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FORECASTING TAIWAN'S GDP BY THE NOVEL WEIGHTED AVERAGE GREY MODEL WITH JUMPING *p*

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Abstract. *In grey forecasting model, a variable p value rolling grey forecasting model shows its high efficiency in forecasting. However, it needs long original series of data to acquire p series. Besides, the theoretical evidence of the regression equation of p on the other variable is not sufficient. Thus, this study proposes the novel weighted average grey model with jumping p and uses the model to forecast Taiwan's GDP. The results indicate that nonlinear grey Bernoulli model (NGBM) has worse forecast performance than all traditional grey forecasting model for out-of-sample test, although NGBM has better forecast performance than all traditional grey forecasting model for in-sample test. The proposed model has better forecasting performance than NGBM for in-sample and out-of-sample period. It means that the proposed method moderates the effect of overestimation and underestimation. The results also show that Taiwan's GDP is steadily growing. It may serve as valuable reference for policy makers and investors.*
Keywords: Grey forecasting, Nonlinear grey Bernoulli model, GDP, Weighted average

1. **Introduction.** Grey forecasting [1] has been developing for over 20 years and its evolution still continues. Deng [2] proposes grey theory in which grey forecasting is suitable for short-term forecasts. Grey forecasting has been successfully applied in numerous fields, including economic indices [3], agriculture [4], transportation [5], electric load [6], semiconductor industry [7], net income [8], etc.

The weakness of the traditional grey forecasting is its low prediction precision. Thus, the researchers develop the hybrid grey model to improve the original model, including grey-Markov model [9], grey-fuzzy model [10], grey-Taguchi model [11], grey Verhulst model [12], Nash nonlinear grey Bernoulli model (NNGBM) [13], etc. Moreover, Chang's [7] jumping p grey model (GM) and Chen's [14,15] NGBM have better prediction precision. On the other hand, some researchers adopt particle swarm optimization algorithm to optimize grey model [16,17].

In Chang's [7] jumping p GM, the coefficient p is a variable for predicting semiconductor production. Chang's article assumes that the value of p is closely related with industrial production growth rate (g). Hence, p can be estimated after the regression of p on g is constructed. The semiconductor production can also be estimated as p is estimated.

In fact, the value of p is only a coefficient, not an economic variable. Moreover, there is no economic theory to show that p has a close relationship with any other variable. Besides, the regression of p on g does not exist when there are few optimal values of p, as the data sequence is short.

For this reason, this paper proposes a different type of jumping p GM, called weighted average GM with jumping p. There are different forecast values as p changes. Therefore, we can find out all forecast values. Check which forecast value with a given p is the closest to the actual value in modeling period. Count the number of occurrence of p and calculate relative frequency as weight of forecast value. Finally, calculate a weighted average of forecast value. All details will be discussed in the next section.

This paper is organized as follows. Section 2 introduces the mathematics of weighted average jumping p GM and defines the relative percentage error. Section 3 uses the proposed method to forecast Taiwan's GDP. Finally, Section 4 presents conclusions.

2. **Mathematical Methodology.** The procedures for deriving GM$(1,1)$ are detailed below:

Step 1: Assume that the original series of data with m entries are:

$$X^{(0)}(1,m) = \left\{x^{(0)}(k)|x^{(0)}(k) \geq 0, k = 1,2,\dots,m\right\} \tag{1}$$

where raw matrix $X^{(0)}(1,m)$ represents the non-negative original time series data.

Step 2: Construct $X^{(1)}(1,m)$ using a one time accumulated generation operation (1-AGO), namely

$$X^{(1)}(1,m) = \left\{x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(k)|x^{(1)}(k) \geq 0, k = 1,2,\dots,m\right\} \tag{2}$$

Step 3: The 1-AGO yields a monotonically increasing sequence similar to the solution curve of the first order linear differential equation. The differential equation has the following form,

$$\frac{d\hat{x}^{(1)}}{dt} + \alpha\hat{x}^{(1)} = \beta\left[\hat{x}^{(1)}\right] \tag{3}$$

The background value is $\hat{x}^{(1)}(t) \cong px^{(1)}(k) + (1-p)x^{(1)}(k+1) = z^{(1)}(k)$, where $p \in [0,1]$.

