ICIC Express Letters
Volume **10**, Number **3**, March **2016**

ICIC International ©2016 ISSN 1881-803X
pp. 575–584

SEVERAL EFFECT PROBABILITIES FOR FUZZY EVENTS AND THEIR APPLICATIONS IN MULTI-OBJECTIVE PROGRAMMING

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Received August 2015; accepted October 2015

Abstract. *Randomness and fuzziness are two common uncertainties in multi-objective problems. They often coexist in real decision process. So it is a hot topic on processing the two uncertainties in academic and application fields. In this paper, for the multi-objective problems with fuzzy goals and random coefficients, we first analyze the essential characteristics of fuzzy decision and review the effect probability's preliminaries of fuzzy events. Then we give several effect probabilities (denoted by EP) formulas with normal distribution for special fuzzy events represented by fuzzy numbers. Further, we establish a multi-objective programming model based on EP by regarding fuzzy goals as fuzzy events. Finally, we illustrate the validity of multi-objective model based on EP through a case.*
Keywords: Fuzzy events, Effect probability, Multi-objective programming, Decision making

1. **Introduction.** Randomness and fuzziness are two common uncertainties in the real problems. Although there exist essential differences, they are always coexistent in complex system optimization, decision analysis and many other fields. For example, one company sets two goals that "the sales are not less than 1000 as far as possible" and "total equipment time is about 40 hours" for a production while the demand and the produce time are random variable obeying some distribution. Obviously, fuzziness and randomness are unavoidable in multi-objective problems and effective processing them is a hot topic in academics and applications. Many scholars have studied on this topic in combination with different methods and theories.

In 1772, Franklin firstly proposed the coordination multi-objective problems with many inconsistent goals. Thereafter, Pareto [1] presented multi-objective optimization decision making problems in 1896. Then many scholars have given deep research on it under different theories and backgrounds. Charnes and Cooper [2] established the multi-objective linear programming based on priority factors in 1961. Yang and Zheng [3] studied on the solutions to multi-objective programming based on investment analysis; Han et al. [4] proposed a multi-objective programming model as an alternative approach for solving network DEA through data envelopment analysis; Dujardin et al. [5] proposed a multi-objective interactive system for adaptive traffic control by regarding the total waiting time, the number of stops for private vehicles and a public transport criterion as decision goals. Besides, stochastic multi-objective programming and fuzzy multi-objective programming were formed by combining with random theory and fuzzy set theory. Hulsurkar et al. [6] proposed a fuzzy programming to solve the multi-objective stochastic programming. Azaron et al. [7] studied on multi-objective programming with uncertain parameters under the supply chain environment and the Pareto solution was given. Cebi and Otay [8] developed multi-objective linear mathematical model for project network problem under fuzziness on the basis of assuming the parameters were uncertain. Bustos et al. [9] established a stochastic multi-objective programming model for transportation problems.

At present, many scholars have studied on the process problems of uncertainties in multi-objective programming, but there still exist two shortcomings: 1) they only considered one uncertainty and ignored the other; 2) they ignored the different effect of fuzzy decision preference in the decision process. In this paper, for the multi-objective problems with fuzzy goals and random coefficients, we mainly do the following contributions: 1) we give several effect probability (denoted as EP) formulas for special fuzzy events; 2) we establish a multi-objective programming model based on EP; 3) we further analyze its characteristics through a case. The rest of the paper is structured with Section 2 reviewing the preliminaries of EP. Section 3 proposed several EP formulas for fuzzy events. Section 4 established a multi-objective programming model based on EP by regarding fuzzy goals as the fuzzy events. Combined with a case, we compared this model with that based on priority factors in Section 5, followed by conclusions in Section 6.

2. **Preliminaries.** For convenience, in this paper: 1) $(\Omega, \mathscr{B}, P)$ denotes the probability space ($\mathscr{B}$ denotes a σ-algebra composed by some subsets of Ω, $P(*) : \mathscr{B} \to [0, 1]$, and satisfies: 1) $P(\phi) = 0$, $P(\Omega) = 1$; 2) $P\left(\bigcup_{n=1}^{\infty} A_i\right) = \sum_{n=1}^{\infty} P(A_i)$, when $\{A_n\}_{n=1}^{\infty} \subset \mathscr{B}$ and any two are incompatible); 2) For $A \in \mathscr{F}(\Omega)$, $A(x)$ denotes the membership function of A, $A_\lambda = \{x|x \in U, A(x) \geq \lambda\}$ is the λ-level set (or λ-set) of A; 3) $\mathscr{F}(\Omega)$ denotes an entirety of fuzzy sets (that is the mapping from Ω to [0,1]).

