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EMBEDDING OF STAR NETWORKS INTO EXCHANGED HYPERCUBES

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Abstract. *As an important variant of hypercube, the exchanged hypercube $EH(s,t)$ not only kept numerous desirable properties of the hypercube, but also reduced the interconnection complexity. In this paper, we analyze important properties related to embedding star networks S_n into $EH(s,t)$ network. The main results are: (1) S_n can be embedded into $EH(s,t)$ with $2 \leq$ expansion < 4, dilation $= N + 2$, congestion $= 1$, load $= 1$ where $s + t = \lceil \log_2(n!) \rceil = N$; (2) S_{4m} can be embedded into $EH(s,t)$ with dilation $= 2d + 6$, where $m > 1$, $d = \lceil \log_2(m!) \rceil$, $s + t + 1 = 4d + 8m - 3$; (3) $S_{2^i m}$ can be embedded into $EH(s,t)$ with dilation $= 2d + 2i + 2$ where $m > 1$, $i \geq 1$, and $d = \lceil \log_2(m!) \rceil$, $s + t + 1 = 2^i d + i2^i m - 2i + 1$.*

Keywords: Star networks, Exchanged hypercube, Dilation, Embedding

1. **Introduction.** It is well known that the versatility is one of the important properties of any general interconnection network model. It refers to the ability of a network model simulating another network model. The nature of the problem is the network embedding, which is also a challenging topic in graph theory. If a network model can be embedded into another network model, it means the latter can simulate the former, and the algorithms originally developed for the former model can be mapped to the latter. Furthermore, one network has better versatility if it can embed more other networks. On the other side, embeddings allow the new architecture to simulate the old one. High efficient embeddings can improve the performance of parallel algorithms. Thus, the embedding of network model is an important issue in parallel computing [1-9]. As one of important applications of a parallel computer network model, hypercube has aroused the interests of many experts and scholars, and then has got a lot of valuable results [10-12]. However, hypercube scales too rapidly as n increases. Existing research has proposed some networks that are variations of the hypercube network variants such as folded hypercube, twisted cube network, cross cube network and so on. The exchanged hypercube [13] $EH(s,t)$ is one of a new variant of the hypercube. It is not only to keep the several desirable properties of the hypercube but also reduces the complexity of the network interconnection. Some research showed $EH(s,t)$ has nice recursiveness and preferable network parameters [14-16]. In particular, the embedding issues of $EH(s,t)$ have become the focus of the researchers for the purpose of improving the versatility. It has already been shown that hypercubes, cycles, and E-2D Mesh networks can be embedded into $EH(s,t)$ [1,17,18]. Since star network represents the communication structures of many applications in scientific computations as well as the topologies of many large-scale interconnection networks [19-21], the star network has such excellent properties as good regularity, symmetry, small diameter, high

reliability, maximally fault tolerant, and it has a lot of mature and effective algorithms. However, the embedding for star networks on $EH(s,t)$ was not well studied, although star networks and $EH(s,t)$ have been widely studied and used in parallel processing and distributed systems. In this paper, we analyze embedding properties of the star networks into $EH(s,t)$ and derive a number of results. Thus, we can find the efficient method for embedding the star networks on $EH(s,t)$, moreover, most mature parallel algorithms for the star networks can be efficiently simulated on the $EH(s,t)$.

The rest of this paper is organized as follows: in the next section, some fundamental definitions and notions are introduced, and main results are proved in Section 3. The last section contains discussions and conclusions.

2. **Preliminaries.** We need some previous definitions and notations concerning interconnection networks and mapping relationship.

