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OBSERVER-BASED ADAPTIVE FUZZY CONTROL FOR A CLASS OF NONLINEAR STRICT-FEEDBACK SYSTEMS

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Abstract. *This paper is concerned with the problem of adaptive fuzzy control via output feedback for a class of nonlinear uncertain single-input single-output strict-feedback systems. Fuzzy logic systems are used to approximate the packaged nonlinear functions, and adaptive backstepping technique is applied to constructing control laws of output feedback and adaptive laws. Furthermore, Lyapunov method is used to prove that all the signals in the closed-loop systems are semi-globally uniformly ultimately bounded (SUUB), and the state variables converge to a small neighborhood of the origin. The simulation results demonstrate the effectiveness of the proposed method.*
Keywords: Fuzzy logic systems, Nonlinear strict-feedback systems, Backstepping, SUUB

1. **Introduction.** In the past decade, adaptive backtepping has aroused wild attention in the field of nonlinear control [1], because it gives an idea for nonlinear systems without satisfying the matching requirements. Earlier classical adaptive backstepping is mainly used for robust control of nonlinear systems with parametric uncertainties. However, a limitation of these work is that they cannot be applied to the nonlinear systems with unknown structural uncertainties. In recent years, due to the approximation capability of fuzzy logic systems and by combining fuzzy logic systems with adaptive backstepping control, various fuzzy adaptive backstepping control approaches were proposed (see, for example, [2,3] and the references therein).

Notice that all the above work is based on the condition that the state variables are measurable, which will limit the applicability of these control methods. As is well known, the state variables of the controlled systems may be unavailable in many control problems. Subsequently, fuzzy/neural adaptive control method via output feedback will become very meaningful. A fuzzy adaptive control approach is developed for a class of SISO strict-feedback nonlinear systems with unmeasured states in [4], and [5] proposes an adaptive fuzzy robust control method for SISO nonlinear systems with nonlinear uncertainties, unmodeled dynamics and dynamic disturbances. In this research, we also consider output-feedback adaptive fuzzy control for nonlinear strict-feedback systems. Unlike the work in [4,5], a linear matrix inequality condition, rather than matrix inequality, is proposed for the stability analysis of the observation error dynamics, and it makes the proposed control strategy easier to be implemented in practice.

The specific scheme of this paper is that an observer is first designed to estimate the immeasurable state vector, and fuzzy logic system is employed to approximate the unknown smooth nonlinear function. Meanwhile, by combining adaptive fuzzy method with backstepping technology, an adaptive fuzzy controller in output feedback form is designed. Simulation example is provided to illustrate the effectiveness of the proposed method. It is worth mentioning that the main contribution of this paper is that a linear matrix inequality is used which extends the range space of practical application, in terms of stability analysis of the observation error dynamics.

2. **Preliminaries.** Consider the following SISO nonlinear systems in strict-feedback form

$$\begin{aligned} \dot{x}_i &= x_{i+1} + f_i(\overline{x}_i), \ 1 \le i \le n-1 \\ \dot{x}_n &= u + f_n(\overline{x}_n) \\ y &= x_1 \end{aligned} \tag{1}$$

where $\overline{x}_i = [x_1, \ldots, x_i]^T \in R^n$ denotes the state vector. $u \in R$ and $y \in R$ are the input and output, respectively. And only output variable $y = x_1$ can be measured directly. $f_i(.)$ $(i = 1, 2, \ldots, n)$ stands for the unknown smooth nonlinear function with $f_i(0) = 0$.

Assumption 2.1. *For $f_i(.)$, there exist known constants $\underline{h}_{ij}$ and $\overline{h}_{ij}$, such that for $1 \le i, j \le n$, $\underline{h}_{ij} \le \frac{\partial f_i}{\partial x_j} \le \overline{h}_{ij}$. It implies that there exists a constant $h_i > 0$, such that $|f_i(x)| \le h_i \|x\|$, which means that the strictly increasing smooth function $\phi_i(s) = h_i(s)$ with $s \in R$ is a bounding function of $f_i(.)$.*

Lemma 2.1. [7] *For $z_i = \hat{x}_i - \alpha_{i-1}$, $i = 1, 2, \ldots, n$, the following holds:*

$$\|\hat{x}\| \le \sum_{i=1}^{n} |z_i| \phi_i \left(\hat{\theta}_i\right)$$

with $\phi_i(\theta_i) = 2\,(1 + k_i + 0.5) + \frac{1}{a_i^2} \hat{\theta}_i S_i^T S_i$.

