## ADAPTIVE NEURAL TRACKING CONTROL FOR PERMANENT MAGNET SYNCHRONOUS MOTORS STOCHASTIC NONLINEAR SYSTEMS

## ZIYANG QU, BING CHEN, SHUHUI GAO, TANTAN YANG AND FATAO SHI

Institute of Complexity Sciences Qingdao University No. 308, Ningxia Road, Qingdao 266071, P. R. China chenbing1958@126.com

Received August 2015; accepted October 2015

ABSTRACT. This paper studies the problem of adaptive neural tracking control for permanent magnet synchronous motors (PMSM) stochastic nonlinear systems. Neural networks are used to approximate the nonlinearities, and adaptive backstepping technique is employed to construct controller. The proposed controller ensures that all signals of the closed loop system remain bounded in probability, and the tracking error converges to an arbitrarily small neighborhood around the origin. Simulation results demonstrate the effectiveness of the proposed approach.

Keywords: Stochastic nonlinear systems, Adaptive control, Neural networks, PMSM

1. Introduction. For many industrial systems, their dynamic models can be described by a set of stochastic differential equations. Recently, with stochastic stability theory [1], lots of results for deterministic system are extended to stochastic systems [2]. Although some basic results of Ito stochastic differential equation and its stability theory [3,4] have already existed, there are only few researches on PMSM stochastic nonlinear systems via neural control approach.

In recent years, the study of electric vehicle drive has been a popular research field. It is necessary that motor drive systems applied in electric vehicle have a high starting torque and wide operating range from standstill to high speed running. PMSM have become more and more attractive for high performance electric vehicle applications because of its high power density, high reliability, high power factor, large torque to inertia ratio and long life over other kinds of motors [5]. Many scholars have proposed some tracking control strategies for PMSM without stochastic disturbance. For example, Baik et al. [6] proposed sliding mode control technique for nonlinear speed control of PMSM, Wang et al. [7] solved chaos synchronization of PMSM with disturbance by using fuzzy adaptive logic, and Cao and Fan [8] studied vector controlled PMSM based on neural network. During the actual running process, stochastic noise is usually unavoidable for PMSM systems due to motor structure and control circuit. So far, there has been no one that reported on how to handle PMSM position control systems with stochastic disturbance.

In this paper, we propose an adaptive neural control scheme for stochastic PMSM systems via backstepping method. In the control design procedure, radial basis function (RBF) neural networks are applied to approximating unknown nonlinear functions, and then adaptive neural network and backstepping technique are combined to construct the desired controller. The proposed controller can guarantee that all the signals in the closed loop system are bounded and the tracking error converges to a small neighborhood of the origin. The simulation results illustrate the effectiveness of the proposed control scheme.

2. Problem Statement and Preliminaries. The dynamic model of PMSM stochastic system under d-q coordinate axis is expressed as follows [9]:

$$d\theta = \omega dt$$

$$d\omega = \frac{1}{J} \left( \frac{3}{2} n_p \left[ \Phi i_q + (L_d - L_q) i_d i_q \right] - B\omega - T_L \right) dt + \psi_1^T dw$$

$$di_q = \frac{1}{L_q} \left( -R_s i_q - n_p \omega L_d i_d - n_p \omega \Phi + u_q \right) dt + \psi_2^T dw$$

$$di_d = \frac{1}{L_d} \left( -R_s i_d + n_p \omega L_q i_q + u_d \right) dt + \psi_3^T dw$$