Definition 2.1. [20]: *Star network is defined as an undirected graph:* $S_m = (V, E)$*,* V *is the set of vertices and* $V = \{(p_1, p_2, \ldots, p_m) | p_i \in <m>, p_i \neq p_j \text{ for } i \neq j\}$*;* E *is the set of edges and* $E = \{((p_1, p_2, \ldots, p_i, \ldots, p_m)(p_i, p_2, \ldots, p_1, \ldots, p_m)) | (p_1, p_2, \ldots, p_i, \ldots, p_m) \in V$ *and* $2 \leq i \leq m\}$*, where* $<m> = \{1, 2, \ldots, m\}$*, and* p *is a permutation of* $<m>$*, that is* $p = (p_1, p_2, \ldots, p_i, \ldots, p_m)$*,* $p_i \in <m>$*.*

In other words, the set S of all permutations constitutes the nodes or the vertices of star network. Two vertices u and v are joined by an edge if and only if $u = v(1, j)$ $(j \geq 2)$, where $(1, j)$ refers to the transposition by interchanging the first and the jth elements of permutation p.

Definition 2.2. [13]: *The exchanged hypercube is defined as an undirected graph:* $EH(s,t) = (V, E)$ $(s \geq 1, t \geq 1)$*,* V *is the set of vertices and* $V = \{a_{s-1} \ldots a_0\, b_{t-1} \ldots b_0 c | a_i, b_j, c \in (0,1), i \in [0, s), j \in [0, t)\}$*; and* E *is the set of edges: (1)* $(v_1, v_2) \in V \times V$ *if* $v_1 \oplus v_2 = 1$*; (2)* $v_1[s+t, t+1] = v_2[s+t, t+1]$*,* $H(v_1[t, 1], v_2[t, 1]) = 1$ *if* $v_1[0] = v_2[0] = 1$*; (3)* $v_1[t, 1] = v_2[t, 1]$*,* $H(v_1[s+t, t+1], v_2[s+t, t+1]) = 1$ *if* $v_1[0] = v_2[0] = 0$*, where* $\oplus$ *denotes the exclusive-OR operator,* $v[x, y]$ *denotes the bit pattern of* v *between dimensions* x *and* y *inclusive.* $H(v_1, v_2)$ *denotes the Hamming distance between vertices* v_1 *and* v_2*, where* $(v_1, v_2) \in V \times V$*.*

Figure 1 shows an exchanged hypercube when $s = 1$, $t = 2$.

Definition 2.3. [21]: *Let* G *and* H *be two undirected graphs. If there exists a mapping pair* $<\varphi, \phi>$*, so that for* $\forall u \in G$*,* $\varphi(u) \in H$*,* $\forall (u, v) \in E(G)$*, there is a mapping* ϕ *of* (u, v) *into the paths* $\phi((u, v))$ *of conjoint* $\varphi(u)$ *and* $\varphi(v)$ *in* H*, and* $|G| \leq |H|$*, then* $<\varphi, \phi>$ *is called an embedded mapping pair of* G *into* H*, that is* G *can be embedded into* H*.*

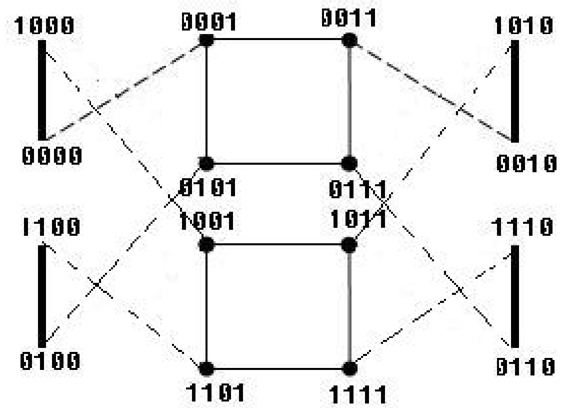

FIGURE 1. An exchanged hypercube, $EH(1, 2)$

Our objective is to develop simulations with small communication delay (measured by the dilation of the embedding) and good processor utilization(measured by the expansion of the embedding) of the star network by $EH(s,t)$.

3. **Embedding of Star Networks.** In this section, we analyze some properties on embedding of star networks into $EH(s,t)$.

