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OSCILLATION CRITERIA FOR CAPUTO IMPULSIVE FRACTIONAL ORDER DIFFERENTIAL EQUATIONS WITH MIXED NONLINEARITIES

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Abstract. *In this paper, by introducing impulse process into fractional order differential equations, a Caputo impulsive fractional order differential system taking the form as*

$$\begin{cases} {}_{t_0}^{C}\mathbf{D}_t^{\alpha} x - p(t)x(t) + q(t)|x(t)|^{\lambda-1}x(t) = e(t),\ t \in (t_0,\infty) \setminus \{t_0, t_1, t_2, \cdots\}, \\ \Delta x|_{t=t_k} = I_k\left(t_k, x(t_k)\right), \\ x\left(t_0^+\right) = x_0, \end{cases}$$

is established, where $0 < \alpha \le 1$, $t_0 < t_1 < t_2 < \cdots$, $\lim_{k\to\infty} t_k = \infty$, $p(t)$, $q(t)$, $e(t)$ *are continuous functions and* λ *is a ratio of odd positive integers. Furthermore, a new oscillation criterion for above mentioned equations is obtained in this paper. Finally, examples are given to illustrate the applicability of the results obtained.*
Keywords: Caputo's derivative, Impulsive equations, Oscillation, Mixed nonlinearity

1. **Introduction.** Recently, the investigation of differential equations of fractional order has received a great deal of attention due to the fact that they have been widely used to model phenomena arising in many branches of science such as physics, biology, and engineering, one can see [1, 2, 3, 4] and the reference therein. Moreover, one of the most important and interesting problems in the analysis of fractional order differential equations is their oscillation criterion. Recent years have witnessed a growing interest in this area, see [5] and [6]. In [5], the authors gave new oscillation criteria for fractional differential equations of the form

$$_a\mathbf{D}_t^q f_1(t,x) = v(t) + f_2(t,x), \tag{1}$$

where the functions f_1, f_2, and v are continuous.

Furthermore, many phenomena may suffer sudden environmental shocks, e.g., earthquakes, hurricanes, and epidemics. However, the model (1) cannot explain such phenomena. To explain these phenomena, introducing the impulse process into differential equations provides a feasible and more realistic model. Guo et al. [7] established an interval oscillation criterion for second-order forced impulsive differential equation with mixed nonlinearities by using arithmetic-geometric mean inequality and Riccati transformation. Motivated by aforementioned work [5, 6, 7], in this paper, we develop the model with impulse condition

$$\begin{cases} {}_{t_0}^{C}\mathbf{D}_t^{\alpha}x - p(t)x(t) + q(t)|x(t)|^{\lambda-1}x(t) = e(t),\ t \in (t_0,\infty)\setminus\{t_0,t_1,t_2,\cdots\}, \\ \Delta x|_{t=t_k} = I_k\left(t_k, x(t_k)\right), \\ x\left(t_0^+\right) = x_0, \end{cases} \tag{2}$$

where $0 < \alpha \le 1$, ${}_{t_0}^{C}\mathbf{D}_t^{\alpha}$ is the Caputo's fractional derivative, $\Delta x|_{t=t_k} = x\left(t_k^+\right) - x\left(t_k^-\right)$, $t_0 < t_1 < t_2 < \cdots$, $\lim_{k\to\infty} t_k = \infty$, $p(t)$, $q(t)$, $e(t)$ are continuous functions, λ is a ratio of odd positive integers,

$$I_k\left(t_k, x(t_k)\right) = a_k|x(t)| - b_k|x(t)|^{\omega}\big|_{t=t_k},\ k = 0,1,2,\cdots, \tag{3}$$

a_k, $b_k > 0$, $k = 1,2,\cdots$, are constants and $\omega > 0$ is a ratio of odd positive integers.

The contents of this paper will be arranged as follows. In Section 2 we collect some preliminaries. Section 3 will be concerned with the new oscillation criteria for Equation (2) and examples are provided to explain our theories in Section 4. Finally, we present the conclusion in Section 5.