The concept of fuzzy event probability was proposed by Zadeh in 1968 [10]. Then the measurement and application have attracted much attention among academic field. In 2008, Chen et al. [11] established the fuzzy event probability measurement by combining with decomposition theorem of fuzzy set.

$$P(A) = \int_0^1 P(A_\lambda) d\lambda. \tag{1}$$

According to the decomposition theorem, if we view $\lambda \in [0, 1]$ as a qualification standard for the elements in Ω during the fuzzy decision making process, then we can regard A_λ as a relatively crisp description of A. The fuzzy event A can be viewed as a family of crisp events $\{A_\lambda|\lambda \in [0, 1]\}$. Obviously, Equation (1) has good structural characteristics. However, it is worth noting that Equation (1) does not consider the nonlinear features of membership states in decision making which can reflect the essence of fuzzy decision under certain degree.

In 2015, Li and Jie [12] analyzed the effect characteristics of different level sets and proposed the level effect function $L(\lambda)$ which maps from $[0, 1]$ to $[0, +\infty)$. And it should satisfy the following basic principles:

Principle 1: The effect of threshold is monotonous, i.e., $L(\lambda)$ is monotone non-decreasing on $[0, 1]$.

Principle 2: The effect of threshold is continuous, i.e., $L(\lambda)$ is continuous on $[0, 1]$.

Principle 3: The sum of effect equals one, i.e., $\int_0^1 L(\lambda) d\lambda = 1$.

Intuitively speaking, $L(\lambda)$ can be understood as a quantitative decision making parameter reflecting the recognized degree of decision making based on the level λ. The size of λ reflects the recognized degree under some extent. The larger (smaller) λ is, the higher (lower) recognized degree is. Li and Jie [12] regarded the recognized effectiveness and the corresponding probability consequence as a local description and built the following fuzzy probability measurement of fuzzy set A according to Equation (1):

$$P(A \oplus L(\lambda)) = \int_0^1 L(\lambda) P(A_\lambda) d\lambda. \quad (2)$$

We call Equation (2) the effect probability based on level effect (denoted as effect probability and EP for short).

It is obvious that if A is a crisp event on the space $(\Omega, \mathscr{B}, P)$ (it means A is a crisp set on Ω and $A \in \mathscr{B}$), then $P(A \oplus L(\lambda)) = P(A)$ for any level effect function $L(\lambda)$. It indicates that Equation (2) is the generalization of the classical probability measurement model. Equation (2) reflects the decision preference of decision makers by the effect of different membership status. It has the theoretical and practical value. Also normal distribution is the most common probability distribution and has good formula and application value. So we will discuss several special effect probability calculations with normal distribution in the next section.

3. **Several Effect Probability Formulas for Fuzzy Events.** Fuzzy number is an effective tool to describe fuzzy information by membership in decision making process. So it is nature to represent fuzzy events by fuzzy number. In the following, we will further discuss the EP with normal distribution for fuzzy events represented by fuzzy number based on double integral.

Theorem 3.1. *Let X be a variable obeying standard normal distribution $N(0,1)$ in probability space $(\Omega, \mathscr{B}, P)$. $L(\lambda) = n\lambda^{n-1}$, $n \geq 1$, $A = (a, b, c)$ and $a < b < c$, then*

$$P(\{X \in A\} \oplus L(\lambda)) = \frac{G(a,b,n)}{(b-a)^n} + \frac{G(-c,-b,n)}{(c-b)^n}. \quad (3)$$

Here, $G(\alpha, \beta, m) = \int_\alpha^\beta (x-\alpha)^m \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$.