w denotes r-dimensional standard Brownian motion.  $u_d$  and  $u_q$  are d-q axis voltages,  $i_d$  and  $i_q$  stand for d-q axis currents.  $\omega$  and  $\theta$  denote rotor angular velocity and rotor position. J,  $n_p$  and B denote inertia, pole pairs and friction factor.  $L_d$  and  $L_q$  are the d-q axis inductance.  $R_s$ ,  $T_L$  and  $\Phi$  are stator resistance, load torque and flux linkage, respectively. For simplicity, introduce the following notations:  $x_1 = \theta$ ,  $x_2 = \omega$ ,  $x_3 = i_q$ ,  $x_4 = i_d$ ,  $a_1 = \frac{3n_p \Phi}{2}$ ,  $a_2 = \frac{3n_p (L_d - L_q)}{2}$ ,  $b_1 = -\frac{R_s}{L_q}$ ,  $b_2 = -\frac{n_p L_d}{L_q}$ ,  $b_3 = -\frac{n_p \Phi}{L_q}$ ,  $b_4 = \frac{1}{L_q}$ ,  $c_1 = -\frac{R_s}{L_d}$ ,  $c_2 = \frac{n_p L_q}{L_d}$ ,  $c_3 = \frac{1}{L_d}$ . Then, the PMSM stochastic system can be described in the following form:

$$\begin{cases} dx_1 = x_2 dt \\ dx_2 = \left(\frac{a_1}{J}x_3 + \frac{a_2}{J}x_3x_4 - \frac{B}{J}x_2 - \frac{T_L}{J}\right) dt + \psi_1^T dw \\ dx_3 = \left(b_1x_3 + b_2x_2x_4 + b_3x_2 + b_4u_q\right) dt + \psi_2^T dw \\ dx_4 = \left(c_1x_4 + c_2x_2x_3 + c_3u_d\right) dt + \psi_3^T dw \end{cases}$$
(1)

For stochastic control system dx = f(x) dt + h(x) dw, where  $f(\cdot)$  and  $h(\cdot)$  are locally Lipchitz functions, the following concepts are proposed.

**Definition 2.1.** For any given V(x), define the differential operator L as follows:

$$LV = \frac{\partial V}{\partial x}f + \frac{1}{2}Tr\left\{h^T\frac{\partial^2 V}{\partial x^2}h\right\}$$
(2)

**Assumption 2.1.** The sign of  $g_i$  which is defined as the coefficient of  $x_i$  does not change, so there exist constants  $b_m$  and  $b_M$  such that for  $1 \le i \le 4$ :  $0 < b_m \le g_i \le b_M < \infty$ .

In this paper, RBF neural networks will be used to approximate continuous function, which are used as the form  $f(Z) = W^T S(Z)$ , with  $Z \in \Omega_Z$  being input vector,  $W = [w_1, w_2, \ldots, w_l]^T$  is weight vector, l > 1 is neural networks node number, and  $S(Z) = [s_1(Z), s_2(Z), \ldots, s_l(Z)]^T$  means basis function vector with  $s_i(Z)$  being used as Gaussian function as follows:  $s_i(Z) = \exp\left[-\frac{(Z-\mu_i)^T(Z-\mu_i)}{\eta_i^2}\right]$ ,  $i = 1, 2, \ldots, l$ , where  $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]^T$  is the center of the receptive field, and  $\eta_i$  is the width of Gaussian function. In [10], it has been shown that for f(Z) over a compact set  $\Omega_Z$  with sufficiently large l, for any  $\varepsilon > 0$ , there exists an RBF neural network  $W^T S(Z)$  such as:

$$f(Z) = W^T S(Z) + \delta(Z), \quad \forall Z \in \Omega_z$$
(3)

where W is ideal weight vector, and  $\delta(Z)$  is approximation error and satisfies  $|\delta(Z)| \leq \varepsilon$ .

3. Adaptive Neural Control via Backstepping. In this section, a backstepping based control design procedure will be developed by the following coordinate transformation:

$$z_i = x_i - \alpha_{i-1}, \quad i = 1, \dots, 4$$
 (4)

with  $\alpha_0 = x_d$ , and  $\alpha_3 = 0$ . The unknown constant  $\theta_i$  is specified as:

$$\theta_i = \frac{1}{b_m} \|W_i\|^2; \quad i = 1, 2$$
(5)

The adaptive laws will be constructed as the following form:

$$\dot{\hat{\theta}}_i = \frac{r_i}{2\lambda_i^2} z_{i+2}^6 S_i^T \left( Z_i \right) S_i \left( Z_i \right) - m_i \hat{\theta}_i \tag{6}$$

where  $\hat{\theta}_i$  is estimation of  $\theta_i$ , and  $\lambda_i$ ,  $m_i$  and  $r_i$  are positive design parameters for i = 1, 2.

