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SINGLE TRANSMIT ANTENNA SELECTION IN FLAT AND SLOW RAYLEIGH FADING CHANNEL

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Abstract. *Single transmit antenna selection is the simplest selection method of antenna selection. In this paper, its performance, through its basic mathematical model and algorithm, is analysed to explore its feasibility in flat and slow Rayleigh fading channel. Through Monte Carlo simulation using MATLAB, its performance can be compared with random antenna selection results. To conclude, the validity, effectiveness and efficiency of single transmit antenna selection, over the random selection, will be assessed. Its applicability, as well as implementation, is also addressed.*
Keywords: Antenna selection, Transmit antenna, MIMO, Rayleigh fading

1. **Introduction.** Multiple-input and multiple-output (MIMO) systems were proposed in 1980s [1] and have become a frequent research topic with indepth investigation [2]. The main advantage of MIMO systems is the exploitation of spatial diversity when multiple channels, among the transmit and receive antennas, are built [3, 4]. However, as addressed in literature [5, 6, 7], RF chain is relatively expensive compared with antenna elements and other processing devices. The price of RF chain does not follow Moore's law [3]. This indicates that whilst a large number of transmit antennas can be used to provide diversity, and thus improve the reliability of wireless communication, there is still a requirement to reduce the number of transmit antennas. The most common method, by which this is possible, is via implementing transmit antenna selection. In its simplest form, transmit antenna selection is a method used to select a set of transmit antennas by a series of criteria and only allows signals to be transmitted by these selected transmit antennas. There are a large number of selection methods, which include single antenna selection,

bulk selection, per-tone selection and joint selection [8, 9, 10]. Considering the system complexity of single transmit antenna selection is low and the performance is acceptable [11], its details are analysed in this paper. Our analysis is supported through ease of implementation by the current progresses on hardware and software in industry [12]. Moreover, the obtained results in this paper can be extended to other relevant fields to improve the system performance [13, 14]. The main contribution of this paper is to, analytically, compare the performances produced by single transmit antenna selection and random selection in flat and slow Rayleigh fading channel. The effects of the numbers of transmit and receive antennas on the error performance are also analyzed in this paper.

The rest of this paper is organized as follows. In Section 2, we first construct the models of a typical MIMO system. The mathematical derivation and analysis of the performance of single transmit antenna selection are then presented in Section 3. Based on the mathematical analysis, a series of simulations are carried out and the simulation results are illustrated and explained in Section 4. The paper is concluded in Section 5.

2. System Model.

2.1. Transmission, propagation and reception of wireless signals.

In general, a MIMO system consists of N_T transmit antennas at the transmitter and N_R receive antennas at the receiver. There are several special cases of a MIMO system. If $N_T = N_R = 1$, then the system is called single-input, single-output (SISO) system; If $N_T = 1$ and $N_R \geq 2$, then the system is termed single-input, multiple-output (SIMO) system and multiple-input, single-output (MISO) system vice versa [15]. A generic and simplified model of MIMO system can be shown in Figure 1. From this figure, it is clear that for a typical MIMO system with N_T transmit antennas and N_R receive antennas, $N_R \times N_T$ channels will be constructed. To model such a system, an $N_R \times N_T$ channel matrix would be useful [16]. Assume $h_{ij}(\tau; t)$ is the equivalent baseband channel impulse response between ith receive antenna and jth transmit antenna, where $i \in \{1, 2, \ldots, N_R\}$, $j \in \{1, 2, \ldots, N_T\}$, τ represents the propagation delay and t is the time variable. The time-varying channel can be characterized by channel matrix

$$\mathbf{H}(\tau; t) = \begin{bmatrix} h_{11}(\tau;t) & h_{12}(\tau;t) & \ldots & h_{1N_T}(\tau;t) \\ h_{21}(\tau;t) & h_{22}(\tau;t) & \ldots & h_{2N_T}(\tau;t) \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R1}(\tau;t) & h_{N_R2}(\tau;t) & \ldots & h_{N_RN_T}(\tau;t) \end{bmatrix}. \tag{1}$$

