APPLICATION OF TISSUE P SYSTEMS IN OPTIMAL IIR FILTER DESIGN

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ABSTRACT. Since the error surface of IIR filter is usually nonlinear and multimodal, classical filter design methods can suffer from some shortcomings: slow convergence and local minimums. P systems are a class of distributed parallel computing models, inspired from the structure and functioning of living cells as well as the interaction of cells in tissues and organs. To overcome the shortcomings in IIR filter design, this paper proposes a novel P systems-based method for optimal IIR filter design. A tissue P system with ring membrane structure is considered as its computing framework, where each object in cells expresses a group of filter coefficients to be optimized. Based on the ring structure, an evolution mechanism is developed for the objects in cells. The proposed method was evaluated on three benchmark examples and compared with three state-of-the-art design methods. The comparison results demonstrate the superiority of the proposed method in terms of effectiveness and robustness.

Keywords: P systems, Tissue P systems, IIR filter design, System identification

1. Introduction. In recent years an important task in digital signal processing has been concerned with design of digital filters. Adaptive infinite impulse response (IIR) filter is one of the most frequently used computing tools in digital signal processing systems. In this filter the output feedback generates an infinite impulse response with only a finite number of parameters. Hence, with same number of coefficients, an adaptive IIR filter performs better than an adaptive finite impulse response (FIR) filter. Alternatively, to achieve a particular level of performance, an IIR filter requires less number of coefficients than the corresponding FIR filter.

The IIR filter design can be considered as an optimization problem, where each requirement contributes with a term to an error function that should be minimized. For the optimization problem, classical gradient-based algorithms can suffer some difficulties: they easily get stuck in local minimum and cannot converge to its global minimum due to its multimodal error surface. For this, some evolutionary algorithms have been introduced to solve the optimization problem in recent years. Genetic algorithms (GA) were first considered to obtain the global optimal coefficients of the adaptive filter [1]. However, GA has two shortcomings: the premature convergence and the lack of good local search ability. An ant colony optimization (ACO)-based IIR filter design method was discussed [2]; however, the existing research has indicated that it still has the search stagnation. Then, other evolutionary algorithms, such as particle swarm optimization (PSO), differential evolution (DE) and artificial bee colony (ABC), have been used to improve the problems stated above. Krusienski and Jenkins [3] discussed a PSO-based design method for IIR filter. Two DE-based and ABC-based design methods that used DE and ABC to optimize the filter's coefficients have been reported in Karaboga [4] and [5], respectively. This paper will focus on how to use P systems to develop a novel design method for the optimal IIR filter design problem.

Membrane computing, as a branch of natural computing, aims to abstract computing models from the structure and functioning of living cells as well as interaction of living cells in tissues and organs. Membrane computing is a class of distributed parallel computing models, known as P systems [6, 7, 8]. A P system usually has three parts: membrane structure, multisets of objects and rules. The multisets of objects are placed in compartments surrounded by membranes, and are evolved by some given rules that can be applied in parallel. In the past years, a variety of variants of P systems have been proposed, such as cell P systems [6], tissue P systems [9, 10], (fuzzy) spiking neural P systems [11, 12, 13, 14], kernel P systems [15] and population P systems [16]. These variants have indicated the parallel computing advantage of P systems and the high effectiveness of solving a lot of difficult problems.

In recent years, application of P systems to solve real-world problems has received a great deal of attention, for example, global optimization problem [17], data clustering [18, 19], fault diagnosis [20, 21], image processing [22, 23], signal processing [24, 25], system and synthetic biology [26]. Especially, the research results on global optimization problems have indicated that compared to the existing methods, P systems offer a more competitive method due to their three advantages: better convergence, stronger robustness and better balance between exploration and exploitation. The motivation behind this work is applying P systems to develop an efficient method for optimal IIR filter design problem. A tissue P system is considered as a computing framework, and its evolution-communication mechanism is developed to find the optimal filter's coefficients for IIR filter design problem. Therefore, most difference of this work with the existing methods stays in use of tissue P systems rather than the existing evolutionary algorithms in the optimal IIR filter design.