2. **Preliminaries.**

Definition 2.1. [8] *A real-valued function $x(t)$ is called a solution of* (2) *on* $[t_0,T)$, $T>0$ *if*

(**i**) $x_{t_0^+} = x_0$;

(**ii**) $x(t)$ *is continuous function and satisfies* (2) *on every subinterval of* $[t_0,T)$ *not containing* $t = t_k$;

(**iii**)

$$x\left(t^+\right) = x(t) + I_k\left(t, x(t)\right) \tag{4}$$

for $t = t_k \in [t_0, t_0+T)$ *at which* $x(t)$ *is assumed to be left continuous, i.e.,* $x\left(t_k^-\right) = x(t_k)$.

A solution $x(t)$ of (2) is called oscillatory if it has arbitrarily large zeros on $[t_0,\infty)$; otherwise, it is called nonoscillatory. A differential equation is called oscillatory if every solution of the equation is oscillatory and nonoscillatory if it has at least one nonoscillatory solution.

Lemma 2.1. [9] *Let* $\alpha > 0$, *then*

$$\mathbf{I}_a^{\alpha}\,{}_a^{C}\mathbf{D}_t^{\alpha}u(t) = u(t) + c_0 + c_1(t-a) + c_2(t-a)^2 + \cdots + c_{n-1}(t-a)^{n-1},$$

for some $c_i \in \mathbb{R}$, $i = 0,1,2,\cdots,n-1$, $n = [\alpha]+1$, *where* $\mathbf{I}_a^{\alpha}$ *denotes the Riemann-Liouville integral operator of order* α.

Lemma 2.2. x *is a solution of the problem* (2) *if and only if the piecewise left continuous function* x *is a solution of the integral equation*

$$x(t_0) = x_0,$$

$$x(t) = \begin{cases} x_0 + \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}f\left(s,x(s)\right)\mathrm{d}s, & t\in(t_0,t_1], \\ x_0 + \frac{1}{\Gamma(\alpha)}\sum\limits_{i=1}^{k}\int_{t_{i-1}}^{t_i}(t_i-s)^{\alpha-1}f\left(s,x(s)\right)\mathrm{d}s & \\ +\frac{1}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}f\left(s,x(s)\right)\mathrm{d}s + \sum\limits_{i=1}^{k}I_i\left(t_i,x(t_i)\right), & t\in(t_k,t_{k+1}], \end{cases} \tag{5}$$

where

$$f\left(t,x(t)\right) = e(t) + p(t)x(t) - q(t)|x(t)|^{\lambda-1}x(t). \tag{6}$$

Proof: Suppose $x(t)$ is a solution of the problem (2). For any $t \in [t_0, t_1]$, noting that $0 < \alpha < 1$, by Lemma 2.1, we have

$$x(t) = \mathbf{I}_{t_0}^{\alpha} f(t, x(t)) + c_0.$$

This, together with $x(t_0) = x_0$, implies

$$c_0 = x_0,$$

then we obtain that

$$x(t) = x_0 + \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} f(s, x(s))\,\mathrm{d}s, \quad t \in (t_0, t_1]$$

and

$$x(t_1) = x\left(t_1^-\right) = x_0 + \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t_1} (t_1-s)^{\alpha-1} f(s, x(s))\,\mathrm{d}s. \tag{7}$$

Similar to that of ([10], Equation (2.10)), for $t \in (t_1, t_2]$, we have

$$x(t) = x\left(t_1^+\right) + \mathbf{I}_{t_1}^{\alpha} f(t, x(t)). \tag{8}$$

Then, combining (6) and (7) with (8), we thus derive that

$$\begin{aligned} x(t) &= x_0 + \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t_1} (t_1-s)^{\alpha-1} f(s, x(s))\,\mathrm{d}s \\ &\quad + \frac{1}{\Gamma(\alpha)} \int_{t_1}^{t} (t-s)^{\alpha-1} f(s, x(s))\,\mathrm{d}s + I_1(t_1, x(t_1)). \end{aligned}$$

The desired assertion then follows by repeating the previous procedures.

On the other hand, if x is a solution of integral Equation (5), it is easy to obtain x is also a solution of (2).