Proof: By $A_\lambda = [a + (b-a)\lambda, c - (c-b)\lambda]$ and the density function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, we know

$$\begin{aligned} P(\{X \in A\} \oplus L(\lambda)) &= \int_0^1 n\lambda^{n-1} \int_{a+(b-a)\lambda}^{c-(c-b)\lambda} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx d\lambda \\ &= \int_a^b \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \int_0^{\frac{x-a}{b-a}} n\lambda^{n-1} d\lambda dx + \int_b^c \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \int_0^{\frac{c-x}{c-b}} n\lambda^{n-1} d\lambda dx \end{aligned}$$

and

$$\int_a^b \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \int_0^{\frac{x-a}{b-a}} n\lambda^{n-1} d\lambda dx = \frac{1}{(b-a)^n} \int_a^b (x-a)^n \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx,$$

$$\begin{aligned} \int_b^c \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \int_0^{\frac{c-x}{c-b}} n\lambda^{n-1} d\lambda dx &= \frac{1}{(c-b)^n} \int_b^c (c-x)^n \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \\ &= \frac{1}{(c-b)^n} \int_{-c}^{-b} (c+x)^n \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx. \end{aligned}$$

Theorem 3.1 holds.

Theorem 3.1 presents the EP formula with standard normal distribution. However, nonstandard normal distribution is more common in real decision making environment. So we present the EP formula with nonstandard normal distribution.

Theorem 3.2. *Let X be a variable obeying normal distribution $N(\mu, \sigma^2)$ in probability space $(\Omega, \mathscr{B}, P)$. $L(\lambda) = n\lambda^{n-1}$, $n \geq 1$, $A = (a, b, c)$ and $a < b < c$, then*

$$P(\{X \in A\} \oplus L(\lambda)) = \frac{\sigma^2 G\left(\frac{a-\mu}{\sigma}, \frac{b-\mu}{\sigma}, n\right)}{(b-a)^n} + \frac{\sigma^2 G\left(\frac{\mu-c}{\sigma}, \frac{\mu-b}{\sigma}, n\right)}{(c-b)^n}. \tag{4}$$

Proof: By the probability density function of $N(\mu, \sigma^2)$ $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, we know

$$\begin{aligned} P(\{X \in A\} \oplus L(\lambda)) &= \int_0^1 n\lambda^{n-1} \int_{a+(b-a)\lambda}^{c-(c-b)\lambda} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx d\lambda \\ &= \int_0^1 n\lambda^{n-1} \int_{\frac{a-\mu+(b-a)\lambda}{\sigma}}^{\frac{c-\mu+(c-b)\lambda}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du d\lambda \\ &= \int_0^1 n\lambda^{n-1} \int_{\frac{a-\mu}{\sigma}+\left(\frac{b-\mu}{\sigma}-\frac{a-\mu}{\sigma}\right)\lambda}^{\frac{c-\mu}{\sigma}-\left(\frac{c-\mu}{\sigma}-\frac{b-\mu}{\sigma}\right)\lambda} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du d\lambda \\ &= P\left(\left\{U \in \left(\frac{a-\mu}{\sigma}, \frac{b-\mu}{\sigma}, \frac{c-\mu}{\sigma}\right)\right\} \oplus L(\lambda)\right). \end{aligned}$$

Theorem 3.2 holds based on Theorem 3.1.

Further, we can get the following corollaries based on Theorem 3.1 and Theorem 3.2.

Corollary 3.1. *Let X be a variable obeying standard normal distribution $N(0, 1)$ in probability space $(\Omega, \mathscr{B}, P)$. $L(\lambda) = n\lambda^{n-1}$, $n \geq 1$, $A = (a, b, c)$ and $a < b < c$, then*

$$P(\{X \in (a, b, b)\} \oplus L(\lambda)) = \frac{G(a, b, n)}{(b-a)^n}, \tag{5}$$

$$P(\{X \in (b, b, c)\} \oplus L(\lambda)) = \frac{G(-c, -b, n)}{(c-b)^n}. \tag{6}$$

