems in which the elements of the coefficient matrix and the rest of the parameters were represented by trapezoidal fuzzy numbers and triangular fuzzy numbers. This problem can be represented with the following model:

$$\begin{cases} \max \widetilde{Z} \approx \widetilde{C}\widetilde{X}, \\ \text{s.t. } \widetilde{A}\widetilde{X} \lessapprox \widetilde{B}, \\ \quad\ \widetilde{X} \gtrapprox \widetilde{0}, \end{cases} \tag{5}$$

where, $\widetilde{C} \in \widetilde{R}^n$, $\widetilde{B} \in \widetilde{R}^m$ and $\widetilde{A} \in \widetilde{R}^{m\times n}$ are given, and $X \in R^n$ is to be determined.

Because the fuzzy numbers do not total order relationship like real numbers, the above model is just a formal model, and cannot be easily solved.

According to the discussions of Section 3, we define a rank for each trapezoidal fuzzy number and triangular fuzzy numbers for comparison purposes. To do this, we substitute the rank order of each fuzzy number for the corresponding rank on Effectiveness Equilibrium Value of fuzzy number in the fuzzy problem under consideration. This leads to an equivalent crisp programming problem as the following model (6), which can be solved with a standard method.

$$\begin{cases} \max E_T\left(\widetilde{Z}\right) = E_T\left(\widetilde{C}\widetilde{X}\right), \\ \text{s.t. } E_T\left(\widetilde{A}\widetilde{X}\right) \le E_T\left(\widetilde{B}\right), \\ \quad\ E_T\left(\widetilde{X}\right) \ge 0. \end{cases} \tag{6}$$

In (5), all algebraic operations in the fuzzy method are performed on the fuzzy numbers. However, all algebraic operations are done on the crisp numbers in our proposed method (6). As a result, the computational effort is decreased significantly.

4.2. **Example analysis.** In the following, we illustrate the proposed method by a numerical example. Consider the following fuzzy programming problem:

$$\begin{cases} \max f(x_1,x_2) = -(0.1,0.3,0.8)x_1^2 - (0.2,0.4,0.7)x_2^2 + (16.1,17,17.3)x_1 + (17.7,18,18.6)x_2, \\ \text{s.t. } (1.4,[2,2.3],2.6)x_1 + (2.7,[2.9,3],3.3)x_2 \lessapprox (47,[49,50],51), \\ \quad (3.8,[3.9,4],4.4)x_1 + (1.6,[2,2],2.2)x_2 \lessapprox (40,[42,44],47), \\ \quad (2.6,[2.7,3],3.2)x_1 + (1.6,[2,2],2.2)x_2 \cong (32,[36,36],40), \\ \quad x_1,x_2 \ge 0. \end{cases} \tag{7}$$

From Corollary 3.1, membership effect-based Effectiveness Equilibrium Value does not meet additivity for trapezoidal fuzzy number, and the sum of two fuzzy numbers is still a fuzzy number. Therefore, we can convert the above model into the following one.

$$\begin{cases} \max f(x_1,x_2) = -(0.1,0.3,0.8)x_1^2 - (0.2,0.4,0.7)x_2^2 + (16.1,17,17.3)x_1 + (17.7,18,18.6)x_2, \\ \text{s.t. } 1.4x_1+2.7x_2,[2x_1+2.9x_2,2.3x_1+3x_2],2.6x_1+3.3x_2 \lessapprox (47,[49,50],51), \\ \quad 3.8x_1+1.6x_2,[3.9x_1+2x_2,4x_1+2x_2],4.4x_1+2.2x_2 \lessapprox (40,[42,44],47), \\ \quad 2.6x_1+1.6x_2,[2.7x_1+2x_2,3x_1+2x_2],3.2x_1+2.2x_2 \cong (32,[36,36],40), \\ \quad x_1,x_2 \ge 0. \end{cases} \tag{8}$$

If using membership effect-based Effectiveness Equilibrium Value to solve the above problem, the computation is extremely complicated. In the following, we will apply the general formulas with different membership effect function obtained in Section 3 to the

Table 1. Different optimization results with different membership effect function

Membership effect function			x_1	x_2	f
$T_1(t) = t^\alpha$	1	$\alpha = 0.5$	5.0486	11.1379	221.1443
	2	$\alpha = 1$	5.0789	11.0262	221.9022
	3	$\alpha = 2$	5.1277	10.8757	222.8499
$T_2(t) = 1 - (1-t)^{\alpha+1}$	4	$\alpha = 0$	5.0789	11.0262	221.9022
	5	$\alpha = 1$	5.0622	11.0999	221.4914
	6	$\alpha = 5$	5.0375	11.2000	220.7695

solving process, which will greatly reduce the time and complexity. The results are listed in Table 1.

All the above results are obtained with Matlab 7.0 by computer. The theories analysis and experiment results indicate that, 1) level cut-based Effectiveness Equilibrium Value model has good interpretability and operability; 2) by using the obtained general formulas, it can greatly reduce the computation time and complexity by computer; 3) through level cut-based Effectiveness Equilibrium Value model, we can get much reasonable optimization results; it can overcome the difficulty in computation with membership effect-based Effectiveness Equilibrium Value model, and plays useful practical role for the algebraic operations of fuzzy numbers.

Remark 4.1. *Here we have considered an example with real decision variables for the sake of clarity and simplicity of exposition. If decision variables are fuzzy numbers in the problem at hand, we also can apply the strategy described above.*

5. **Conclusions.** In this paper, for the ranking and algebraic operation of fuzzy numbers, we firstly analyze the features and shortcomings of the existing methods, then establish level cut-based Effectiveness Equilibrium Value model, and give the general operation formulas for trapezoidal fuzzy number and triangular fuzzy number; we secondly discuss the applications in optimization problems combined with a concrete example. At the same time, we solve the nonadditivity of membership effect-based Effectiveness Equilibrium Value model, and also we verify the operability of our model through different membership effect function. The theories analysis and experiment results indicate that level cut-based Effectiveness Equilibrium Value model not only has good interpretability and application background, but also can solve the algebraic operation of fuzzy numbers. Therefore, the discussions in this paper can enrich the existing fuzzy metric theories to a certain degree, and can be widely applied in many fields such as resources allocation, fuzzy decision-making, fuzzy optimization. It can provide significant insights for constructing fuzzy optimization methods under complex environments.

In our further study, we will discuss the ranking methods of fuzzy numbers with any form under decision preference and also apply them to fuzzy programming for complex systems optimization.

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