The rest of paper is organized as follows. Section 2 introduces the optimal IIR design problem. The proposed design method based P systems is discussed in detail in Section 3. Section 4 presents experimental results to illustrate the efficiency of the proposed method. Finally, Section 5 includes the conclusions.

2. **Problem Statement.** An IIR system can be described by the following transfer function

$$H_s(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_L z^{-L}}{1 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}$$
(1)

where L and M are the orders of Z-domain feed-forward and feed-back coefficient polynomials of the IIR system respectively; a_i and b_i denote the corresponding feed-forward and feed-back coefficients. The feed-back filter order M is greater than the feed-forward filter order L. The IIR system can also be expressed by the following difference equation

$$y(k) = \sum_{i=1}^{M} b_i(k)y(k-i) + \sum_{i=1}^{L} a_i(k)x(k-i)$$
(2)

Figure 1 shows the block diagram of IIR system identification by an adaptive IIR filter. For the identification problem. adaptive algorithms can adjust the feed-forward and feedback coefficients of the adaptive filter to minimize the error e(k) = d(k) - y(k), where d(k) denotes the output of the IIR system. Thus, the optimal design problem of IIR filter can be considered as an optimization problem, where a mean square error (MSE) is used as its objective function

$$\min_{w} J(w) = \min_{w} \frac{1}{N} \sum_{k=1}^{N} (d(k) - y(k))^2$$
(3)

where $w = (a_0, a_1, \ldots, a_L, b_1, b_2, \ldots, b_M)$ denotes the coefficient vector of the IIR filter, and N is the number of input samples.

In this work, a tissue P system will be designed to optimize the filter's coefficient vector for the optimal IIR filter design problem.



FIGURE 1. IIR system identification based on an adaptive IIR filter

3. Proposed Design Method Based on P Systems. Tissue P systems are a kind of P systems, which are inspired from the behaviours of multiple one-membrane cells evolved in a public environment. Tissue P system has a net-like structure where each cell is regarded as an information processor that processes the objects and communicates them between the cells along the channels assigned in advance. The P systems have two object's processing mechanisms: evolution and communication mechanisms. More details of tissue P systems can be found in literature [7] and [9].

3.1. A tissue P system. The considered P system in this work is a tissue P system of degree q and can be formally defined as follows,

$$\Pi = (O, R_1, R_2, \dots, R_q, R', i_o)$$

where

(1) O is the set of objects in cells;

(2) R_i is the set of evolution rules in *i*th cell, $1 \le i \le q$;

(3) R' is the set of communication rules between the q cells;

(4) $i_o = 0$ indicates that the environment is output region of the system.



FIGURE 2. The designed tissue P system

Figure 2 shows the membrane structure of the tissue P system. In the P system, a ring membrane structure is considered to realize a special object evolution, where arrows denote the communication channels of objects. Objects are evolved in the cells and communicated along the channels. The environment labeled by 0 is assigned as its output region.

3.2. **Objects.** To apply P system to solve the optimization problem, its each object is designed to denote a coefficient vector of adaptive IIR filter,

$$X = (x_1, x_2, \dots, x_D) = (a_0, a_1, \dots, a_L, b_1, \dots, b_M)$$
(4)

where the dimension of the object is D = L + M + 1.

Suppose that the q cells have the same number of objects, denoted by m. The environment (i.e., output region) stores a global best object (denoted by X_{gbest}), which is the found object with the lowest MSE value in the q cells so far. X_{gbest} will be updated in each computing step, and it is also the final computing result when the system halts.

3.3. Evolution mechanism. The evolution rules are considered to generate new objects used in next computing step. During the evolution, each cell maintains the same size (the same number of objects). In this work, three known genetic operations (selection, crossover and mutation) are used to evolve the objects in cells. In a computing step, all objects (located in object pool) in each cell and the best object (located in external pool) from its adjacent cell constitute a matching pool. The object in external pool is actually the best object communicated from its adjacent cell in previous computing step. The objects in matching pool will be evolved by executing selection, crossover and mutation operations in turn. To maintain the size of objects in each cell, truncation operation is used to constitute new object pool according to the MSE values of objects. The objects in new object pool will be regarded as the objects to be evolved in next computing step. Figure 3 shows the evolution procedure of objects in cells.