Step 1: According to the coordinate transformation (4), it follows from the first differential equation of system (1) that:

$$dz_1 = (x_2 - \dot{x}_d) dt \tag{7}$$

Then we choose a Lyapunov function candidate as follows:

$$V_1 = \frac{1}{4}z_1^4 \tag{8}$$

By Equations (7) and (8), we can get  $\dot{V}_1 = z_1^3 (z_2 + \alpha_1 - \dot{x}_d)$ , where  $z_2 = x_2 - \alpha_1$ . According to Young's inequality, one has  $z_1^3 z_2 \leq \frac{3}{4} z_1^4 + \frac{1}{4} z_2^4$ . Consequently, with  $v_1 = k_1 - \frac{3}{4} > 0$ , it can be verified by choosing  $\alpha_1 = -k_1 z_1 + \dot{x}_d$  that:

$$\dot{V}_1 \le -v_1 z_1^4 + \frac{1}{4} z_2^4 \tag{9}$$

Step 2: Similar to Step 1, we can obtain:

$$dz_2 = \left(\frac{a_1}{J}x_3 + \frac{a_2}{J}x_3x_4 - \frac{B}{J}x_2 - \frac{T_L}{J} - \dot{\alpha}_1\right)dt + \psi_1^T dw$$
(10)

Also, choose the Lyapunov function candidate as:  $V_2 = V_1 + \frac{1}{4}z_2^4$ .

By Equations (2) and (10), we have:

$$LV_2 = \dot{V}_1 + z_2^3 \left( \left( \frac{a_1}{J} + \frac{a_2}{J} x_4 \right) x_3 - \frac{B}{J} x_2 - \frac{T_L}{J} - \dot{\alpha}_1 \right) + \frac{3}{2} z_2^2 \psi_1^T \psi_1 \tag{11}$$

According to Young's inequality, the inequality  $\frac{3}{2}z_2^2\psi_1^T\psi_1 \leq \frac{3}{4}l_1^{-2}z_2^4 \|\psi_1\|^4 + \frac{3}{4}l_1^2$  holds, where  $l_1$  is a designed positive constant. Substituting (9) into (11) gives:

$$LV_{2} \leq -v_{1}z_{1}^{4} + z_{2}^{3} \left( \left( \frac{a_{1}}{J} + \frac{a_{2}}{J}x_{4} \right) (z_{3} + \alpha_{2}) + \frac{1}{4}z_{2} + \frac{3}{4}l_{1}^{-2}z_{2} \|\psi_{1}\|^{4} - \frac{B}{J}x_{2} - \frac{T_{L}}{J} - \dot{\alpha}_{1} \right) + \frac{3}{4}l_{1}^{2}z_{2} \|\psi_{1}\|^{4} - \frac{B}{J}x_{2} - \frac{T_{L}}{J} - \dot{\alpha}_{1} \right) + \frac{3}{4}l_{1}^{2}z_{2} \|\psi_{1}\|^{4} - \frac{B}{J}x_{2} - \frac{T_{L}}{J} - \dot{\alpha}_{1} \right) + \frac{3}{4}l_{1}^{2}z_{2} \|\psi_{1}\|^{4} - \frac{B}{J}x_{2} - \frac{T_{L}}{J} - \dot{\alpha}_{1} \right) + \frac{3}{4}l_{1}^{2}z_{2} \|\psi_{1}\|^{4} - \frac{B}{J}x_{2} - \frac{T_{L}}{J} - \dot{\alpha}_{1} \right) + \frac{3}{4}l_{1}^{2}z_{2} \|\psi_{1}\|^{4} - \frac{B}{J}x_{2} - \frac{T_{L}}{J} - \dot{\alpha}_{1} \right) + \frac{3}{4}l_{1}^{2}z_{2} \|\psi_{1}\|^{4} - \frac{B}{J}x_{2} - \frac{T_{L}}{J} - \dot{\alpha}_{1} \right) + \frac{3}{4}l_{1}^{2}z_{2} \|\psi_{1}\|^{4} + \frac{B}{J}x_{2} - \frac{T_{L}}{J} - \dot{\alpha}_{1} \right) + \frac{3}{4}l_{1}^{2}z_{2} \|\psi_{1}\|^{4} + \frac{B}{J}x_{2} - \frac{T_{L}}{J} - \dot{\alpha}_{1} \right) + \frac{3}{4}l_{1}^{2}z_{2} \|\psi_{1}\|^{4} + \frac{B}{J}x_{2} - \frac{T_{L}}{J} - \dot{\alpha}_{1} \right) + \frac{3}{4}l_{1}^{2}z_{2} \|\psi_{1}\|^{4} + \frac{B}{J}x_{2} - \frac{T_{L}}{J} - \dot{\alpha}_{1} \right) + \frac{3}{4}l_{1}^{2}z_{2} \|\psi_{1}\|^{4} + \frac{B}{J}x_{2} - \frac{T_{L}}{J} - \dot{\alpha}_{1} \right) + \frac{3}{4}l_{1}^{2}z_{2} \|\psi_{1}\|^{4} + \frac{B}{J}x_{2} - \frac{T_{L}}{J} - \dot{\alpha}_{1} + \frac{B}{J}x_{2} + \frac{B}{J}z_{2} + \frac{B}{J}z_$$

where  $z_3 = x_3 - \alpha_2$ . By using Young's inequality again, it is obtained that:

$$\left(\frac{a_1}{J} + \frac{a_2}{J}x_4\right)z_2^3 z_3 \le \frac{3}{4}\left(\frac{a_1}{J} + \frac{a_2}{J}x_4\right)z_2^4 + \frac{1}{4}\left(\frac{a_1}{J} + \frac{a_2}{J}x_4\right)z_3^4$$

Choose  $\alpha_2 = \frac{1}{\left(\frac{a_1}{J} + \frac{a_2}{T}x_4\right)} \left(-k_2 z_2 - \frac{1}{4} z_2 - \frac{3}{4} l_1^{-2} z_2 \|\psi_1\|^4 + \frac{B}{J} x_2 + \frac{T_L}{J} + \dot{\alpha}_1\right)$ , and we can easily get:

$$LV_2 \le -v_1 z_1^4 - v_2 z_2^4 + \frac{1}{4} \left(\frac{a_1}{J} + \frac{a_2}{J} x_4\right) z_3^4 + \frac{3}{4} l_1^2 \tag{12}$$

with  $v_2 = k_2 - \frac{3}{4} \left( \frac{a_1}{J} + \frac{a_2}{J} x_4 \right) > 0.$ **Step 3:** By (2) and (4), the following equation is obtained easily:

$$dz_3 = (b_1 x_3 + b_2 x_2 x_4 + b_3 x_2 + b_4 u_q - L\alpha_2) dt + \left(\psi_2 - \frac{\partial \alpha_2}{\partial x_2}\psi_1\right)^T dw$$
(13)

where

$$L\alpha_2 = \sum_{i=1}^2 \frac{\partial \alpha_2}{\partial x_i} \dot{x}_i + \sum_{i=0}^2 \frac{\partial \alpha_2}{\partial x_d^{(i)}} x_d^{(i+1)} + \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 + \frac{1}{2} \sum_{p,q=1}^2 \frac{\partial^2 \alpha_2}{\partial x_p \partial x_q} \psi_p^T \psi_q \tag{14}$$