Corollary 3.2. *Let X be a variable obeying normal distribution $N(\mu, \sigma^2)$ in probability space $(\Omega, \mathscr{B}, P)$. $L(\lambda) = n\lambda^{n-1}$, $n \geq 1$, $A = (a, b, c)$ and $a < b < c$, then*

$$P(\{X \in (a, b, b)\} \oplus L(\lambda)) = \frac{\sigma^2 G\left(\frac{a-\mu}{\sigma}, \frac{b-\mu}{\sigma}, n\right)}{(b-a)^n}, \tag{7}$$

$$P(\{X \in (b, b, c)\} \oplus L(\lambda)) = \frac{\sigma^2 G\left(\frac{\mu-c}{\sigma}, \frac{\mu-b}{\sigma}, n\right)}{(c-b)^n}. \tag{8}$$

Note 1: For $L(\lambda) = n\lambda^{n-1}$, $n \geq 1$, n reflects the accepted degree of decision makers on the membership state. The value of n reflects different fuzziness processing preference. The bigger (smaller) n is, the higher (lower) the importance of membership is. Too much bigger or smaller n embodies an extreme preference decision. So the value n should be in the interval $[1, 3]$.

Note 2: In reality, randomness and fuzziness always occur together in the decision problems (such as: one factory makes a sales goal "the sales amount is not less than 100 hundred as far as possible". In this problem, the market demand is a random variable obeying some probability distribution and the sales goal is a fuzzy event). The fact that "some quantity indices are not less (more) than the given value b" is the most common case in multi-objective programming problems. If X denotes the random variable, B denotes fuzzy events "not less than b as far as possible", then the probability of B can be shown as (here, $L(\lambda)$ is the level effect function; δ is the allowable limit of less than b):

$$P(\{X \in B \oplus L(\lambda)\}) = P(\{X \in (b-\delta, b, b) \oplus L(\lambda)\}) + P(\{X \geq b\}). \tag{9}$$

It indicates that our discussions are the theoretical foundation for the multi-objective programming under complex environment.

Obviously, normal distribution has wide applications in real life. And the EP formulas are the foundation of EP applications in multi-objective problems. So we can build a new method to multi-objective programming by combining with the above EP formulas under the random environment.

4. **Multi-Objective Programming Based on EP.** Multi-objective programming is a common problem in complex system optimization and it has the basic form as [13]:

$$\begin{aligned} &\min f(x) = (f_1(x), f_2(x), \cdots, f_s(x)) \\ &\text{s.t. } g_i(x) \le 0,\ i = 1, 2, \cdots, m. \end{aligned} \tag{10}$$

Here, $x = \{x_1, x_2, \cdots, x_n\}$ is the decision variable; $f_k(x)$ is the sth decision goal consisting of decision goal function $h_k(x)$ and satisfied value V_k, i.e., $f_k(x) = \{h_k(x), V(k)\}$, $k = 1, 2, \cdots, s$; $g_i(x) \le 0$, $i = 1, 2, \cdots, m$ are constraint functions. And we call Equation (10) a certain multi-objective programming when $f_1(x), f_2(x), \cdots, f_s(x)$ and $g_1(x), g_2(x), \cdots, g_m(x)$ are real functions; we call Equation (10) an uncertain multi-objective programming when there exist some uncertainties in $f_1(x), f_2(x), \cdots, f_s(x)$ or $g_1(x), g_2(x), \cdots, g_m(x)$.

Obviously, it is difficult to absolutely satisfy all the goals because the decision goals and constraint often have some conflict in real problems. So Equation (10) is a formal model and has no direct solving methods. The core is to convert Equation (10) into a single goal programming by some strategies.

Multi-objective programming based on priority factors is one of the most common methods to multi-objective programming problems. It was proposed by Charnes and Cooper in 1961, to deal with the quantitative multi-objective problems. So Equation (10) is converted a single goal programming by introducing priority factors of every decision goals [2]:

$$\begin{aligned} &\min z = \sum_{i=1}^{l} Q_k \left(w_k^- d_k^- + w_k^+ d_k^+ \right) \\ &\text{s.t.} \begin{cases} h_k(x) + d_k^- - d_k^+ = V_k,\ k = 1, 2, \cdots, s, \\ \sum\limits_{j=1}^{n} c_{jk} x_j = h_k(x),\ k = 1, 2, \cdots, s, \\ \sum\limits_{j=1}^{n} a_{ij} x_j \le (=, \ge)\, b_i,\ i = 1, 2, \cdots, m, \\ x_j \ge 0,\ j = 1, 2, \cdots, n, \\ d_k^-, d_k^+ \ge 0,\ k = 1, 2, \cdots, s. \end{cases} \end{aligned} \tag{11}$$