FIGURE 3. Evolution procedure of objects in a cell

In this work, selection operation uses usual rotating wheel method, while crossover operation uses singlepoint crossover in which the position of crossover point is determined according to crossover probability p_c . The single-point mutation is used to realize the mutations of objects. If v is a mutation point determined according to mutation probability p_m , its value becomes, after mutating,

$$v = \begin{cases} v \pm 2\delta v, & v \neq 0\\ v \pm 2\delta, & v = 0 \end{cases}$$
(5)

where the signs "+" or "-" occur with equal probability, and δ is real number in the range [0, 1], generated with uniform distribution.

3.4. Communication mechanism. Communication mechanism is designed to exchange the objects between two adjacent cells or between each cell and the environment. Each cell communicates its best object, X_{lbest} , into its subsequent cell in the ring membrane structure. Moreover, Z_{lbest} is transmitted into the environment to update the global best object, X_{gbest} . The tissue P system has two types of communication rules:

- Rule $(i, Z_{lbest}/\lambda, j)$, where j = i + 1 for $\forall i \in \{1, 2, \dots, q-1\}$ or j = 1 for i = nThe rule communicates the local best object in cell i, Z_{lbest}^i , into its subsequent cell j. Thus, cell j will receive the object as its external best object.
- Rule $(i, Z_{lbest}^i / \lambda, 0)$, where $i = 1, 2, \dots, q$

The rule transmits the local best object in cell i, Z_{lbest}^{i} , into the environment and updates the global best object of the system, Z_{gbest} . The updating strategy can be described as follows:

$$Z_{gbest} = \begin{cases} Z_{lbest}^{i}, & \text{if } f(Z_{lbest}^{i}) < f(Z_{gbest}) \\ Z_{gbest}, & \text{otherwise} \end{cases}$$
(6)

where $f(\cdot)$ denotes the MSE value of an object.

3.5. Halting and output. As usual in P system, the q cells, as parallel computing units, will run independently. In addition, the environment always stores the best object found in the system so far, X_{gbest} .

In this work, maximum execution step number T_{max} is used as the halting condition of the tissue P system, that is, the tissue P system will continue to execute until it reaches the maximum execution step number. When the system halts, the best object X_{gbest} stored in the environment will be regarded as the output of whole system, namely, the found optimal filter's coefficients.

4. Experiments and Analysis. To test the performance of the proposed design method based on tissue P system, three benchmark examples were used in the experiment, shown in Table 1. For example 1, the filter with same order was employed to identify the IIR system. However, two filters with the reduced orders are used to identify the IIR systems in example 2 and example 3, respectively. Thus, the error surfaces of the last two examples are multimodal, so they have much local minimums.

Examples	Transfer functions		
	IIR system	Filter	
1	$H_s(z) = \frac{1}{1 - 1.2z^{-1} + 0.6z^{-2}}$	$H_f(z) = \frac{1}{1 + b_1 z^{-1} + b_2 z^{-2}}$	
2	$H_s(z) = \frac{0.05 - 0.4z^{-1}}{1 - 1.1314z^{-1} + 0.25z^{-2}}$	$H_f(z) = \frac{a_0}{1+b_1 z^{-1}}$	
3	$H_s(z) = \frac{1 - 0.4z^{-2} - 0.65z^{-4} + 0.26z^{-6}}{1 - 0.77z^{-2} - 0.8498z^{-4} + 0.648z^{-6}}$	$H_f(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}{1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}$	

TABLE 1. Three benchmark test examples of IIR systems

In the experiment, the proposed method was compared with three existing methods [5], ABC, PSO and LSQ-nonlin. The results of three methods for the three examples are from literature [5].