3. **Main Results.** In this section, we will propose an output feedback adaptive fuzzy control methodology via backstepping for system (1).

3.1. **Observer design.** The control strategy begins with a state observer as follows:

$$\begin{aligned} \dot{\hat{x}}_i &= \hat{x}_{i+1} - l_i(y - \hat{x}_1), \ 1 \le i \le n-1 \\ \dot{\hat{x}}_n &= u - l_n(y - \hat{x}_1) \end{aligned} \tag{2}$$

where $\hat{x}_i$ is the estimation of x_i, $(i = 1, \ldots, n)$. l_i is the design parameter such that $A_o = \begin{bmatrix} L & I_{n-1} \\ l_n & 0 \end{bmatrix} \left(L^T = [l_1, \ldots, l_{n-1}]\right)$ is a strict Hurwitz matrix. Define $e_i = x_i - \hat{x}_i$, $1 \le i \le n$, and then, from (1) and (2), the error dynamic can be expressed as follows

$$\dot{e} = A_o e + F(x) \tag{3}$$

where $e = [e_1, \ldots, e_n]^T$, $F(x) = [f_1(\overline{x}_1), \ldots, f_n(\overline{x}_n)]^T$.

3.2. **Control design and stability analysis.** Backstepping design is based on sets of coordinate transformations, i.e., $z_i = \hat{x}_i - \alpha_{i-1}$, for $1 \le i \le n$, with $\alpha_0 = 0$. The virtual control signal is designed as

$$\alpha_i = -(k_i + 0.5) z_i - \frac{1}{2a_i^2} z_i \hat{\theta}_i S_i^T(Z_i) S_i(Z_i) \tag{4}$$

The control law is

$$u = -(k_n + 0.5) z_n - \frac{1}{2a_n^2} z_n \hat{\theta}_n S_n^T(Z_n) S_n(Z_n) \tag{5}$$

where k_i and a_i are the positive design parameters, $Z_i = [\hat{x}_1, \ldots, \hat{x}_i, \theta_1, \ldots, \theta_i]^T$. $\hat{\theta}_i$ is an estimation of the unknown constant θ_i, and it satisfies the following differential equation:

$$\dot{\hat{\theta}}_i = \frac{r_i}{2a_i^2} z_i^2 S_i^T S_i - \sigma_i \hat{\theta}_i \tag{6}$$

where r_i and σ_i are the positive design parameters.

To construct a control law via backstepping, consider the Lyapunov function as $V = V_e + V_z + V_\theta$, where $V_e = e^T P e$, $V_z = \sum_{i=1}^n \frac{1}{2} z_i^2$, $V_\theta = \sum_{i=1}^n \frac{1}{2r_i} \tilde{\theta}_i^2$ with $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. According to (3), one has

$$\dot{V}_e = e^T \left(PA_o + A_o^T P\right) e + 2e^T P(F(x) - F(\hat{x})) + 2e^T PF(\hat{x}) \tag{7}$$

$$2e^T P(F(x) - F(\hat{x})) = 2e^T PJe \tag{8}$$

where J is the Jacobi matrix, its element is $[J]_{ij}$, if $j \le i$, $[J]_{ij} = \frac{\partial f_i}{\partial x_j}$, then $[J]_{ij} = 0$. Furthermore, by Assumption 2.1, $\frac{\partial f_i}{\partial x_j}$ can be expressed as a convex combination of $\underline{h}_{ij}$ and $\overline{h}_{ij}$. Namely, there exists a function $0 \le \mu_{ij}(t) \le 1$ such that

$$\frac{\partial f_i}{\partial x_j} = \mu_{ij} \underline{h}_{ij} + (1 - \mu_{ij}) \overline{h}_{ij} \tag{9}$$

Substituting (9) into (8) results in

$$2e^T P(F(x) - F(\hat{x})) = 2e^T P \left(\underline{H} + \overline{H}\right) e \tag{10}$$

where $\left[\underline{H}\right]_{ij} = \mu_{ij} \underline{h}_{ij}$, $\left[\,\overline{H}\,\right]_{ij} = (1 - \mu_{ij}) \overline{h}_{ij}$.

By Assumption 2.1 and Lemma 2.1, one can get

$$2e^T PF(\hat{x}) \le \varepsilon_0 e^T e + c \sum_{i=1}^n z_i^2 \phi_i^2 \left(\hat{\theta}_i\right) \tag{11}$$

with $c = n\varepsilon_0{}^{-1} \left\|P\right\|^2 \sum_{i=1}^n h_i^2$.