Lemma 3.1. *(see [20]) In any embedding G into H with dilation 1, the degree of G should be less than or equal to the degree of H.*

Theorem 3.1. *There is no embedding existing for S_n into $EH(s,t)$ with dilation 1, when* $\max(s,t) < n-2$, $(s+t+1 = \lceil \log_2(n!) \rceil)$.

Proof: Obviously, the degree of S_n is $n-1$. We discuss the degree of $EH(s,t)$ in two cases. (1) When $s \geq t$, the degree of $EH(s,t)$ is $s+1$, according to Lemma 3.1, thus $n-1 \leq s+1 \Rightarrow s > n-2$, and this result is in conflict with known conditions $t \leq s \leq n-2$, so there is no embedding existing for S_n into $EH(s,t)$ with dilation 1 when $t \leq s \leq n-2$. (2) When $t \geq s$, the degree of $EH(s,t)$ is $t+1$, according to Lemma 3.1, thus $n-1 \leq t+1 \Rightarrow t > n-2$, and this result is in conflict with known conditions $s \leq t \leq n-2$, so there is no embedding existing for S_n into $EH(s,t)$ with dilation 1 when $s \leq t \leq n-2$. From the above, there is no embedding existing for S_n into $EH(s,t)$ with dilation 1 when $\max(s,t) < n-2$, $(s+t+1 = \lceil \log_2(n!) \rceil)$.

Theorem 3.2. *For S_n and $EH(s,t)$, where $s+t = \lceil \log_2(n!) \rceil = N$, there is an embedding mapping pair $< \varphi, \phi >$ existing for $2 \leq$ expansion < 4, dilation $= N+2$, congestion $=$ 1, load $=$ 1.*

Proof: At first we prove $2 \leq$ expansion < 4. Since expansion $= |EH(s,t)| \,/\, |S_n| = 2^{s+t+1}/n!$, and $s+t = \lceil \log_2(n!) \rceil$, thus, $n! \cdot 2 \leq 2^{s+t+1} < n! \cdot 4$, and then $2 \leq$ expansion < 4. Obviously load $= 1$, next we prove dilation $= N+2$ in detail. Let $\varphi : V(S_n) \rightarrow V(EH(s,t))$, for $\forall u, v \in V(S_n)$, $e = (u,v)$, $\varphi(u) = a_{s-1}a_{s-2}\cdots a_1a_0b_{t-1}b_{t-2}\cdots b_1b_0c$, $\varphi(v) = a'_{s-1}a'_{s-2}\cdots a'_1a'_0b'_{t-1}b'_{t-2}\cdots b'_1b'_0c'$, and there exists an injection ϕ, such that $\phi(e)$ is a shortest path from $\varphi(u)$ to $\varphi(v)$ in $EH(s,t)$. There are four cases.

(1) When $c = c' = 0$, by the definition of $EH(s,t)$, we know $\phi(e)$ is a shortest path from $\varphi(u)$ to $\varphi(v)$ that must pass though $p = a'_{s-1}a'_{s-2}\cdots a'_1a'_0b_{t-1}b_{t-2}\cdots b_1b_00$, $p' = a'_{s-1}a'_{s-2}\cdots a'_1a'_0b_{t-1}b_{t-2}\cdots b_1b_01$, $q' = a'_{s-1}a'_{s-2}\cdots a'_1a'_0b'_{t-1}b'_{t-2}\cdots b'_1b'_01$, so the path of $\phi(e)$ denotes $\varphi(u) \rightarrow p \rightarrow p' \rightarrow q \rightarrow \varphi(v)$, and then we can obtain dilation $= s+t+1+1 = N+2$.

(2) When $c = 0$, $c' = 1$, by Definition 2.2, we know $\phi(e)$ is a shortest path from $\varphi(u)$ to $\varphi(v)$ must pass though $p = a'_{s-1}a'_{s-2}\cdots a'_1a'_0b_{t-1}b_{t-2}\cdots b_1b_00$, $p' = a'_{s-1}a'_{s-2}\cdots a'_1a'_0b_{t-1}$ $b_{t-2}\cdots b_1b_01$, that is the path of $\phi(e)$ denotes $\varphi(u) \rightarrow p \rightarrow p' \rightarrow \varphi(v)$. Hence dilation $= s+t+1 = N$.