Lemma 2.3. [11] *Suppose X, Y and U, V are nonnegative, then*

$$\text{(I)} \quad \lambda X Y^{\lambda-1} - X^{\lambda} \le (\lambda-1) Y^{\lambda}, \quad \lambda > 1, \tag{9}$$

$$\text{(II)} \quad \mu U V^{\mu-1} - U^{\mu} \ge (\mu-1) V^{\mu}, \quad 0 < \mu < 1, \tag{10}$$

where each equality holds if and only if $X = Y$ or $U = V$.

3. **Main Results.**

Theorem 3.1. *Suppose that $p(t) > 0$, $q(t) > 0$, $\lambda > 1$, $\omega > 1$, if*

$$\begin{aligned} \liminf_{k\to\infty} &\left(\sum_{i=1}^{k} \int_{t_{i-1}}^{t_i} (t_i - s)^{\alpha-1} \left((\lambda-1) \left(\frac{p(s)}{\lambda} q^{-\frac{1}{\lambda}}(s) \right)^{\frac{1}{\lambda-1}} + e(s) \right) \mathrm{d}s \right. \\ &\left. + \sum_{i=1}^{k} \left(\frac{a_k}{\omega} b_k^{-\frac{1}{\omega}} \right)^{\frac{1}{\omega-1}} \right) = -\infty \end{aligned} \tag{11}$$

and

$$\begin{aligned} \liminf_{k\to\infty} &\left(\sum_{i=1}^{k} \int_{t_{i-1}}^{t_i} (t_i - s)^{\alpha-1} \left((\lambda-1) \left(\frac{p(s)}{\lambda} q^{-\frac{1}{\lambda}}(s) \right)^{\frac{1}{\lambda-1}} - e(s) \right) \mathrm{d}s \right. \\ &\left. + \sum_{i=1}^{k} \left(\frac{a_k}{\omega} b_k^{-\frac{1}{\omega}} \right)^{\frac{1}{\omega-1}} \right) = -\infty \end{aligned} \tag{12}$$

hold, then Equation (2) is oscillatory.

Proof: Suppose there exists a nonoscillatory solution of Equation (2). Without losing the generality, assume that $x(t) > 0$ for all $t \geq t_{k_0} \geq 0$, where $k_0 \geq 1$ depends on the solution $x(t)$. For $t \in (t_k, t_{k+1}]$, $k \geq k_0 + 1$, using Lemma 2.2, we derive at

$$\begin{aligned} x(t) &= x_0 + \frac{1}{\Gamma(\alpha)} \sum_{i=1}^{k} \int_{t_{i-1}}^{t_i} (t_i - s)^{\alpha-1} f\left(s, x(s)\right) \mathrm{d}s \\ &\quad + \frac{1}{\Gamma(\alpha)} \int_{t_k}^{t} (t-s)^{\alpha-1} f\left(s, x(s)\right) \mathrm{d}s + \sum_{i=1}^{k} I_i\left(t_i, x(t_i)\right) \\ &= x_0 + \frac{1}{\Gamma(\alpha)} \sum_{i=1}^{k_0} \int_{t_{i-1}}^{t_i} (t_i - s)^{\alpha-1} f\left(s, x(s)\right) \mathrm{d}s \\ &\quad + \frac{1}{\Gamma(\alpha)} \sum_{i=k_0+1}^{k} \int_{t_{i-1}}^{t_i} (t_i - s)^{\alpha-1} f\left(s, x(s)\right) \mathrm{d}s \\ &\quad + \frac{1}{\Gamma(\alpha)} \int_{t_k}^{t} (t-s)^{\alpha-1} f\left(s, x(s)\right) \mathrm{d}s + \sum_{i=1}^{k} I_i\left(t_i, x(t_i)\right). \end{aligned} \tag{13}$$