Choose stochastic Lyapunov function candidate as  $V_3 = V_2 + \frac{1}{4}z_3^4 + \frac{b_m}{2r_1}\tilde{\theta}_1^2$ . By Equations (2) and (13), one has

$$LV_{3} = LV_{2} + z_{3}^{3} \left( b_{1}x_{3} + b_{2}x_{2}x_{4} + b_{3}x_{2} + b_{4}u_{q} - L\alpha_{2} \right) + \frac{3}{2}z_{3}^{2} \left( \psi_{2} - \frac{\partial\alpha_{2}}{\partial x_{2}}\psi_{1} \right)^{T} \left( \psi_{2} - \frac{\partial\alpha_{2}}{\partial x_{2}}\psi_{1} \right) - \frac{b_{m}}{r_{1}}\tilde{\theta}_{1}\dot{\hat{\theta}}_{1}$$
(15)

Note that  $\frac{3}{2}z_3^2 \left(\psi_2 - \frac{\partial\alpha_2}{\partial x_2}\psi_1\right)^T \left(\psi_2 - \frac{\partial\alpha_2}{\partial x_2}\psi_1\right) \leq \frac{3}{4}l_2^{-2}z_3^4 \left\|\psi_2 - \frac{\partial\alpha_2}{\partial x_2}\psi_1\right\|^4 + \frac{3}{4}l_2^2$  with  $l_2 > 0$  being a designed constant. Then, using the above inequality and substituting (12) and (14) into (15) shows:

$$LV_{3} \leq -v_{1}z_{1}^{4} - v_{2}z_{2}^{4} + \frac{3}{4}\sum_{i=1}^{2}l_{i}^{2} + z_{3}^{3}\left(b_{4}u_{q} + \bar{f}_{1}\left(Z_{1}\right)\right) - \frac{3}{4}z_{3}^{4} - \frac{b_{m}}{r_{1}}\tilde{\theta}_{1}\dot{\hat{\theta}}_{1}$$
(16)

where  $\bar{f}_1(Z_1) = \frac{1}{4} \left( \frac{a_1}{J} + \frac{a_2}{J} x_4 \right) z_3 + b_1 x_3 + b_2 x_2 x_4 + b_3 x_2 - \sum_{i=1}^2 \frac{\partial \alpha_2}{\partial x_i} \dot{x}_i - \sum_{i=0}^2 \frac{\partial \alpha_2}{\partial x_d^{(i)}} x_d^{(i+1)} - \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 - \sum_{i=0}^2 \frac{\partial \alpha_2}{\partial x_d^{(i)}} \dot{x}_d^{(i+1)} - \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 - \sum_{i=0}^2 \frac{\partial \alpha_2}{\partial x_d^{(i)}} \dot{x}_d^{(i+1)} - \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 - \sum_{i=0}^2 \frac{\partial \alpha_2}{\partial x_d^{(i)}} \dot{x}_d^{(i+1)} - \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 - \sum_{i=0}^2 \frac{\partial \alpha_2}{\partial x_d^{(i)}} \dot{x}_d^{(i+1)} - \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 - \sum_{i=0}^2 \frac{\partial \alpha_2}{\partial x_d^{(i)}} \dot{x}_d^{(i+1)} - \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 - \sum_{i=0}^2 \frac{\partial \alpha_2}{\partial x_d^{(i)}} \dot{x}_d^{(i+1)} - \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 - \sum_{i=0}^2 \frac{\partial \alpha_2}{\partial x_d^{(i)}} \dot{x}_d^{(i+1)} - \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 - \sum_{i=0}^2 \frac{\partial \alpha_2}{\partial x_d^{(i)}} \dot{x}_d^{(i+1)} - \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 - \sum_{i=0}^2 \frac{\partial \alpha_2}{\partial x_d^{(i)}} \dot{x}_d^{(i+1)} - \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 - \sum_{i=0}^2 \frac{\partial \alpha_2}{\partial x_d^{(i)}} \dot{x}_d^{(i+1)} - \sum_{i=0}^2 \frac{\partial \alpha_2}{\partial x_d^{(i+1)}}$ 