(3) When $c = 1$, $c' = 0$, from the above analyses of (2), hence dilation $= N$.

(4) When $c = c' = 1$, according to Definition 2.2, we know $\phi(e)$ is a shortest path from $\varphi(u)$ to $\varphi(v)$ that must pass though $r = a_{s-1}a_{s-2}\cdots a_1a_0b'_{t-1}b'_{t-2}\cdots b'_1b'_01$, $r' = a_{s-1}a_{s-2}\cdots a_1a_0$ $b'_{t-1}b'_{t-2}\cdots b'_1b'_00$, $m = a'_{s-1}a'_{s-2}\cdots a'_1a'_0b'_{t-1}b'_{t-2}\cdots b'_1b'_00$, that is the path of $\phi(e)$ denoting $\varphi(u) \rightarrow r \rightarrow r' \rightarrow m \rightarrow \varphi(v)$, hence dilation $= s+t+1+1 = N+2$. From the above, S_n can be embedded into $EH(s,t)$ with expansion $= 2$, dilation $= N+2$, congestion $= 1$, load $= 1$.

In order to reduce the congestion of embedding, we decompose vertices set into the set of odd permutations and the set of even permutations, and then obtain the minimum congestion of odd-even embedding mapping. From the following examples, we know the cost to achieve smaller congestion is adding the number of bit in $EH(s,t)$.

We represent the nodes of S_n by a permutation of $S = \{1, 2, 3, \ldots, 4m-1, 4m\}$, so we decompose the set S as follows: $S = A \cup B \cup C \cup D$, $A = \{1, 2, \ldots, m-1, m\}$,

$B = \{m+1, m+2, \ldots, 2m-1, 2m\}$, $C = \{2m+1, 2m+2, \ldots, 3m-1, 3m\}$, $D = \{3m+1,\ 3m+2, \ldots, 4m-1, 4m\}$, and $p(j)$ denotes the element which is position j in the permutation p; let $A_p = \{x \in A \text{ and } p(x) \notin A\}$, $B_p = \{x \in B \text{ and } p(x) \notin B\}$, $C_p = \{x \in C \text{ and } p(x) \notin C\}$, $D_p = \{x \in D \text{ and } p(x) \notin D\}$, m_p is defined as a binary string of length $8m-3$, where $m_p(j)$ denotes two bits, namely the $(2j-1)$th and $(2j)$th bits of $m_p(j)$, for all $1 \le j \le 4m-2$: (1) $m_p(j) = 00$ if $p(j) \in A$; (2) $m_p(j) = 01$ if $p(j) \in B$; (3) $m_p(j) = 10$ if $p(j) \in C$; (4) $m_p(j) = 11$ if $p(j) \in D$.

Let the last bit of the string m_p denoted by *last*(m_p), be given by

$$last\,(m_p) = \begin{cases} 0 & \text{if } p(4m-1) \in A \text{ and } p(4m) \in A \cup B \cup C \cup D \\ & \text{or } p(4m-1) \in B \text{ and } p(4m) \in B \cup C \cup D \\ & \text{or } p(4m-1) \in C \text{ and } p(4m) \in C \cup D \\ & \text{or } p(4m-1) \in D \text{ and } p(4m) \in D \\ 1 & \text{otherwise} \end{cases}$$