If the fading channel between the transmitter and the receiver is flat, i.e. all frequency components in the transmitted signal experience the same attenuation and phase shift, the channel matrix $\mathbf{H}(\tau; t)$ can be reduced to [15]

$$\mathbf{H}(0; t) \equiv \mathbf{H}(t). \tag{2}$$

Moreover, if the fading channel varies slowly in comparison to the symbol duration T of the transmitted signal, (2) can be further reduced to [15]

$$\mathbf{H}(0) \equiv \mathbf{H}. \tag{3}$$

It is clear that $\mathbf{H}$ is a random, but quasi-static channel matrix used to model the network with flat and slow fading channels.

Therefore, for each received signal at a single receive antenna, we have [15]

$$r_i(t) = \sum_{j=1}^{N_T} h_{ij} s_j g(t) + z_i(t), \qquad 0 \leq t \leq T \tag{4}$$

where $\{h_{ij}\}$ represents the i.i.d. channel gain between the jth transmit antenna and the ith receive antenna; $\{s_j\}$ represents the information symbol transmitted by the jth

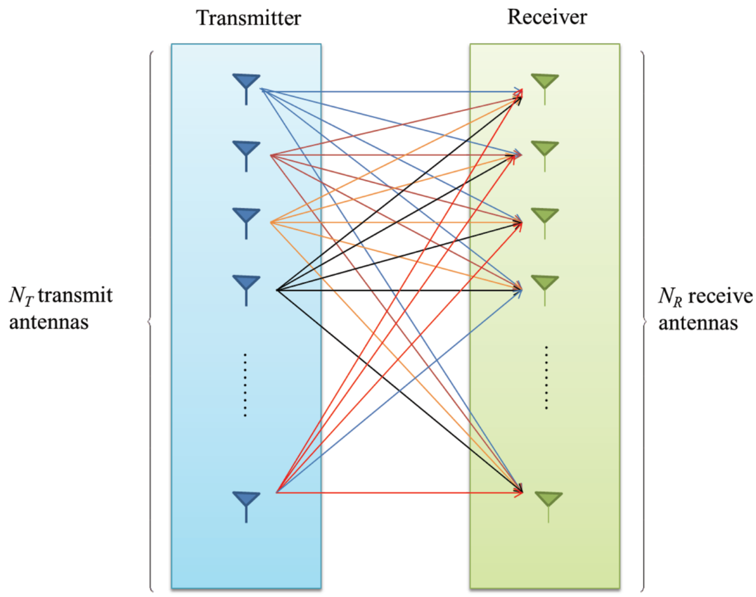

FIGURE 1. A generic and simplified model of MIMO system

transmit antenna; $g(t)$ is the basic pulse adopted for modulation; $\{z_i(t)\}$ represents the i.i.d. AWGN term for the ith receive antenna.

In addition, the basic pulse $g(t)$ is designed and adopted by the modulation scheme and, thus, is known at both transmitting and receiving ends. Therefore, by employing a matched filter with impulse response $g^*(T-t)$ at the receive antenna, the optimal detection output of the ith receive antenna can be expressed by [15]

$$\begin{aligned} y_i(t) &= \left[\int_0^T r_i(t)g^*[T-(\tau-t)]\mathrm{d}t\right]_{\tau=T} \\ &= \int_0^T \left[\sum_{j=1}^{N_T} h_{ij}s_jg(t)+z_i(t)\right] g^*(t)\mathrm{d}t \\ &= \sum_{j=1}^{N_T} h_{ij}s_j\left[\int_0^T g(t)g^*(t)\mathrm{d}t\right] + \int_0^T z_i(t)g^*(t)\mathrm{d}t. \end{aligned} \tag{5}$$