Step 4: A discrete form of Equation (3) is described as:

$$x^{(0)}(k) + \alpha z^{(1)}(k) = \beta\left[z^{(1)}(k)\right], \quad k = 2,3,4,\dots \tag{4}$$

Using the least square method, the above model parameters α and β can be written as

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \left(\mathrm{Z}^T\mathrm{Z}\right)^{-1}\mathrm{Z}^T X, \tag{5}$$

where Z and X are defined as follows.

$$\mathrm{Z} = \begin{bmatrix} -z^{(1)}(2) & \left[z^{(1)}(2)\right] \\ -z^{(1)}(3) & \left[z^{(1)}(3)\right] \\ \vdots & \vdots \\ -z^{(1)}(m) & \left[z^{(1)}(m)\right] \end{bmatrix}, \quad X = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(m) \end{bmatrix} \tag{6}$$

Step 5: The corresponding particular solution of Equation (3) is

$$\hat{x}^{(1)}(k+1) = \left[\left(x^{(0)}(1) - \frac{\beta}{\alpha}\right)e^{-\alpha k} + \frac{\beta}{\alpha}\right], \quad k = 1,2,3,\dots \tag{7}$$

Step 6: To compare with the nonlinear grey Bernoulli model [14,15], we also calculate the corresponding particular solution of NGBM$(1,1)$

$$\hat{x}^{(1)}(k+1) = \left[\left(x^{(0)}(1)^{(1-n)} - \frac{\beta}{\alpha}\right)e^{-\alpha(1-n)k} + \frac{\beta}{\alpha}\right]^{1/(1-n)}, \; n \neq 1, \; k = 1,2,3,\dots \tag{8}$$

Step 7: Obtain $\hat{x}^{(0)}(k+1)$ which is defined as

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \tag{9}$$

Step 8: In the grey model, the main criterion for assessing forecasting accuracy is relative percentage error between the actual and forecast values. The relative percentage error (RPE) $\xi(k)$ is defined as

$$\xi(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100\% \tag{10}$$

and the average relative percentage error (ARPE) $\zeta(k)$ is defined as

$$\zeta(k) = \frac{1}{m-1}\sum_{k=2}^{m} |\xi(k)| \tag{11}$$

Step 9: Set $p \in \{p_1 \ldots p_j \ldots p_J\}$. For a given p_j, calculate the forecast value $x_{p_j}(k+1)$. Find the fitted value which has the smallest relative percentage error to each actual value. Then count cumulative relative frequency (W_j) of p_j as the weights of forecast value.

Step 10: Calculate the weighted average of the forecast value $x(k+1)$. Thus,

$$\hat{x}^{(0)}(k+1) = \sum \left[W_j * x_{p_j}(k+1)\right] \tag{12}$$

3. **Forecasting Taiwan's GDP.** Forecasting Twain's GDP is selected as a case study. GDP annual data is obtained from the website of the Ministry of Economics Affairs of Taiwan. The data period is from 2009 to 2014. The unit of GDP is million US dollars. This study assumes that the values of p_i are 0.1, 0.3, 0.5, 0.7, and 0.9.

Firstly, the data from 2009 to 2013 are used to construct grey model GM$(1,1)$. The data in 2014 acts as out-of-sample test. The simulation results will compare with the outcomes from Chen's nonlinear grey Bernoulli model [14,15].

In 2010, the forecast value (446817.00) is the closest to the actual value (446105) when GM's p is 0.1. In 2011, the fitted value (482690.48) is the closest to the actual value (485653) as GM's p is 0.9. In 2012, the forecast value (494402.72) is the closest to the actual value (495845) as GM's p is 0.5. In 2013, the fitted value (510716.44) is the closest to the actual value (511293) when GM's p is 0.3. Thus, the cumulative relative frequency are 1/4, 1/4, 1/4, 0 and 1/4 when the values of p are 0.1, 0.3, 0.5, 0.7 and 0.9, respectively. Thus, we can obtain Taiwan's forecasting function as follows.

$$\begin{aligned}\hat{X}_0(k+1) = &(1/4)\hat{X}_0(k+1|p=0.1) + (1/4)\hat{X}_0(k+1|p=0.3) \\ &+ (1/4)\hat{X}_0(k+1|p=0.5) + (1/4)\hat{X}_0(k+1|p=0.9)\end{aligned} \tag{13}$$

For in-sample tests, the ARPE of GM$(1,1)$ are 1.8583%, 1.3988%, 1.3544%, 1.6874% and 2.1737% when the values of p are 0.1, 0.3, 0.5, 0.7 and 0.9, respectively. The ARPE of NGBM is 0.6163%. Clearly, NGBM has smaller modeling error than any GM$(1,1)$ for in-sample tests. The results are the same as Chen's findings [14,15]. When we use the weighted average GM$(1,1)$ with jumping p, the ARPE decreases to 0.3952%. The proposed method performs better than NGBM$(1,1)$.

For the out-of-sample test, the RPE of weighted average GM$(1,1)$ with jumping p is 1.3086% while the RPE of NGBM$(1,1)$ is 3.1960%. NGBM is even worse than all traditional GM$(1,1)$. It indicates that NGBM$(1,1)$ is not always reliable in out-of-sample test although NGBM has the best forecast performance in sample period. In this case, NGBM's RPE in 2014 is the greatest. Thus, the proposed weighted average GM$(1,1)$ with jumping p moderates the effect of overestimation and underestimation. All results are listed in Table 1.