Here, V_k denotes the satisfying goal value of the kth decision goal; d_k^+, d_k^- denote the part more than or less than the satisfying goal value (called as the positive and negative deviation); w_k^-, w_k^+ denote the weight of positive and negative deviation in the same priority respectively; Q_k denotes the priority level of kth decision goal, $\{q_1, q_2, \cdots, q_L\}$ are all the values of Q_k and satisfied that $q_1 \gg q_2 \gg \cdots \gg q_L$ (here, $a \gg b$ shows that a is much greater than b).

Obviously, priority factors in Equation (11) have two effects: 1) It describes the importance order of decision goals in the decision process; 2) It converts the multi-objective programming into a single goal linear programming. These effects make Equation (11) a good calculation formula. However, this model lacks system and universality which show that: 1) It cannot solve nonlinear problems with uncertainty; 2) The weight of different dimensional goals are difficult to determine when different type goals have distinctions in the same degree (or in the same priority level).

However, real multi-objective environment is often random. So Equation (11) cannot solve the multi-objective programming problems in random environment, effectively.

However, we can construct the corresponding multi-objective model by expectation model [14] and Chance-constrained model [15].

And, expectation model [11] can convert a random problem into a concrete model by representing random uncertain parameters by the expectation values. So the multi-objective expectation model can be described as model (12) for the multi-objective problems in random environment [14]:

$$\begin{aligned} &\min E(f(x)) = (E(f_1(x)), E(f_2(x)), \cdots, E(f_s(x))) \\ &\text{s.t. } E(g_i(x)) \le 0, \ i = 1, 2, \cdots, m. \end{aligned} \tag{12}$$

Here, $E(\eta)$ denotes the expectation value of random variable η.

Equation (12) converts the random model into a certain model, effectively. So the certain solving solutions to (10) are also suitable to Equation (12). Although, Equation (12) makes the random multi-objective programming problem easy to solve results, it also embodies two shortcomings: 1) the quality of decision results cannot be guaranteed when the randomness is big; 2) Equation (12) cannot effectively solve the decision making problems in different risks. For these two shortcomings, Charnes and Cooper proposed the chance-constrained model for the random problems in 1959 [15]. And multi-objective chance-constrained model was presented in combination with chance-constrained model [15]:

$$\begin{aligned} &\min \left(\overline{f}_1, \overline{f}_2, \cdots, \overline{f}_s\right) \\ &\text{s.t. } \begin{cases} P\left(h_k(x) \le \overline{h}_k\right) \ge \beta_k, \ k = 1, 2, \cdots, s, \\ P(g_i(x) \le 0) \ge \alpha, \ i = 1, 2 \cdots, m. \end{cases} \end{aligned} \tag{13}$$

Here, $P(A)$ denotes the probability of event A, β_k is the satisfied threshold of kth decision goal, α denotes the satisfied threshold of $g_i(x) \le 0$, $\overline{f}_k$ denotes the decision goal consisting of objective function $\overline{h}_k$ and satisfied value $V(k)$, i.e., $\overline{f}_k = \left\{\overline{h}_k, V(k)\right\}$, $k = 1, 2, \cdots, s$.

Equation (13) can effectively solve the multi-objective problems in different risks even the randomness is big. However, this model is complex when there are many decision goals. The decision makers have to make suitable probability values α and β_k for every decision goal.