As the term $\int_0^T g(t)g^*(t)\mathrm{d}t$ is designed artificially and, also, fixed, without loss of generality, it can be assumed that $\int_0^T g(t)g^*(t)\mathrm{d}t = 1$. Furthermore, the term $\int_0^T z_i(t)g^*(t)\mathrm{d}t$ is the processed noise term and is still random and Gaussian distributed [17], which could be denoted as n_i. As a result, (5) can be rewritten as

$$y_i(t) \equiv y_i = \sum_{j=1}^{N_T} h_{ij}s_j + n_i. \tag{6}$$

From the viewpoint of matrix, a neat form of (6) can be given for all received signals at these N_R receive antennas

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{7}$$

where $\mathbf{y} = [y_1, y_2, \ldots, y_{N_R}]^T$; $\mathbf{s} = [s_1, s_2, \ldots, s_{N_T}]^T$; $\mathbf{n} = [n_1, n_2, \ldots, n_{N_R}]^T$; superscript $(\cdot)^T$ represents the transpose of a matrix.

2.2. **Single transmit antenna selection strategy.** As can be seen from the modelling procedure presented in the previous section, this MIMO system is inefficient as N_T RF chains are involved. Alternatively, we can select a 'best' transmit antenna from those N_T transmit antennas according to specific criteria and, thus, the required RF chains can be reduced from N_T to 1. This is the basic idea of single transmit antenna selection. Although it is a trade-off between performance and implementation cost, it can be deemed necessary when the number of RF chains is strictly limited.

In this paper, we select the 'best' antenna according to the optimization objective [18]

$$m_t = \arg \max_{j\in\{1,2,\ldots,N_T\}} \sum_{i=1}^{N_R} |h_{ij}|^2. \tag{8}$$

3. **Performance Analysis.** In Rayleigh fading channel, the probability density function (PDF) of the normalized independent and identically distributed (i.i.d.) $|h_{ij}|$ is

$$f_h(h) = \begin{cases} he^{-h^2/2} & h > 0 \\ 0 & \text{otherwise} \end{cases}. \tag{9}$$

It can then be proved that $\sum_{i=1}^{N_R} |h_{ij}|^2$ is gamma distributed with parameters N_R and 2, denoted by $\Gamma(N_R, 2)$ [19]. Therefore, the PDF of $\sum_{i=1}^{N_R} |h_{ij}|^2$ can be expressed by

$$f_{\Sigma h}(x) = \begin{cases} \frac{x^{N_R-1}e^{-\frac{x}{2}}}{2^{N_R}\Gamma(N_R)} & x > 0 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where $\Gamma(\cdot)$ is the gamma function.

Therefore, if a transmit antenna is randomly selected without following the optimization objective given in (8), the average $\sum_{i=1}^{N_R} |h_{ij}|^2$ can be determined by

$$\bar{X} = E\left[\sum_{i=1}^{N_R} |h_{ij}|^2\right] = \int_0^{+\infty} x f_{\Sigma h}(x)\mathrm{d}x = 2N_R. \tag{11}$$

Considering the single transmit antenna selection, we should first obtain the PDF of $\max_{j\in\{1,2,\ldots,N_T\}} \sum_{i=1}^{N_R} |h_{ij}|^2$. For i.i.d. $\{h_{ij}\}$, by applying simple statistic theories on (10), we can determine the PDF of $\max_{j\in\{1,2,\ldots,N_T\}} \sum_{i=1}^{N_R} |h_{ij}|^2$ by [20]

$$f_{\max\Sigma h}(y) = \begin{cases} N_T \left[\frac{\gamma(N_R, \frac{y}{2})}{\Gamma(N_R)}\right]^{N_T-1} \frac{y^{N_R-1}e^{-\frac{y}{2}}}{2^{N_R}\Gamma(N_R)} & y > 0 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

where $\gamma(\cdot,\cdot)$ is the lower, incomplete gamma function.