Substituting (6), (3) into (13) gives that

$$\begin{aligned} x(t) &= x_0 + \frac{1}{\Gamma(\alpha)} \sum_{i=1}^{k_0} \int_{t_{i-1}}^{t_i} (t_i - s)^{\alpha-1} f\left(s, x(s)\right) \mathrm{d}s \\ &\quad + \frac{1}{\Gamma(\alpha)} \sum_{i=k_0+1}^{k} \int_{t_{i-1}}^{t_i} (t_i - s)^{\alpha-1} \left(p(s)x(s) - q(s)x^{\lambda}(s)\right) \mathrm{d}s \\ &\quad + \frac{1}{\Gamma(\alpha)} \int_{t_k}^{t} (t-s)^{\alpha-1} \left(p(s)x(s) - q(s)x^{\lambda}(s)\right) \mathrm{d}s \\ &\quad + \sum_{i=1}^{k} \left(a_k|x(t_k)| - b_k|x(t_k)|^{\omega}\right) \\ &\quad + \frac{1}{\Gamma(\alpha)} \sum_{i=k_0+1}^{k} \int_{t_{i-1}}^{t_i} (t_i - s)^{\alpha-1} e(s) \mathrm{d}s + \frac{1}{\Gamma(\alpha)} \int_{t_k}^{t} (t-s)^{\alpha-1} e(s) \mathrm{d}s \\ &:= \Phi(k_0) + \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4, \end{aligned} \tag{14}$$

where

$$\Phi(k_0) = x_0 + \frac{1}{\Gamma(\alpha)} \sum_{i=1}^{k_0} \int_{t_{i-1}}^{t_i} (t_i - s)^{\alpha-1} f\left(s, x(s)\right) \mathrm{d}s,$$

$$\Phi_1 = \frac{1}{\Gamma(\alpha)} \sum_{i=k_0+1}^{k} \int_{t_{i-1}}^{t_i} (t_i - s)^{\alpha-1} \left(p(s)x(s) - q(s)x^{\lambda}(s)\right) \mathrm{d}s,$$

$$\Phi_2 = \frac{1}{\Gamma(\alpha)} \int_{t_k}^{t} (t-s)^{\alpha-1} \left(p(s)x(s) - q(s)x^{\lambda}(s)\right) \mathrm{d}s,$$

$$\Phi_3 = \sum_{i=1}^{k} \left(a_k|x(t_k)| - b_k|x(t_k)|^{\omega}\right),$$

$$\Phi_4 = \frac{1}{\Gamma(\alpha)} \sum_{i=k_0+1}^{k} \int_{t_{i-1}}^{t_i} (t_i - s)^{\alpha-1} e(s) \mathrm{d}s + \frac{1}{\Gamma(\alpha)} \int_{t_k}^{t} (t-s)^{\alpha-1} e(s) \mathrm{d}s.$$

Using Lemma 2.3 (I) for Φ_1, Φ_2, Φ_3, we have $\Phi_1 \le \frac{1}{\Gamma(\alpha)}\sum_{i=k_0+1}^{k}\int_{t_{i-1}}^{t_i}(t_i-s)^{\alpha-1}(\lambda-1)\left(\frac{p(s)}{\lambda}q^{-\frac{1}{\lambda}}(s)\right)^{\frac{1}{\lambda-1}}\mathrm{d}s$, $\Phi_2 \le \frac{1}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}\left(\frac{p(s)}{\lambda}q^{-\frac{1}{\lambda}}(s)\right)^{\frac{1}{\lambda-1}}\mathrm{d}s$, $\Phi_3 \le \sum_{i=1}^{k}\left(\frac{a_k}{\omega}b_k^{-\frac{1}{\omega}}\right)^{\frac{1}{\omega-1}}$, which follows from (14) that

$$\begin{aligned} x(t) \le \Phi(k_0) + \Phi_4 + \frac{1}{\Gamma(\alpha)}\sum_{i=k_0+1}^{k}\int_{t_{i-1}}^{t_i}(t_i-s)^{\alpha-1}(\lambda-1)\left(\frac{p(s)}{\lambda}q^{-\frac{1}{\lambda}}(s)\right)^{\frac{1}{\lambda-1}}\mathrm{d}s \\ + \frac{1}{\Gamma(\alpha)}\int_{t_k}^{t}(t-s)^{\alpha-1}\left(\frac{p(s)}{\lambda}q^{-\frac{1}{\lambda}}(s)\right)^{\frac{1}{\lambda-1}}\mathrm{d}s + \sum_{i=1}^{k}\left(\frac{a_k}{\omega}b_k^{-\frac{1}{\omega}}\right)^{\frac{1}{\omega-1}}. \end{aligned} \tag{15}$$

Taking the limit inferior of both sides of (15) as $t \to \infty$, we get a contradiction to condition (11). In the case of $x(t) < 0$, we use the function $y(t) = -x(t)$ as a positive solution of the differential Equation (2) and repeating the above procedure leads to a contradiction with (12). This completes the proof.