 $\frac{1}{2} \sum_{p,q=1}^{2} \frac{\partial^2 \alpha_2}{\partial x_p \partial x_q} \psi_p^T \psi_q + \frac{3}{4} l_2^{-2} z_3 \left\| \left( \psi_2 - \frac{\partial \alpha_2}{\partial x_2} \psi_1 \right) \right\|^4 + \frac{3}{4} z_3. \text{ According to (3), there exists a neural network such that } \bar{f}_1 \left( Z_1 \right) = W_1^T S_1 \left( Z_1 \right) + \delta_1 \left( Z_1 \right) \text{ with } \left| \delta_1 \left( Z_1 \right) \right| \le \varepsilon_1. \text{ Furthermore, it follows from Young's inequality and (5) that:}$ 

$$z_{3}^{3}\bar{f}_{1}(Z_{1}) \leq \frac{b_{m}}{2\lambda_{1}^{2}} z_{3}^{6}\theta_{1} S_{1}^{T} S_{1} + \frac{1}{2}\lambda_{1}^{2} + \frac{3}{4}z_{3}^{4} + \frac{1}{4}\varepsilon_{1}^{4}$$

$$\tag{17}$$

Substituting (17) into (16) gives:

$$LV_3 \le -v_1 z_1^4 - v_2 z_2^4 + \frac{3}{4} \sum_{j=1}^2 l_j^2 + z_3^3 b_4 u_q + \frac{b_m}{2\lambda_1^2} z_3^6 \theta_1 S_1^T S_1 + \frac{1}{2} \lambda_1^2 + \frac{1}{4} \varepsilon_1^4 - \frac{b_m}{r_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1 \qquad (18)$$

The control input  $u_q$  can be chosen as  $u_q = -k_3 z_3 - \frac{1}{2\lambda_1^2} z_3^3 \hat{\theta}_1 S_1^T S_1$ . Further using (6), (18) can be expressed as  $LV_3 \leq -\sum_{i=1}^3 v_i z_i^4 + \frac{1}{2}\lambda_1^2 + \frac{3}{4}\sum_{i=1}^2 l_i^2 + \frac{1}{4}\varepsilon_1^4 + \frac{b_m m_1}{r_1} \tilde{\theta}_1 \hat{\theta}_1$  with  $v_3 = k_3 b_4 > 0$ . **Step 4:** By Equation (4), one has:

$$dz_4 = (c_1 x_4 + c_2 x_2 x_3 + c_3 u_d) dt + \psi_3^T dw$$
(19)

Choose stochastic Lyapunov function candidate as  $V_4 = V_3 + \frac{1}{4}z_4^4 + \frac{b_m}{2r_2}\tilde{\theta}_2^2$ . By Equations (2) and (19), the following equation can be obtained:

$$LV_4 = LV_3 + z_4^3(c_1x_4 + c_2x_2x_3 + c_3u_d) + \frac{3}{2}z_4^2\psi_3^T\psi_3 - \frac{b_m}{r_2}\tilde{\theta}_2\dot{\theta}_2$$

Then, by repeating the same line in Step 3, the following inequality can be verified:

$$LV_{4} \leq -\sum_{i=1}^{3} v_{i} z_{i}^{4} + \frac{1}{2} \lambda_{1}^{2} + \frac{3}{4} \sum_{i=1}^{3} l_{i}^{2} + \frac{1}{4} \varepsilon_{1}^{4} + \frac{b_{m} m_{1}}{r_{1}} \tilde{\theta}_{1} \hat{\theta}_{1} + z_{4}^{3} \left( c_{3} u_{d} + \bar{f}_{2} \left( Z_{2} \right) \right) - \frac{3}{4} z_{4}^{4} - \frac{b_{m}}{r_{2}} \tilde{\theta}_{2} \hat{\theta}_{2}$$

$$\tag{20}$$

where  $\bar{f}_2(Z_2) = c_1 x_4 + c_2 x_2 x_3 + \frac{3}{4} l_3^{-2} z_4 \|\psi_3\|^4 + \frac{3}{4} z_4$ . Similarly, one has:

$$z_4^3 \bar{f}_2(Z_2) \le \frac{b_m}{2\lambda_2^2} z_4^6 \theta_2 S_2^T S_2 + \frac{1}{2}\lambda_2^2 + \frac{3}{4}z_4^4 + \frac{1}{4}\varepsilon_2^4$$