The parameters of tissue P system are chosen: q = 8, m = 20, $p_c = 0.8$ and $p_m = 0.2$. For example 1 and example 2, N = 100 and $T_{\text{max}} = 100$ are chosen, while N = 200 and $T_{\text{max}} = 500$ were considered in example 3. For each example, MSE value is used to evaluate the algorithm's performance. The proposed algorithm was executed 50 times for each example, and then mean value and standard deviation of MSE values for the 50 runs were computed.

Table 2 provides comparison results of the proposed method with other three methods in terms of mean and standard deviation of MSE. For example 1, tissue P system and ABC can attain lowest MSE = 0, while PSO and LSQ-nonlin cannot get the best MSE

TABLE 2. Comparison results of MSE for four methods on the benchmark test examples

Examples	Tissue P system	ABC	PSO	LSQ-nonlin
1	0.00	0.00	2.00e-4	5.73e-2
	(± 0.00)	(± 0.00)	$(\pm 1.00e-4)$	$(\pm 7.17e-2)$
0	2.06e-2	6.10e-2	6.46e-2	2.56e-1
Δ	$(\pm 5.36e-4)$	$(\pm 1.21e-2)$	$(\pm 1.85e-2)$	$(\pm 1.17e-1)$
3	1.16e-3	1.50e-3	5.60e-3	4.20e-2
	$(\pm 9.97e-5)$	$(\pm 5.00e-4)$	$(\pm 1.80e-3)$	$(\pm 6.11e-2)$

value. And tissue P system and ABC have standard deviation with 0, which indicates that the two methods can robustly obtain the optimal filter's coefficients.

For example 2, tissue P system has the lowest MSE = 2.06e-2, MSE values of ABC and PSO are 6.10e-2 and 6.46e-2 respectively, and LSQ-nonlin has the highest MSE = 2.56e-1. Since there are much local minimums in example 2, the comparison results illustrate that tissue P system can obtain the global optimal filter's coefficients while LSQ-nonlin may get stuck in some local minimum. In addition, it can be found that tissue P system has the smallest standard deviation, which indicates that the proposed design method is robust.

For example 3, MSE values of tissue P system and ABC are 1.16e-3 and 1.50e-3, which are better than those of other two methods. The comparison results indicate that tissue P system and ABC can obtain the optimal filter's coefficients. Since in example 3 the reduced order's filter is used to identify the IIR system, it also has much local minimums. It is obvious that LSQ-nonlin gets stuck in some local minimum. In addition, it can be observed that compared with PSO and LSQ-nonlin, tissue P system and ABC have better robustness. Table 3 also provides the filter's coefficients estimated by the four methods for example 3. Note that the coefficients of tissue P system are corresponding to MSE = 0.00116, which are the best filter's coefficients in the four methods for example 3.

Coefficients	Tissue P system	ABC	PSO	LSQ-nonlin
Coefficients	(0.00116)	(0.0015)	(0.0056)	(0.042)
a_0	1.0114	0.995	1.000	0.9994
a_1	0.086	0.103	0.119	0.0003
a_2	0.3404	0.282	0.286	0.3414
a_3	-0.0243	-0.073	-0.067	-0.0044
a_4	-0.4171	-0.347	-0.339	-0.3940
b_1	0.0018	0.086	0.086	0.08
b_2	-0.0026	-0.066	-0.067	-0.0048
b_3	-0.078	-0.078	-0.078	-0.0023
b_4	-0.8672	-0.798	-0.798	-0.8597

TABLE 3. The filter's coefficients estimated by the four methods for example 3

5. Conclusions. This paper discussed the application of tissue P systems to solve optimal IIR filter design problem and presented a novel design method. In experiment the proposed method has been applied to identify three benchmark IIR systems. The proposed method was compared with three existing methods, ABC, PSO and LSQ-nonlin. The comparison results on the three examples demonstrate that the proposed method can obtain the global optimal filter's coefficients. The research clearly indicates that compared to the existing design methods, tissue P systems can provide a competitive method for IIR filter design problem.

In a number of real-world engineering problems, some special filters, such as low pass (LP), high pass (HP), band pass (BP) and band stop (BS) filters, have been widely used. Our further work is to develop a novel design method based on P system for the special filters.

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