Obviously, Equation (12) and Equation (13) cam solve random multi-objective programming problems. However, both of them have some shortcomings. Simultaneously, real decision environment is often random rather than certain and the decision goal values are fuzzy rather than the certain values. So the existing methods to multi-objective programming are lack of systematic processing mechanism. And for the multi-objective problems with both fuzziness and randomness, the reliability of decision results is the key that we must consider in the decision process. So, for the multi-objective programming model (10) in random environment, we can regard the satisfied goal $f_k(x)$ as the fuzzy event and regard the level effect function $L(\lambda)$ as the treatment measures to fuzziness when the satisfied values V_k are fuzzy. Then we can represent $P(\{h_k(x) \in V_k\} \oplus L(\lambda)) \triangleq p_k(x)$ as the case of achieving the kth decision goal; further Equation (10) can be converted into the multi-objective programming based on EP as the following:

$$\begin{aligned} &\min \widetilde{p}(x) = (\widetilde{p}_1(x), \widetilde{p}_2(x), \cdots, \widetilde{p}_s(x)) \\ &\text{s.t. } \begin{cases} p_k(x) = P(\{h_k(x) \in V_k\} \oplus L(\lambda)), \ k = 1, 2, \cdots, s, \\ g_i(x) \le 0, \ i = 1, 2, \cdots, m. \end{cases} \end{aligned} \tag{14}$$

Here, $g_i(x)$, $i = 1, 2, \cdots, m$ are the real functions; $\widetilde{p}_k(x)$ denotes the decision goal with decision function $p_k(x)$ satisfying value 1 (that is $\widetilde{p}_k(x) = \{p_k(x), 1\}$, $k = 1, 2, \cdots, s$). If $g_i(x)$, $i = 1, 2, \cdots, m$ are random, we can convert them into the quantitative constraint combining with expectation model [14], Chance-constrained model [15] and Quasi-linear stochastic programming model [16].

It is easy to see that $P(\{h_k(x) \in V_k\} \oplus L(\lambda)) \triangleq p_k(x)$ is a dimensionless pattern of decision goal $f_k(x)$. It is easy to establish the solving methods by combining with different theory methods, such as, if $w_1, w_2, \cdots, w_s$ are the weights of the decision goals $f_1(x), f_2(x), \cdots, f_s(x)$, there exists:

$$\begin{aligned} &\max p(x) = \sum_{k=1}^{s} w_k p_k(x) \\ &\text{s.t.} \begin{cases} p_k(x) = P(\{h_k(x) \in V_k\} \oplus L(\lambda)),\ k = 1, 2, \cdots, s, \\ g_i(x) \le 0,\ i = 1, 2, \cdots, m. \end{cases} \end{aligned} \tag{15}$$

Here Equation (15) is a concrete decision model by making the comprehensive probability maximum of all the decision goals.

Due to the fact that "not more (or less) than b as far as possible" is the most common form in the multi-objective problems and δ is the greatest degree more (or less) than the quantitative, we can represent these events as $(-\infty, b) \cup (b, b, b+\delta)$ (or $(b-\delta, b, b) \cup [b, +\infty)$). So $P(\{X \in (a, b, c)\} \oplus L(\lambda))$ is the most common EP calculation form and we will discuss the application of this model in the next part.

5. **The Application of EP in Multi-Objective Programming.** In this section, we will discuss the practicability and effectiveness of Equation (14) and compare it with Equation (11) through a case.

Case description: Some company successfully developed two new products A and B by adding materials C and D. And every ton A and B need to add C 2 kg/t and 1 kg/t, D 1 kg/t and 3 kg/t, respectively. The profits of A and B are variables ξ_1, ξ_2 obeying normal distribution $N(800, 20^2)$ and $N(1000, 35^2)$, respectively. This company decides to produce these two new products in a workshop and have a deep research on the equipment product time. Then getting the equipment time for A and B are variables η_1, η_2 obeying normal distribution $N(5, 1)$ and $N(8, 2)$ because of the operating conditions. In the next production cycle, there are C and D 60 kg respectively. The best equipment time is 200 hours. And 20 hours is the maximum over or short for the best equipment time. The company wants to recieve total profits more than 30000 yuan as far as possible. If short, 28000 yuan is the minimum total profits. How to arrange the production of A and B is suitable?