Substituting (10) and (11) into (7), one can get

$$\dot{V}_e \le e^T \left(PA_o + A_o^T P + P\left(\underline{H} + \overline{H}\right) + \left(\underline{H} + \overline{H}\right)^T P + \varepsilon_0 I\right) e + c \sum_{i=1}^n z_i^2 \phi_i^2 \left(\hat{\theta}_i\right) \tag{12}$$

Letting

$$V_z = \sum_{i=1}^n V_i = \sum_{i=1}^n \frac{1}{2} z_i^2 \tag{13}$$

one can get

$$\dot{V}_i = z_i(\alpha_i - l_i e_1 - \dot{\alpha}_{i-1}) + z_i z_{i+1} \tag{14}$$

From (4), one can get

$$\dot{\alpha}_i = \sum_{j=1}^i \left(\frac{\partial \alpha_i}{\partial \hat{x}_j} (\hat{x}_{j+1} - l_j e_1) + \frac{\partial \alpha_i}{\partial \hat{\theta}_j} \left(\frac{r_j}{2a_j^2} z_j^2 S_j^T S_j - \sigma_j \hat{\theta}_j\right)\right) \tag{15}$$

Furthermore, $\dot{V}_i$ can be expressed as

$$\begin{aligned} \dot{V}_i \;=\; & z_i \left(\alpha_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{x}_j} \hat{x}_{j+1} - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_j} \left(\frac{r_j}{2a_j^2} z_j^2 S_j^T S_j - \sigma_j \hat{\theta}_j\right)\right) \\ & + z_i \left(\sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{x}_j} l_j e_1 - l_i e_1\right) + z_i z_{i+1} \end{aligned} \tag{16}$$

For the crossing term $z_i \left(\sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{x}_j} l_j e_1 - l_i e_1\right)$, it satisfies

$$z_i \left(\sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{x}_j} l_j e_1 - l_i e_1\right) \le \frac{1}{2\beta_i} z_i^2 \left(\sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{x}_j} l_j - l_i\right)^2 + \frac{1}{2} \beta_i e_1^2 \tag{17}$$

where β_i is a positive constant. Taking (16) with (17) into account gives

$$\dot{V}_i \le z_i\left(\alpha_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_i}{\partial\hat{x}_j}\hat{x}_{j+1} - \sum_{j=1}^{i-1}\frac{\partial\alpha_i}{\partial\hat{\theta}_j}\left(\frac{r_j}{2a_j^2}z_j^2S_j^TS_j - \sigma_j\hat{\theta}_j\right)\right) + \frac{1}{2\beta_i}z_i^2\left(\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}l_j - l_i\right)^2 + \frac{1}{2}\beta_ie_1^2 + z_iz_{i+1} \quad (18)$$

Especially, the time derivative of $V_n = \frac{1}{2}z_n^2$ is given by

$$\dot{V}_n = z_n(u - \dot{\alpha}_{n-1} - l_ne_1) \quad (19)$$

At the present stage, define

$$\overline{f}_i = -\sum_{j=1}^{i-1}\frac{\partial\alpha_i}{\partial\hat{x}_j}\hat{x}_{j+1} - \sum_{j=1}^{i-1}\frac{\partial\alpha_i}{\partial\hat{\theta}_j}\left(\frac{r_j}{2a_j^2}z_j^2S_j^TS_j - \sigma_j\hat{\theta}_j\right) + \frac{1}{2\beta_i}z_i\left(\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}l_j - l_i\right)^2 + cz_i\phi_i^2\left(\hat{\theta}_i\right) + z_{i-1} \quad (20)$$

$$\overline{f}_n = z_{n-1} - \dot{\alpha}_{n-1} - l_ne_1 + cz_n\phi_n^2\left(\hat{\theta}_n\right) \quad (21)$$

with $z_0 = 0$. Thus, taking (13), (18)-(21) into account together gives

$$\dot{V}_z \le \sum_{i=1}^{n} z_i\left(\alpha_i + \overline{f}_i\right) + \sum_{i=1}^{n-1}\frac{1}{2}\beta_ie_1^2 - c\sum_{i=1}^{n} z_i^2\phi_i^2\left(\hat{\theta}_i\right) \quad (22)$$

with $\alpha_n = u$. According to [6], fuzzy logic system $W_i^TS_i(Z_i)$ is now utilized to approximate the unknown function $\overline{f}_i$ such that for any given $\varepsilon_i > 0$, $\overline{f}_i = W_i^TS_i(Z_i) + \delta_i(Z_i)$ with δ_i satisfying $|\delta_i| \le \varepsilon_i$ being an approximation error. Thus, one has, for $1 \le i \le n$,

$$z_i\overline{f}_i \le \frac{1}{2a_i^2}z_i^2\theta_iS_i^TS_i + \frac{1}{2}a_i^2 + \frac{1}{2}z_i^2 + \frac{1}{2}\varepsilon_i^2 \quad (23)$$

where $\theta_i = \|W_i\|^2$.