Observe that the string m_p unambiguously identifies which elements of S are in A_p, B_p, C_p, D_p, respectively, and moreover, indicates to which set A, B, C, or D each such element is mapped. The single bit at the end of m_p is sufficient, as p is one-to-one and, therefore, the number of elements mapped by p to each set A, B, C, or D must be identical. So, if one knows which set contains the image of everything except the last two elements of S, one can deduce the sets containing the images of the last two elements unambiguously with the aid of the defined ending bit, *last*(m_p). For example, let $4m = 8$ and consider $p = (3, 5, 6, 2, 4, 8, 1, 7)$, where p is a permutation on $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$, let $A = \{1, 2\}$, $B = \{3, 4\}$, $C = \{5, 6\}$, $D = \{7, 8\}$, thus $m_p(1) = 01$ and $p(1) = 3$, $m_p(2) = 10$ and $p(2) = 5$, $m_p(3) = 10$ and $p(3) = 6$, and similarly, $m_p(4) = 00$ and $p(4) = 2$, $m_p(5) = 01$ and $p(5) = 4$, $m_p(6) = 11$ and $p(6) = 8$, and *last*(m_p) $= 0$. So, m_p is the string 01,10,10,00,01,11,0, which has length $8m-3 = 13$. Let $A_p^C = \{x \in (S-A) \text{ and } p(x) \in A\}$, $B_p^C = \{x \in (S-B) \text{ and } p(x) \in B\}$, $C_p^C = \{x \in (S-C) \text{ and } p(x) \in C\}$, $D_p^C = \{x \in (S-D) \text{ and } p(x) \in D\}$, and as p is a one-to-one function, $|A_p| = \left|A_p^c\right|$. Place each of the eight sets A_p, B_p, C_p, D_p, A_p^c, B_p^c, C_p^c, D_p^c in increasing order. Let a_i, b_i, c_i, d_i denote the ith element of A_p, B_p, C_p, D_p, respectively, and let $\overline{a_i}$, $\overline{b_i}$, $\overline{c_i}$, $\overline{d_i}$ denote the ith element of A_p^c, B_p^c, C_p^c, D_p^c. Then, we define the function M_p (a permutation on the symbols in S) by: (1) $M_p(x) = x$ if $x \notin A_P^C \cup B_P^C \cup C_P^C \cup D_P^C$; (2) $M_p(x) = a_i$ if $x = \overline{a}_i$; (3) $M_p(x) = b_i$ if $x = \overline{b_i}$; (4) $M_p(x) = c_i$ if $x = \overline{c_i}$; (5) $M_p(x) = d_i$ if $x = \overline{d_i}$. By the above definitions, we can obtain four permutations: p_A, p_B, p_C, p_D, and then deduce $p = p_A p_B p_C p_D$. $p_A = (M_p(p^{-1}(1), p^{-1}(2), p^{-1}(3), \ldots, p^{-1}(m)))$, $p_B = (M_p(p^{-1}(m+1), p^{-1}(m+2), p^{-1}(m+3), \ldots, p^{-1}(2m)))$, $p_C = (M_p(p^{-1}(2m+1), p^{-1}(2m+2), p^{-1}(2m+3), \ldots, p^{-1}(3m)))$, $p_D = (M_p(p^{-1}(3m+1), p^{-1}(3m+2), p^{-1}(3m+3), \ldots, p^{-1}(4m)))$.

Theorem 3.3. *For $m > 1$, $d = \lceil \log_2(m!) \rceil$, $s+t+1 = 4d+8m-3$, there is an embedding mapping φ existing for S_{4m} into $EH(s,t)$ with dilation $= 2d+6$.*