Obviously, the problem is a multi-objective problem with two decision goals. Supposing x_1, x_2 are the number of A and B, $f_1(x_1, x_2)$, $f_2(x_1, x_2)$ are the decision goals and $h_1(x_1, x_2)$, $h_2(x_1, x_2)$ are the decision goal functions. Then there exists the following model:

$$\begin{aligned} &\min f(x_1, x_2) = (f_1(x_1, x_2), f_2(x_1, x_2)) \\ &\text{s.t.} \begin{cases} f_1(x_1, x_2) = \{h_1(x_1, x_2), 30000\}, \\ f_2(x_1, x_2) = \{h_2(x_1, x_2), 200\}, \\ h_1(x_1, x_2) = \xi_1 x_1 + \xi_2 x_2, \\ h_2(x_1, x_2) = \eta_1 x_1 + \eta_2 x_2, \\ 2x_1 + x_2 \le 60, \\ x_1 + 3x_2 \le 60, \\ x_1, x_2 \ge 0. \end{cases} \end{aligned} \tag{16}$$

Obviously, this is a multi-objective problem with random environment and fuzzy decision goals. So we can solve it by two methods.

Method 1: Multi-objective programming based on priority factors. Obviously, Equation (14) is a random programming problem. We can convert it to a quantitative decision problem by Equation (12):

$$\min f(x_1,x_2) = (f_1(x_1,x_2), f_2(x_1,x_2))$$
$$\text{s.t.}\ \begin{cases} f_1(x_1,x_2) = \{h_1(x_1,x_2), 30000\}, \\ f_2(x_1,x_2) = \{h_2(x_1,x_2), 200\}, \\ 800x_1 + 1000x_2 = h_1(x_1,x_2), \\ 5x_1 + 8x_2 = h_2(x_1,x_2), \\ 2x_1 + x_2 \le 60, \\ x_1 + 3x_2 \le 60, \\ x_1, x_2 \ge 0. \end{cases} \tag{17}$$

Obviously, Equation (17) is a certain multi-objective programming model. So, we can further establish the following model in combination with Equation (11) and decision goals:

$$\min z = Q(1)d_1^- + Q(2)\left(d_2^+ + d_2^-\right)$$
$$\text{s.t.}\ \begin{cases} h_1(x_1,x_2) + d_1^- - d_1^+ = 30000, \\ h_2(x_1,x_2) + d_2^- - d_2^+ = 200, \\ 800x_1 + 1000x_2 = h_1(x_1,x_2), \\ 5x_1 + 8x_2 = h_2(x_1,x_2), \\ 2x_1 + x_2 \le 60, \\ x_1 + 3x_2 \le 60, \\ x_1, x_2 \ge 0. \end{cases} \tag{18}$$

We can apply the simplex method to (18) and the results are shown as Table 1.

TABLE 1. The results of (18)

Coefficients	x_1	x_2	d_1^-	d_2^-	d_2^+
$Q(1) \ll Q(2)$	25	10	0	0	6
$Q(1) \gg Q(2)$	25.4545	9.0909	545.4545	0	0

The results show that we only can get a satisfied solution not an optimal solution satisfying both the decision goals simultaneously. And we still cannot get the probability of this satisfied solution. However, the probability of random problem is an important indicator in real decision process. We should get the probability of satisfied solution and make an appropriate decision.

Method 2: Multi-objective programming model based on EP. According to Section 4, we can regard two decisions as two fuzzy events and denote them by fuzzy number $V_1 = (28000, 30000, 30000) \cup (30000, +\infty)$ and $V_2 = (180, 200, 220)$. Then we can establish the following model for this problem according to Equation (14):

$$\min \widetilde{p}(x_1,x_2) = (\widetilde{p}_1(x_1,x_2), \widetilde{p}_2(x_1,x_2))$$
$$\text{s.t.}\ \begin{cases} p_k(x_1,x_2) = P(\{h_k(x_1,x_2) \in V_k\} \oplus L(\lambda)),\ k = 1,2, \\ h_1(x_1,x_2) = \xi_1 x_1 + \xi_2 x_2, \\ h_2(x_1,x_2) = \eta_1 x_1 + \eta_2 x_2, \\ 2x_1 + x_2 \le 60, \\ x_1 + 3x_2 \le 60, \\ x_1, x_2 \ge 0. \end{cases} \tag{19}$$