Remark 3.1. *When* (3) *is replaced by* $I_k(t_k, x(t_k)) = 0$, *i.e., the equation is not added the impulse condition, the similar result is obtained by* [5].

Remark 3.2. *From Theorem 3.1 Equation* (15), *we remark that the equations which do not impose impulse condition can be oscillatory although they are nonoscillatory when imposing impulse condition.*

Corollary 3.1. *If the conditions of Theorem 3.1 hold and* $\max_{0\le i\le\infty}(t_{i+1}-t_i) \le \Delta$, *and there exists* $\beta : 0 < \beta < \alpha$, *such that*

$$\begin{aligned} \liminf_{k\to\infty}\Bigg[\frac{k}{\Gamma(\alpha)}\left(\frac{1-\beta}{\alpha-\beta}\right)^{1-\beta}\Delta^{\alpha-\beta}\sum_{i=1}^{k}\left(\int_{t_{i-1}}^{t_i}(Z(s))^{\frac{1}{\beta}}\mathrm{d}s\right)^{\beta} \\ + \sum_{i=1}^{k}\int_{t_{i-1}}^{t_i}(t_i-s)^{\alpha-1}(\lambda-1)e(s)\mathrm{d}s + \sum_{i=1}^{k}\left(\frac{a_k}{\omega}b_k^{-\frac{1}{\omega}}\right)^{\frac{1}{\omega-1}}\Bigg] = -\infty \end{aligned}$$

and

$$\begin{aligned} \liminf_{k\to\infty}\Bigg[\frac{k}{\Gamma(\alpha)}\left(\frac{1-\beta}{\alpha-\beta}\right)^{1-\beta}\Delta^{\alpha-\beta}\sum_{i=1}^{k}\left(\int_{t_{i-1}}^{t_i}(Z(s))^{\frac{1}{\beta}}\mathrm{d}s\right)^{\beta} \\ - \sum_{i=1}^{k}\int_{t_{i-1}}^{t_i}(t_i-s)^{\alpha-1}(\lambda-1)e(s)\mathrm{d}s + \sum_{i=1}^{k}\left(\frac{a_k}{\omega}b_k^{-\frac{1}{\omega}}\right)^{\frac{1}{\omega-1}}\Bigg] = -\infty \end{aligned}$$

hold, where $Z(s) = (\lambda-1)\left(\frac{p(s)}{\lambda}q^{-\frac{1}{\lambda}}(s)\right)^{\frac{1}{\lambda-1}}$, *then Equation* (2) *is oscillatory.*

Proof: It follows from Hölder inequality that

$$\int_{t_{i-1}}^{t_i}(t_i-s)Z(s)\mathrm{d}s \le \left(\int_{t_{i-1}}^{t_i}(t_i-s)^{\frac{1}{1-\beta}}\mathrm{d}s\right)^{1-\beta}\left(\int_{t_{i-1}}^{t_i}(Z(s))^{\frac{1}{\beta}}\mathrm{d}s\right)^{\beta}.$$

Then, by Theorem 3.1, the result is obtained.

Remark 3.3. *Corollary 3.1 is satisfied the equations with invariable step-size impulses, i.e.,* $t_k = t_0 + k\Delta$.