From (4), one can get

$$z_nu = -(k_n + 0.5)z_n^2 - \frac{1}{2a_n^2}z_n^2\hat{\theta}_nS_n^TS_n \quad (24)$$

Substituting (4), (23) and (24) into (22), one can get

$$\dot{V}_z \le -\sum_{i=1}^{n}k_iz_i^2 + \sum_{i=1}^{n}\frac{1}{2a_i^2}z_i^2\tilde{\theta}_iS_i^TS_i + \sum_{i=1}^{n}\frac{1}{2}\left(a_i^2 + \varepsilon_i^2\right) + \sum_{i=1}^{n-1}\frac{1}{2}\beta_ie_1^2 - c\sum_{i=1}^{n}z_i^2\phi_i^2\left(\hat{\theta}_i\right) \quad (25)$$

For $V_\theta = \sum_{i=1}^{n}\frac{1}{2r_i}\tilde{\theta}_i^2$, its time derivative is

$$\dot{V}_\theta = -\sum_{i=1}^{n}\frac{1}{r_i}\tilde{\theta}_i\dot{\hat{\theta}}_i \quad (26)$$

Thus, from (12), (25) and (26), one can get

$$\dot{V} \le e^T\left(PA_o + A_o^TP + P\left(\underline{H} + \overline{H}\right) + \left(\underline{H} + \overline{H}\right)^TP + \varepsilon_0I + \beta\right)e - \sum_{i=1}^{n}k_iz_i^2 + \sum_{i=1}^{n}\frac{1}{2}\left(a_i^2 + \varepsilon_i^2\right) + \sum_{i=1}^{n}\frac{1}{r_i}\tilde{\theta}_i\left(\frac{r_i}{2a_i^2}z_i^2S_i^TS_i - \dot{\hat{\theta}}_i\right) \quad (27)$$

with $\beta = diag\left[\sum_{i=1}^{n-1}\frac{1}{2}\beta_i, 0, \ldots, 0\right]$. Thus, we can get the results in the following theorem.

Theorem 3.1. *Consider system (1) satisfying Assumption 2.1, and construct observer (2). Suppose that the packaged nonlinear function $\overline{f}_i$ can be approximated by fuzzy logic systems in the sense that the approximate errors are bounded. If there exists definitive positive matrix P, such that*

$$PA_o + A_o^T P + PM_\alpha + M_\alpha^T P + \varepsilon_0 I + \beta < 0$$

where $M_\alpha \in \Xi = \left\{M|[M]_{ij} = \underline{h}_{ij},\ or\ [M]_{ij} = \overline{h}_{ij},\ 1 \le i, j \le n\right\}$, and $\alpha = 2^{\frac{n(n+1)}{2}}$. The control law (5) and the adaptive law (6) guarantee that all the closed-loop signals are SUUB and all the variables of the closed-loop system can converge to a small enough neighborhood around the origin by choosing appropriate design parameters.

Proof: By the definition of $\dot{\hat{\theta}}_i$, the following holds

$$\sum_{i=1}^{n} \frac{1}{r_i}\tilde{\theta}_i\left(\frac{r_i}{2a_i^2}z_i^2 S_i^T S_i - \dot{\hat{\theta}}_i\right) = \sum_{i=1}^{n} \frac{\sigma_i}{r_i}\tilde{\theta}_i\hat{\theta}_i \le \sum_{i=1}^{n} \frac{\sigma_i}{r_i}\left(-\frac{1}{2}\tilde{\theta}_i^2 + \frac{1}{2}\theta_i^2\right) \tag{28}$$

Thus, taking (26) into account with (28), one can get

$$\begin{aligned}\dot{V} \ \le\ & e^T\left(PA_o + A_o^T P + P\left(\underline{H} + \overline{H}\right) + \left(\underline{H} + \overline{H}\right)^T P + \varepsilon_0 I + \beta\right)e \\ & - \sum_{i=1}^{n} k_i z_i^2 + \sum_{i=1}^{n}\frac{1}{2}\left(a_i^2 + \varepsilon_i^2 + \frac{\sigma_i}{r_i}\theta_i^2\right) - \sum_{i=1}^{n}\frac{\sigma_i}{2r_i}\tilde{\theta}_i^2\end{aligned} \tag{29}$$

with $\underline{H}$ and $\overline{H}$ being the time-varying matrices, and they lead to the result that stability analysis becomes very difficult. To overcome this difficulty, we will develop the time-independent LMI stability conditions for the error dynamic system in the following.