The average $\max_{j\in\{1,2,\ldots,N_T\}} \sum_{i=1}^{N_R} |h_{ij}|^2$ can be determined by

$$\begin{aligned} \bar{Y} &= E\left[\max_{j\in\{1,2,\ldots,N_T\}} \sum_{i=1}^{N_R} |h_{ij}|^2\right] = \int_0^{+\infty} y f_{\max\Sigma h}(y)\mathrm{d}y \\ &= \frac{N_T}{2^{N_R}\Gamma(N_R)} \sum_{l=0}^{N_T-1}\sum_{k=0}^{N_R-1} \frac{(-1)^l(N_T-1)!}{l!(N_T-1-l)!2^k k!} \\ &\quad \times \left(\frac{1}{2}+\frac{1}{2l}\right)^{-\frac{N_R+kl+l}{l}} \Gamma\left(1+k+\frac{N_R}{l}\right) \end{aligned} \tag{13}$$

where $\Gamma(\cdot)$ is the Gamma function.

Further assuming $\mathcal{E}_b$ is the energy per bit, the average normalized signal-to-noise power ratio for both cases can be expressed as

$$\mathrm{SNR}_{rs} = 2\bar{X}\mathcal{E}_b \tag{14}$$

and

$$\mathrm{SNR}_{sas} = 2\bar{Y}\mathcal{E}_b. \tag{15}$$

Assuming that the transmitter is equiprobable and the antipodal PAM is employed for modulation, the error probability of both cases can be determined by

$$P_{e-rs} = \int_{\sqrt{2\bar{X}\mathcal{E}_b}}^{\infty} \frac{1}{\sqrt{\pi}} e^{-n^2} \mathrm{d}n = Q\left(\sqrt{2\bar{\mathrm{X}}\mathcal{E}_{\mathrm{b}}}\right) \tag{16}$$

and

$$P_{e-sas} = \int_{\sqrt{2\bar{Y}\mathcal{E}_b}}^{\infty} \frac{1}{\sqrt{\pi}} e^{-n^2} \mathrm{d}n = Q\left(\sqrt{2\bar{\mathrm{Y}}\mathcal{E}_{\mathrm{b}}}\right) \tag{17}$$

where $Q(\cdot)$ is the tail probability of the standard normal distribution.

4. **Simulation and Analysis.** By Monte Carlo method, manipulate $(N_T, N_R) = (4, 4)$, $(N_T, N_R) = (16, 4)$, $(N_T, N_R) = (4, 16)$ and $(N_T, N_R) = (16, 16)$. The simulation results are presented in Figure 2, Figure 3, Figure 4, Figure 5 respectively. From these figures, the results can be summarized. Firstly, as long as $N_T N_R \neq 1$, single transmit antenna selection always outperforms random selection. In one sense, an increase in N_T will only benefit the case with the single transmit antenna selection. An increase in N_R will result in a better performance of both selection strategies. However, the improvement efficiencies of N_T and N_R are not symmetrical. In general, the improvement of an increase in N_R is more significant than the improvement of an increase in N_T by the same level. The diversity gain is more significant than the coding gain in terms of the improvement of system reliability. Therefore, if the total number of antennas is specified and bounded, in order to obtain a better performance, they should be placed at the receiving end.