Theorem 3.2. *Suppose that $p(t) < 0$, $q(t) < 0$, $\lambda < 1$, $\omega > 1$, if*

$$\begin{aligned}&\liminf_{k\to\infty}\left(\sum_{i=1}^{k}\int_{t_{i-1}}^{t_i}(t_i-s)^{\alpha-1}\left((1-\lambda)\left(\frac{|p(s)|}{\lambda}|q(s)|^{-\frac{1}{\lambda}}\right)^{\frac{1}{\lambda-1}}+e(s)\right)\mathrm{d}s\right.\\&\left.+\sum_{i=1}^{k}\left(\frac{a_k}{\omega}b_k^{-\frac{1}{\omega}}\right)^{\frac{1}{\omega-1}}\right)=-\infty\end{aligned}\tag{16}$$

and

$$\begin{aligned}&\liminf_{k\to\infty}\left(\sum_{i=1}^{k}\int_{t_{i-1}}^{t_i}(t_i-s)^{\alpha-1}\left((1-\lambda)\left(\frac{|p(s)|}{\lambda}|q(s)|^{-\frac{1}{\lambda}}\right)^{\frac{1}{\lambda-1}}-e(s)\right)\mathrm{d}s\right.\\&\left.+\sum_{i=1}^{k}\left(\frac{a_k}{\omega}b_k^{-\frac{1}{\omega}}\right)^{\frac{1}{\omega-1}}\right)=-\infty\end{aligned}\tag{17}$$

hold, then Equation (2) is oscillatory.

Proof: We only need to note that, if $x(t) > 0$, by Lemma 2.3

$$\left(p(s)x(s)-q(s)x^{\lambda}(s)\right)=-\left(|p(s)|x(s)-|q(s)|x^{\lambda}(s)\right)\le(1-\lambda)\left(\frac{|p(s)|}{\lambda}|q(s)|^{-\frac{1}{\lambda}}\right)^{\frac{1}{\lambda-1}},$$

then the desired result follows by repeating the procedures of Theorem 3.1.

4. **Example.** Consider the following fractional differential equations

$$\begin{cases}{}_0^C\mathbf{D}_t^{\frac{1}{2}}x-t^2x(t)+t^3|x(t)|x(t)=\sin t,\ t\in(0,\infty)\setminus\{0,2\pi,4\pi,\cdots\},\\ \Delta x|_{t=2k\pi}=I_k\left(2k\pi,x(2k\pi)\right)=a_k|x(2k\pi)|-b_k|x(2k\pi)|^2,\\ x\left(0^+\right)=0,\end{cases}\tag{18}$$

Here

$$\alpha=\frac{1}{2},\ p(t)=t^2,\ q(t)=t^3,\ e(t)=\sin t,\ \lambda=2,\ \omega=2,\ t_k=2k\pi,\ k=0,1,\cdots.$$

Case 1: Choosing $a_k = k^{\frac{1}{2}}$, $b_k = k^4$. Then

$$\begin{aligned}&\liminf_{k\to\infty}\left(\sum_{i=1}^{k}\int_{t_{i-1}}^{t_i}(t_i-s)^{\alpha-1}\left((\lambda-1)\left(\frac{p(s)}{\lambda}q^{-\frac{1}{\lambda}}(s)\right)^{\frac{1}{\lambda-1}}+e(s)\right)\mathrm{d}s\right.\\&\left.+\sum_{i=1}^{k}\left(\frac{a_k}{\omega}b_k^{-\frac{1}{\omega}}\right)^{\frac{1}{\omega-1}}\right)\\&=\liminf_{k\to\infty}\left(\sum_{i=1}^{k}\int_{2(i-1)\pi}^{2i\pi}(2i\pi-s)\left(\frac{1}{2}s^{\frac{1}{2}}+\sin s\right)\mathrm{d}s+\sum_{i=1}^{k}\frac{k^{-\frac{3}{2}}}{2}\right)\\&=-\infty\end{aligned}$$

and

$$\begin{aligned}&\liminf_{k\to\infty}\left(\sum_{i=1}^{k}\int_{t_{i-1}}^{t_i}(t_i-s)^{\alpha-1}\left((\lambda-1)\left(\frac{p(s)}{\lambda}q^{-\frac{1}{\lambda}}(s)\right)^{\frac{1}{\lambda-1}}-e(s)\right)\mathrm{d}s\right.\\&\left.+\sum_{i=1}^{k}\left(\frac{a_k}{\omega}b_k^{-\frac{1}{\omega}}\right)^{\frac{1}{\omega-1}}\right)\\&=\liminf_{k\to\infty}\left(\sum_{i=1}^{k}\int_{2(i-1)\pi}^{2i\pi}(2i\pi-s)\left(\frac{1}{2}s^{\frac{1}{2}}-\sin s\right)\mathrm{d}s+\sum_{i=1}^{k}\frac{k^{-\frac{3}{2}}}{2}\right)\\&=-\infty.\end{aligned}$$