For notation simplicity, let I_{ij} denote the n-order square matrix which element $[I_{ij}]_{mk} = 1$ for $m = i$ and $k = j$, and others are 0. Thus, according to the definition of $\underline{H}$ and $\overline{H}$, the following equality holds

$$\underline{H} + \overline{H} = \sum_{i,j=1}^{n}\left(\mu_{ij}\underline{h}_{ij} + (1-\mu_{ij})\overline{h}_{ij}\right)I_{ij} \tag{30}$$

where μ_{ij} is a function and satisfies $0 \le \mu_{ij} \le 1$. As a result,

$$PA_o + A_o^T P + P\left(\underline{H} + \overline{H}\right) + \left(\underline{H} + \overline{H}\right)^T P + \varepsilon_0 I + \beta < 0 \tag{31}$$

is equivalent to

$$\begin{aligned} & PA_o + A_o^T P + \sum_{i,j=1}^{n}\left(\mu_{ij}\underline{h}_{ij} + (1-\mu_{ij})\overline{h}_{ij}\right)PI_{ij} \\ & + \sum_{i,j=1}^{n}\left(\mu_{ij}\underline{h}_{ij} + (1-\mu_{ij})\overline{h}_{ij}\right)I_{ij}^T P + \varepsilon_0 I + \beta < 0\end{aligned} \tag{32}$$

According to convex combination theory, (32) is equivalent to

$$PA_o + A_o^T P + PM_\alpha + M_\alpha^T P + \varepsilon_0 I + \beta < 0 \tag{33}$$

In addition, when inequality (33) holds, there exists a constant $\alpha > 0$, such that for all α

$$e^T\left(PA_o + A_o^T P + P(\underline{H} + \overline{H}) + \left(\underline{H} + \overline{H}\right)^T P + \varepsilon_0 I + \beta\right)e \le -\frac{\alpha}{\lambda_M(P)}e^T Pe \tag{34}$$

where $\lambda_M(P)$ denotes the maximal eigenvalue of matrix P.

Thus, from (29) and (34), one can get

$$\dot{V} \le -\frac{\alpha}{\lambda_M(P)}e^T Pe - \sum_{i=1}^{n} k_i z_i^2 - \sum_{i=1}^{n}\frac{\sigma_i}{2r_i}\tilde{\theta}_i^2 + \sum_{i=1}^{n}\frac{1}{2}\left(a_i^2 + \varepsilon_i^2 + \frac{\sigma_i}{r_i}\theta_i^2\right) \tag{35}$$

Now, take $a_0 = \min\left\{\frac{\alpha}{\lambda_M(P)}, 2k_i, \sigma_i | i = 1, 2, \ldots, n\right\}$, $b_0 = \sum_{i=1}^{n}\frac{1}{2}\left(a_i^2 + \varepsilon_i^2 + \frac{\sigma_i}{r_i}\theta_i^2\right)$.

Then one can get

$$\dot{V} \leq -a_0 V + b_0 \tag{36}$$

which implies that all the closed-loop signals are bounded. Especially, for any given $\eta > 0$, by appropriately choosing a_i, ε_i, σ_i to be sufficiently small, as well as r_i to be sufficiently large, it is possible to let $b_0/a_0 \leq \eta$.

4. **Simulation Example.** The feasibility of the proposed method is tested by an example.