The control input  $u_d$  can be chosen as  $u_d = -k_4 z_4 - \frac{1}{2\lambda_2^2} z_4^3 \hat{\theta}_2 S_2^T S_2$ . Then, with  $v_4 = k_4 c_3 > 0$ , the inequality (20) can be expressed as:

$$LV_4 \le -\sum_{i=1}^4 v_i z_i^4 + \frac{1}{2} \sum_{i=1}^2 \lambda_i^2 + \frac{3}{4} \sum_{i=1}^3 l_i^2 + \frac{1}{4} \sum_{i=1}^2 \varepsilon_i^4 + \sum_{i=1}^2 \frac{b_m m_i}{r_i} \tilde{\theta}_i \hat{\theta}_i$$
(21)

4. Stability Analysis. For stability analysis of the closed-loop system, we choose Lyapunov function as  $V = V_4$ . For the term  $\frac{m_i b_m}{r_i} \tilde{\theta}_i \hat{\theta}_i$ , the following inequality is obvious:

$$-\frac{m_i b_m}{r_i} \tilde{\theta}_i^2 + \frac{m_i b_m}{r_i} \tilde{\theta}_i \theta_i \le -\frac{m_i b_m}{2r_i} \tilde{\theta}_i^2 + \frac{m_i b_m}{2r_i} \theta_i^2$$
(22)

Furthermore, substituting (22) into (21), we have:

$$LV \le -\sum_{i=1}^{4} v_i z_i^4 + \frac{1}{2} \sum_{i=1}^{2} \lambda_i^2 + \frac{3}{4} \sum_{i=1}^{3} l_i^2 + \frac{1}{4} \sum_{i=1}^{2} \varepsilon_i^4 + \sum_{i=1}^{2} \left( -\frac{b_m m_i}{2r_i} \tilde{\theta}_i^2 + \frac{b_m m_i}{2r_i} \theta_i^2 \right)$$
(23)

Let  $a_0 = \min\{4v_1, 4v_2, 4v_3, 4v_4, m_1, m_2\}$  and  $b_0 = \frac{1}{2}\sum_{i=1}^2 \lambda_i^2 + \frac{3}{4}\sum_{i=1}^3 l_i^2 + \frac{1}{4}\sum_{i=1}^2 \varepsilon_i^4 + \sum_{i=1}^2 \frac{b_m m_i}{2r_i}\theta_i^2$ , (23) can be rewritten as the following form:

$$LV \le -a_0 V + b_0, \quad t \ge 0 \tag{24}$$

Therefore,  $z_i$  and  $\tilde{\theta}_i$  are bounded in probability.  $\alpha_i$  is also bounded in probability because  $||S_i|| \leq s$ . Consequently, all the signals in the closed loop system remain bounded in the sense of probability. Furthermore, (24) and [3] (Th. 4.1) imply that:

$$\frac{dE\left[V\left(t\right)\right]}{dt} \le -a_0 E\left[V\left(t\right)\right] + b_0$$

Thus, to guarantee that the tracking error converges to a small neighborhood around the origin, we can properly adjust the parameters  $a_0$  and  $b_0$ .

5. Simulation Example. In order to illustrate the effectiveness of the proposed approach, the simulation is run for PMSM with parameters as follows: J = 0.003798, B = 0.001158,  $a_1 = 0.56025$ ,  $a_2 = -0.00135$ ,  $b_1 = -215.873$ ,  $b_2 = -2.714$ ,  $b_3 = -118.571$ ,  $b_4 = 317.460$ ,  $c_1 = -238.596$ ,  $c_2 = 3.316$ ,  $c_3 = 350.877$ ,  $\psi_1 = 0.25$ ,  $\psi_2 = 0.15 \cos x_2$ ,  $\psi_3 = 0.15 \sin x_3$ .