Then by Theorem 3.1, Equation (18) is oscillatory.
Case 2: Choosing $a_k = k$, $b_k = k^{-1}$. Then

$$\begin{aligned}&\liminf_{k\to\infty}\left(\sum_{i=1}^{k}\int_{t_{i-1}}^{t_i}(t_i-s)^{\alpha-1}\left((\lambda-1)\left(\frac{p(s)}{\lambda}q^{-\frac{1}{\lambda}}(s)\right)^{\frac{1}{\lambda-1}}+e(s)\right)\mathrm{d}s\right.\\&\left.+\sum_{i=1}^{k}\left(\frac{a_k}{\omega}b_k^{-\frac{1}{\omega}}\right)^{\frac{1}{\omega-1}}\right)\\=&\liminf_{k\to\infty}\left(\sum_{i=1}^{k}\int_{2(i-1)\pi}^{2i\pi}(2i\pi-s)\left(\frac{1}{2}s^{\frac{1}{2}}+\sin s\right)\mathrm{d}s+\sum_{i=1}^{k}\frac{k^{\frac{3}{2}}}{2}\right)\\&>0.\end{aligned}$$

So the condition (11) and (12) are not satisfied. However, when Equation (18) is not subject to impulse condition, the equation is oscillatory, i.e., impulse condition can destroy the oscillator of the equation.

5. **Conclusions.** This paper has investigated the oscillation problem for Caputo impulsive fractional order differential equations with mixed nonlinearities. By exploiting some basic inequalities, a new oscillation criterion for Caputo impulsive fractional order differential equations has been established. Finally, examples are given to show the effectiveness of proposed criteria. Future work includes time delay system case.

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REFERENCES

[1] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego, 1999.
[2] V. Lakshmikantham, Theory of fractional functional differential equations, *Nonlinear Anal.*, vol.69, no.10, pp.3337-3343, 2008.
[3] A. El-Sayed and A. Ibrahim, Multivalued fractional differential equations, *Appl. Math. Comput.*, vol.68, no.1, pp.15-25, 1995.
[4] W. Glockle and T. Nonnenmacher, A fractional calculus approach of self-similar protein dynamics, *Biophys. J.*, vol.68, no.1, pp.46-53, 1995.
[5] S. Grace, R. Agarwal, P. Wang and A. Zafer, On the oscillation of fractional differential equations, *Fract. Calc. Appl. Anal.*, vol.15, no.2, pp.222-231, 2012.
[6] M. Muayyad, A study on the conditions of oscillation of solutions of second order impulsive delay differential equations, *Raf. J. Comp. Maths.*, vol.9, no.1, pp.129-137, 2012.
[7] Z. Guo, X. Zhou and W. Ge, Interval oscillation criteria for second-order forced impulsive differential equations with mixed nonlinearities, *J. Math. Anal. Appl.*, vol.381, no.1, pp.187-201, 2011.
[8] A. Samoilenko and N. Perestyuk, *Impulsive Differential Equations*, World Scientific, San Diego, 1995.
[9] S. Zhang, Positive solutions for boundary-value problems of nonlinear fractional differential equations, *Electron. J. Differ. Eq.*, vol.2006, no.36, pp.1-12, 2006.
[10] A. Chen and Y. Chen, Existence of solutions to anti-periodic boundary value problem for nonlinear fractional differential equations with impulses, *Adv. in Differ. Equations*, pp.1-17, 2011.
[11] G. Hardy, J. Littlewood and G. Polya, *Inequalities*, Cambridge University Press, Cambridge, 1988.