The above results show that the proposed method works well. Thus, this study uses GDP data of 2010-2014 to construct weighted average GM$(1,1)$ with jumping p. The forecast function is

$$\hat{X}_0(k+1) = (1/4)\hat{X}_0(k+1|p=0.3) + (2/4)\hat{X}_0(k+1|p=0.5) + (1/4)\hat{X}_0(k+1|p=0.7) \tag{14}$$

TABLE 1. Example for demonstrating that weighted average GM(1, 1) with jumping p gives more precision than NGBM(1, 1)

Year	2009	2010	2011	2012	2013	$\zeta(k)\%$	2014
Actual GDP	392065	446105	485653	495845	511293		529587
Forecast GDP ($p = 0.1$)	392065	446817.00	465700.89	485382.87	505896.68	1.8583	527277.46
$\xi(k)\%$	0	−0.1596	4.1083	2.1100	1.0554		0.3461
Forecast GDP ($p = 0.3$)	392065	450656.03	469847.30	489855.84	510716.44	1.3988	532465.39
$\xi(k)\%$	0	−1.0202	3.2545	1.2079	0.1128		−0.5435
Forecast GDP ($p = 0.5$)	392065	454554.52	474060.11	494402.72	515618.26	1.3544	537744.18
$\xi(k)\%$	0	−1.8941	2.3871	0.2909	−0.8459		−1.5403
Forecast GDP ($p = 0.7$)	392065	458513.66	478340.70	499025.11	520603.95	1.6874	543115.91
$\xi(k)\%$	0	−2.7816	1.5057	−0.6414	−1.8211		−2.5546
Forecast GDP ($p = 0.9$)	392065	462534.66	482690.48	503724.63	525675.38	2.1737	548582.67
$\xi(k)\%$	0	−1.9602	−1.0173	−1.1731	−1.5328		−1.9131
Forecast GDP GM with jumping p	392065	446817.00 ($p = 0.1$)	482690.48 ($p = 0.9$)	494402.72 ($p = 0.5$)	510716.44 ($p = 0.3$)		536517.43
$\xi(k)\%$	0	−0.1596	−1.0173	0.2909	0.1128	0.3952	1.3086
NGBM(1, 1) ($n = 0.2$)	392065	444792.11	480964.36	499820.93	509228.91		512661.36
$\xi(k)\%$	0	0.2943	0.9654	−0.8018	0.4036	0.6163	3.1960

TABLE 2. The forecast value of Taiwan's GDP using the weighted average GM(1, 1) with jumping p

Year	2010	2011	2012	2013	2014	2015	2016	2017	$\zeta(k)\%$
Actual GDP	446105	485653	495845	511293	529587				
Forecast GDP ($p = 0.1$)	446105	477951.40	491955.53	506369.99	521206.80	536478.33	552197.32	568376.88	1.2289
$\xi(k)\%$	0	1.5858	0.7844	0.9629	1.5824				
Forecast GDP ($p = 0.3$)	446105	480770.31	494939.06	509525.37	524541.55	540000.28	555914.59	572297.91	0.6216
$\xi(k)\%$	0	1.0054	0.1827	0.3457	0.9527				
Forecast GDP ($p = 0.5$)	446105	483622.26	497958.51	512719.74	527918.55	543567.90	559681.16	576272.06	0.3592
$\xi(k)\%$	0	0.4181	−0.4262	−0.2790	0.3150				
Forecast GDP ($p = 0.7$)	446105	486507.81	501014.54	515953.83	531338.58	547182.07	563497.99	580300.42	0.6152
$\xi(k)\%$	0	−0.1760	−1.0426	−0.9116	−0.3307				
Forecast GDP ($p = 0.9$)	446105	489427.57	504107.80	519228.36	534802.46	550843.70	567366.08	584384.06	1.2451
$\xi(k)\%$	0	−0.7772	−1.6664	−1.5520	−0.9848				
Forecast GDP GM with jumping p	446105	486507.81 ($p = 0.7$)	494939.06 ($p = 0.3$)	512719.74 ($p = 0.5$)	527918.55 ($p = 0.5$)	543579.54	559693.72	576285.61	0.2382
$\xi(k)\%$	0	−0.1760	0.1827	−0.2790	0.3150				

The predicted values of Taiwan's GDP are 543579.54, 559693.72 and 576285.61 from 2015 to 2017. The ARPE of the proposed model is 0.2382% which is less than any other GM(1, 1). All results are listed in Table 2. The evidences show that Taiwan's GDP is growing steadily.

4. **Conclusions.** When the original data is limited, a variable p value rolling grey forecasting model cannot be obtained. Besides, the regression of p on the other variable needs more theoretical evidences. For this reason, this study proposes the weighted average GM(1, 1) with jumping p. In a case study, the proposed method performs better than NGBM(1, 1) for in-sample and out-of-sample tests. Finally, the forecast function is obtained and the forecast outcomes show that Taiwan's GDP are steadily growing. It may serve as valuable information for policy makers and investors.

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