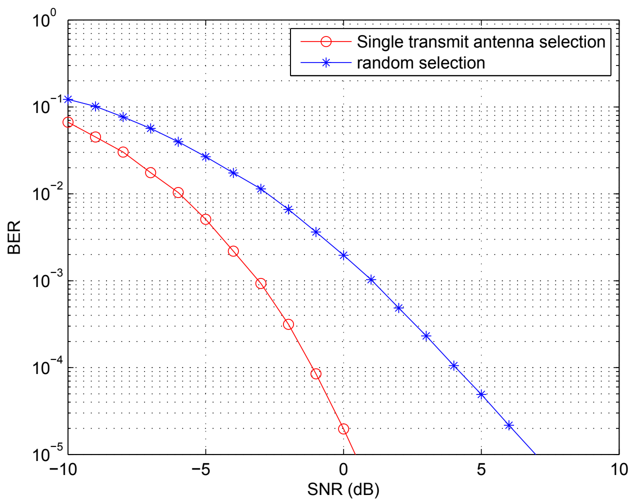

FIGURE 2. System performance, given $(N_T, N_R) = (4, 4)$

5. **Conclusion.** In conclusion, the feasibility of single transmit antenna selection in flat and slow Rayleigh fading channel has been analysed. With the mathematical derivation and simulations by Monte Calro method, the superiority of single transmit antenna selection over random selection has been illustrated and validated. The influencing mechanisms of both N_T and N_R have also been illustrated. Suggestions for implementation, regarding single transmit antenna selection, have also been summarized from the simulation results. Future work will focus on the comparison of alternative antenna selection methods and the implementation of single transmit antenna selection.

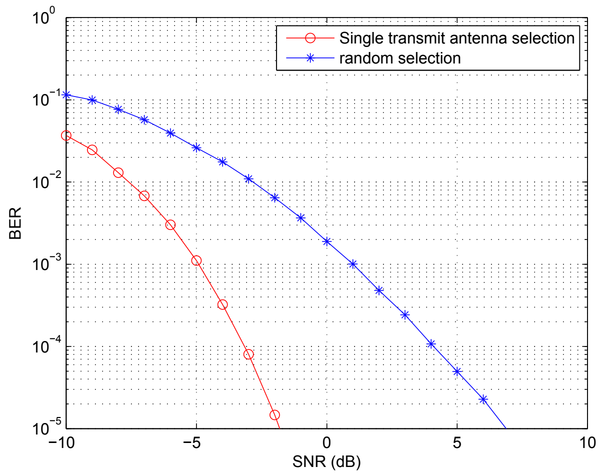

FIGURE 3. System performance, given $(N_T, N_R) = (16, 4)$

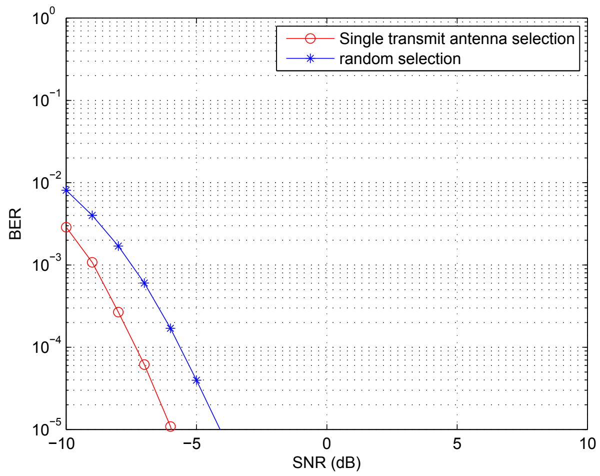

FIGURE 4. System performance, given $(N_T, N_R) = (4, 16)$

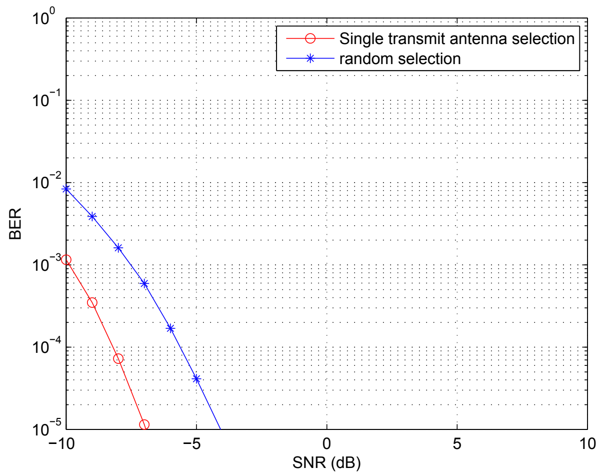

FIGURE 5. System performance, given $(N_T, N_R) = (16, 16)$

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