The simulation is carried out under zero initial condition for PMSM. Reference signal is taken as  $x_d = \sin(t)$  and load torque  $T_L = \begin{cases} 1.5, & 0 \le t \le 5, \\ 2, & t > 5 \end{cases}$ . Neural networks  $W_1^T S_1(Z_1)$  and  $W_2^T S_2(Z_2)$  contain eleven nodes with centers spaced evenly in the interval [-5, 5]. The following control parameters are chosen:

$$k_1 = 4, \ k_2 = 10, \ k_3 = 14, \ k_4 = 10, \ r_1 = r_2 = 2.5, \ \lambda_1 = \lambda_2 = 2, \ m_1 = 0.5, \ m_2 = 0.005.$$

The simulation results are shown by Figures 1 and 2. Figure 1 displays system output and reference signal, and Figure 2 shows trajectories of input signals. From Figures 1 and 2, it is seen clearly that the proposed controller can track reference signal well and controller is bounded.



(dotted line)

(dotted line)

6. **Conclusions.** This paper studies adaptive neural networks backstepping position tracking control method for PMSM stochastic nonlinear systems. The proposed controller can guarantee that all signals of the closed loop system are bounded and the tracking error converges to an arbitrarily small neighborhood of the origin. The simulation results illustrate the effectiveness of the proposed control scheme. It should be pointed out that the work in this paper does not consider the problem of input saturation. So how to control a nonlinear system with input saturation is our future research direction.

Acknowledgment. This work is partially supported by the National Natural Science Foundation of China under Grant 61473160 and Grant 61174033. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

## REFERENCES

- H. Deng, M. Krstić and R. J. Williams, Stabilization of stochastic nonlinear systems driven by noise of unknown covariance, *IEEE Trans. Automatic Control*, vol.46, no.8, pp.1237-1253, 2001.
- [2] S. J. Liu, J. F. Zhang and Z. P. Jiang, Decentralized adaptive output-feedback stabilization for large-scale stochastic nonlinear systems, *Automatica*, vol.43, no.2, pp.238-251, 2007.
- [3] R. Khasminskii, Stochastic Stability of Differential Equations, Springer Science & Business Media, 2011.
- [4] C. Li, L. Chen and K. Aihara, Stochastic stability of genetic networks with disturbance attenuation, IEEE Trans. Circuits and Systems II: Express Briefs, vol.54, no.10, pp.892-896, 2007.
- [5] Z. Q. Zhu and D. Howe, Electrical machines and drives for electric, hybrid, and fuel cell vehicles, Proc. of the IEEE, vol.95, no.4, pp.746-765, 2007.
- [6] I. C. Baik, K. H. Kim and M. J. Youn, Robust nonlinear speed control of PM synchronous motor using boundary layer integral sliding mode control technique, *IEEE Trans. Control Systems Technology*, vol.8, no.1, pp.47-54, 2000.
- [7] L. Wang, Y. Li, X. Zhu and J. Zhang, Chaos synchronization of permanent magnet synchronous motor with disturbance using fuzzy adaptive logic, *Power System Protection and Control*, vol.39, no.11, pp.33-37, 2011.
- [8] X. Cao and L. Fan, Vector controlled permanent magnet synchronous motor drive based on neural network and multi-fuzzy controllers, *The 5th International Conference on Fuzzy Systems and Knowledge Discovery*, vol.3, pp.254-258, 2008.
- [9] J. Yu, J. Gao, Y. Ma and H. Yu, Adaptive fuzzy tracking control for a permanent magnet synchronous motor via backstepping approach, *Mathematical Problems in Engineering*, 2010.
- [10] M. Karabacak and H. I. Eskikurt, Design, modelling and simulation of a new nonlinear and full adaptive backstepping speed tracking controller for uncertain PMSM, *Applied Mathematical Modelling*, vol.36, no.11, pp.5199-5213, 2012.