By $\xi_1 \sim N(800, 20^2)$, $\xi_2 \sim N(1000, 35^2)$, $\eta_1 \sim N(5,1)$ and $\eta_2 \sim N(8,2)$, we know $h_1(x_1,x_2) \sim N(800x_1 + 1000x_2, 20^2x_1^2 + 35^2x_2^2)$ and $h_2(x_1,x_2) \sim N(5x_1 + 8x_2, x_1^2 + 2x_2^2)$. If $w_1, w_2 > 0$ as the weights of the decision goals, then we can convert (19) into a concrete model through Equation (15):

$$\max p(x_1,x_2) = w_1p_1(x_1,x_2) + w_2p_2(x_1,x_2)$$
$$\text{s.t.} \begin{cases} p_1(x_1,x_2) = P(\{h_1(x_1,x_2) \in V_1\} \oplus L(\lambda)), \\ p_2(x_1,x_2) = P(\{h_2(x_1,x_2) \in V_2\} \oplus L(\lambda)), \\ h_1(x_1,x_2) = \xi_1x_1 + \xi_2x_2, \\ h_2(x_1,x_2) = \eta_1x_1 + \eta_2x_2, \\ 2x_1 + x_2 \le 60, \\ x_1 + 3x_2 \le 60, \\ x_1, x_2 \ge 0. \end{cases} \tag{20}$$

The company further invites experts repeatedly on the decision consciousness of decision-makers and get the level effect function is $L(\lambda) = n\lambda^{n-1}$, $n \ge 1$. We can get the results shown as Table 2 under different coefficients by genetic algorithm (The parameters setting is stated as follows: the coding length is 20; the size of population is 80; the maximum generation is 100; the crossover probability is 1 and the mutation probability is 0.001.).

Table 2. The results of Equation (18)

Coefficients		x_1	x_2	The comprehensive probability
$n=1$	$w_1 = 0.9$, $w_2 = 0.1$	23.9883	11.9941	0.93
	$w_1 = 0.7$, $w_2 = 0.3$	23.7439	12.0821	0.7970
	$w_1 = 0.5$, $w_2 = 0.5$	23.5239	12.1554	0.6650
	$w_1 = 0.3$, $w_2 = 0.7$	23.5239	12.1408	0.5345
	$w_1 = 0.1$, $w_2 = 0.9$	22.8886	12.3607	0.4059
$n=2$	$w_1 = 0.9$, $w_2 = 0.1$	23.7439	12.0821	0.9233
	$w_1 = 0.7$, $w_2 = 0.3$	23.9638	12.0088	0.7938
	$w_1 = 0.5$, $w_2 = 0.5$	23.6217	12.1261	0.6563
	$w_1 = 0.3$, $w_2 = 0.7$	23.9638	12.0088	0.5284
	$w_1 = 0.1$, $w_2 = 0.9$	22.6950	12.0968	0.3922

Obviously, this model achieves the coordination between the goals by maximizing the probability of each decision goal. It is a general multi-objective programming model. By comparing Method 1 and Method 2, we can get the following conclusions: 1) fuzziness essentially affects the decision result in the decision making process; 2) Equation (11) cannot embody the probability of results and it is only suitable for the quantitative multi-objective problems; 3) Equation (14) can effectively deal with the fuzziness and randomness simultaneously; 4) Equation (14) is a dimensionless model, so it can eliminate the dimension influence by converting the dimensional goal into the probability issue; 5) the results of Equation (14) can reflect the comprehensive probability, which can be the decision foundation; 6) the importance of decision goals also affect the decision results even the same result may lead to different probability in different decision importance among goals.

6. **Conclusions.** Multi-objective decision problems are common in complex system optimization, management decision and many other fields. In real multi-objective decision problems, fuzziness and randomness often coexist. In this paper, for the multi-objective problems with fuzzy goals and random coefficients, we establish a multi-objective model based on EP by regarding the fuzzy goals as fuzzy events. And several effect probabilities for special fuzzy events are further given with normal distribution. Finally, we

compare the model with the other methods through a case. Theory analysis and experiment show that our method based on EP not only has good structure characteristics and interpretability, but also can merge fuzzy preference into the decision-making process. Although we discuss the multi-objective programming with random goal functions, it is more common that the constraints have randomness. We will discuss the multi-objective programming with fuzzy goals and random constraints in the next direction.

Acknowledgment. This work is supported by the National Natural Science Foundation of China (71371064) and the Natural Science Foundation of Hebei Province (F2015208099, F2015208100).

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