Proof: Let p be any permutation on $S = \{1, 2, 3, \ldots, 4m-1, 4m\}$, and from the above analyses, we can partition S into the four sets A, B, C, D. Let w be the embedding of S_m into $EH(s,t)$, where $s+t+1 = d$, $d = \lceil \log_2(m!) \rceil$. Then, define the embedding φ for S_{4m} into $EH(s,t)$ by $\varphi(p) = w(p_A)w(p_B)w(p_C)w(p_D)m_p$, where p_A, p_B, p_C, p_D and m_p are as described above. It follows that φ is one-to-one and $\varphi(p)$ is a binary string of length $4d+8m-3$, that is $s+t+1 = 4d+8m-3$. So for the two adjacent nodes p and q in S_n, it must have $p = q \cdot (1,j)$, where $(1,j)$ denotes a transposition. Moreover, any transposition of the form $(1,j)$ changes the set A_p^c, B_p^c, C_p^c, D_p^c, that is: one of the sets A_p^c, B_p^c, C_p^c, D_p^c add or reduce one element since any two permutations of p_A, p_B, p_C, p_D have deferent two bits. By the definition of m_p, we know $w(p_A)$ and one other, say $w(p_B)$, change, plus at most four bits in the string m_p. Consequently, for $p = q \cdot (1,j)$, the string $\varphi(p)$ and $\varphi(q)$ differ in at most $2d + 4$ bits, and by the definition of $EH(s,t)$, the shortest distance of $\varphi(p)$ and $\varphi(q)$ in $EH(s,t)$ is $2d+4+2$ at most, and thus dilation $= 2d + 4 + 2 = 2d + 6$.

From the above, S_{4m} can be embedded into $EH(s,t)$ with dilation $= 2d + 6$.

Using the same strategy, and partitioning the set S into 2^i equal size subsets, one obtains the following more general statement.

Theorem 3.4. *For $m > 1$, $i \geq 1$, and $d = \lceil \log_2(m!) \rceil$, $s + t + 1 = 2^i d + i2^i m - 2i + 1$, $S_{2^i m}$ can be embedded into $EH(s,t)$ with dilation $= 2d + 2i + 2$.*

Proof: The proof of Theorem 3.4 follows in the same manner as described earlier for Theorem 3.3. Let a permutation p on the set $S = \{1, 2, 3, \ldots, 2^i m - 1, 2^i m\}$, and it can be embedded into $EH(s,t)$ by $\varphi(p) = w(p_1)w(p_2)w(p_3)\cdots w(p_{2^i})m_p$, where p_1, p_2, $p_3, \cdots p_{2^{i-1}}, p_{2^i}, w$ and m_p are defined as above. Using the same strategy, when $1 \leq j \leq 2^i m - 2$, $m(j)$ is a binary string of length i, and then, the last bit of m_p is $last(m_p)$ that the last two bits of $p(2^i m - 1, 2^i m)$ are mapped to, so we obtain m_p is a binary string of length $i2^i m - 2i + 1$, and moreover, $\varphi(p)$ is a binary string of length $2^i d + i2^i m - 2i + 1$, that is $s + t + 1 = 2^i d + i2^i m - 2i + 1$.

Similar to prove Theorem 3.3, for the two adjacent nodes p and q in S_n, it must have $p = q \cdot (1, j)$. Any transposition of the form $(1, j)$ changes at most two of the component permutations, say p_1, p_K, and changes at most in two bits. As each membership in p_K that maps into m_p is binary string of length i, for $p = q \cdot (1, j)$, the string $\varphi(p)$ and $\varphi(q)$ differ in at most $2d + 2i$ bits, and by the definition of $EH(s,t)$, the shortest distance of $\varphi(p)$ and $\varphi(q)$ in $EH(s,t)$ is $2d + 2i + 2$ at most, and thus dilation $= 2d + 2i + 2$.

From the above, $S_{2^i m}$ can be embedded into $EH(s,t)$ with dilation $= 2d + 2i + 2$.

4. **Conclusions.** $EH(s,t)$ is an important variant of hypercube basing on link removal from hypercube. For the purpose of extensively investigating the versatility of $EH(s,t)$, we present several strategies and performance about embedding star networks into $EH(s, t)$, and evaluate the embedding efficiency measured by dilation, expansion, load and congestion respectively. These results show that most star networks can be able to embed into $EH(s,t)$ with low overheads. Thus, mature algorithms of star networks can run in $EH(s,t)$ effectively and the latter can simulate the former with small communication delay and good processor utilization.

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