Example 4.1. *Consider the following single-link robot system in [8].*

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{u - 0.5mgl\sin(x_1)}{M} \\ y &= x_1 \end{aligned} \tag{37}$$

with $m = 1$ *kg,* $M = 0.5$ *kg/*m^2*,* $l = 1$ *m. The observer gain is* $L = [-6.5751,\ -57.1814]^T$ *and the control parameters are* $a_1 = a_2 = 1$*,* $r_1 = r_2 = 3$*,* $k_1 = k_2 = 20$ *and* $\sigma_1 = \sigma_2 = 0.005$*. As done in [8], the initial conditions are chosen as* $x_1(0) = -1.2$*,* $x_2(0) = 0.8$*,* $\hat{x}_1(0) = \hat{x}_2(0) = 0$ *and* $\theta_1(0) = 0$*.*

Figures 1 and 2 show the state and observed state responses. Figure 3 illustrates the trajectory of adaptive parameter θ_1*. Figure 4 plots the trajectory of control input signal* u*.*

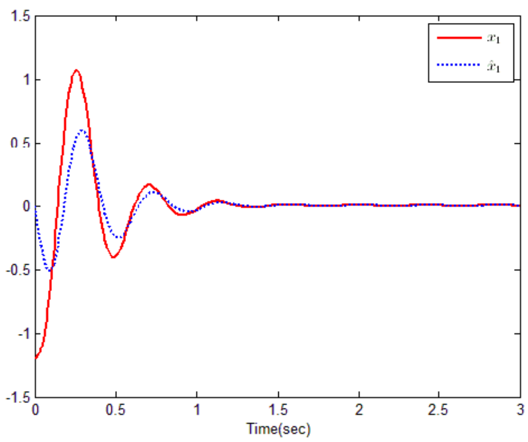

FIGURE 1. x_1 ("-") and $\hat{x}_1$ ("- -")

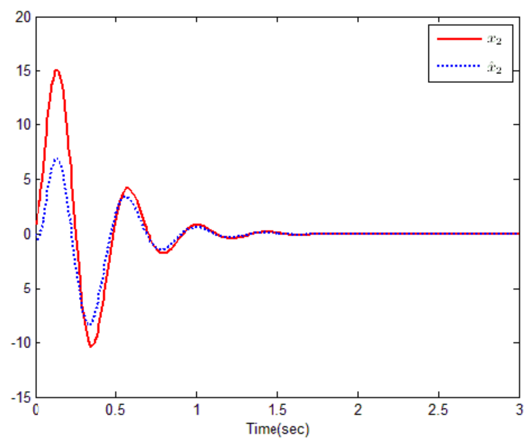

FIGURE 2. x_2 ("-") and $\hat{x}_2$ ("- -")

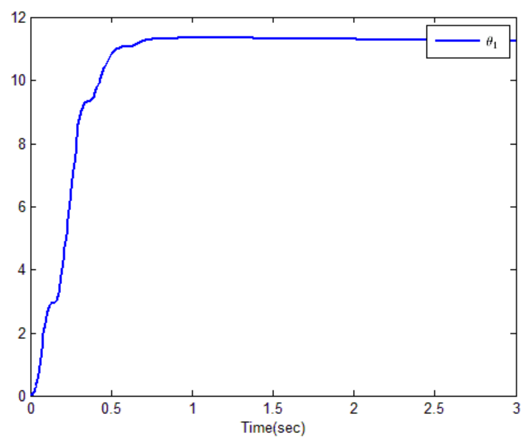

FIGURE 3. Adaptive parameter θ_1

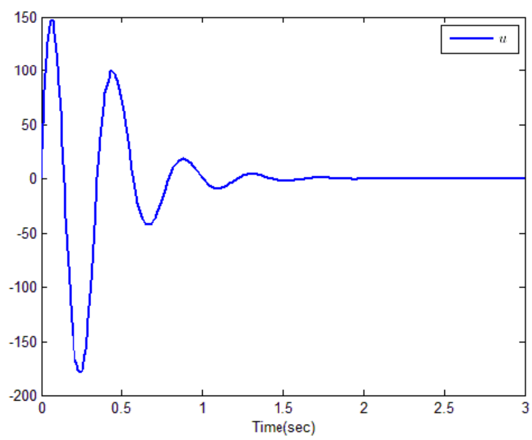

FIGURE 4. Control input signal u

From the simulation results, we can see that the state variables converge to a small neighborhood of the origin. In other words, the simulation results illustrate the effectiveness of the proposed scheme and control performances can be achieved well as in [8]. However, the number of adaptive parameter is only one in our paper, while the number is 7 in [8]. Thus, the burden computation burden is alleviated.

5. **Conclusions.** In this paper, adaptive fuzzy output feedback control is considered for a class of single-input single-output nonlinear strict-feedback systems. In the process of controller design, fuzzy logic system is used to approximate unknown smooth nonlinear function and by combining backstepping technology, an adaptive fuzzy logic system is designed. The scheme guarantees all the signals of close-system are SUUB.

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