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INVENTORY CONTROL MODEL FOR PERISHABLE ITEMS WITH TIME-VARYING DEMAND RATE AND FIXED DETERIORATION

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Abstract. *Due to that perishable items deteriorate and the demand rate is time-varying, this paper presents a retailer inventory control model with shortage point and deterioration point. According to whether the perishable items have deteriorated or are out of shortage, different scenarios are modeled based on which we derive the ordering cost, the holding cost, the shortage loss cost and the deterioration cost. Then the mathematical model is constructed by summing up all costs and aims to optimize the shortage point by minimizing the total cost over a finite horizon. Finally, the sensitivity analysis of the optimal solution to the model parameters is carried out in the numerical example.*
Keywords: Perishable items, Deterioration, Inventory model, Time-varying demand

1. **Introduction.** The inventory management of perishable items has attracted attention from managers and scholars recently. Due to that perishable items are characterized by the shorter life cycle, deterioration, uncertainty demand, numerous models have been developed to optimize the inventory control policy [1-3]. The previous work shares an assumption that the demand is stationary, i.e., it is constant, so these work studied the discrete-time inventory model [4-6]. Liu et al. studied a discrete-time inventory model where the vendor announces a price escalation [4]. Aliyu and Boukas modeled for the discrete-time inventory control problem with deterministic or stochastic demand [5]. Atıcı et al. developed a discrete-time model of inventory where time points may be unevenly spaced [6]. However, time-varying demand is more common since the uncertainty is always inevitable. So the inventory policy with time-varying demand for perishable items is general [7-9]. We will consider an inventory control policy with time-varying demand.

The change of the deterioration rate has impact on the inventory control policy and the previous work can be classified into two types: constant and random deterioration rate. For the scenario with constant deterioration rate, Moon et al. constructed the economic order quantity models by considering inflation and time discounting [9]; Chu et al. considered the permissible delay in payments [10]; Change proposed the optimal economic order model that considered the inflation and allowed the deferred payment [11]. However, some studies on random deterioration rate have also been presented, for example, Lin et al. established an inventory replenishment policy with the assumption that the deterioration rate increases linearly with time [8]; Wee assumed the deterioration rate follows to a two-parameter Weibull distribution [12]. The inventory control policy with random deterioration rate is more complicated in the modeling, so in our model the constant deterioration rate is considered to simplify the problem. It is reasonable since the constant deterioration rate can be also regarded as the expected value of the random deterioration rate.

Shortage is not allowed in some inventory models, which limits the contribution in the practical application. Therefore, some models considering backlogging rate have been studied. Papachristos and Skouri considered the backlogging rate which reduces with the customer waiting time [7], and Skouri et al. further studied an inventory model with general ramp type demand rate, time dependent deterioration rate and partial backlogging [13]. Chang and Dye deal with an inventory model with a varying deterioration rate and partial backlogging rate [14]. In view of these work, the following model deals with shortages with the backlogging rate. The main contribution of this work is that the replenishment cycle is divided into different periods according to the sequence of the shortage point and the deterioration point.

The outline of the remaining part is organized as follows: Section 2 introduces the problem description and notations for modeling; the inventory control model is developed in Section 3 and a numerical example is given in Section 4; finally, Section 5 concludes the paper.

2. Problem Description and Notations.

(1) The inventory system is operating in a finite time horizon; namely H time units.

(2) The replenishment policy is carried out with the interval T and it is assumed that there are n replenishment cycles $n = H/T$; the replenishment point is $T_i = (i-1)H/n = (i-1)T$.

(3) The demand rate of retailers is a continues linearly function depending on the inventory level $I(t)$; let $f(t)$ denote the demand rate of retailers at time t:

$$f(t) = \begin{cases} D + \delta I(t) & I(t) \geq 0 \\ D & I(t) < 0 \end{cases} \tag{1}$$

where D is the customers' demand rate at time t, δ is an impact coefficient of the inventory level on the demand rate of retailers, and D and δ are constant ($D > 0$, $\delta > 0$).

(4) The shortage appears when the retailers' inventory holding cannot meet the demand from customers. The shortage in the last cycle is not allowed. Set the backlogging rate that customers are willing to wait for delivery during the shortage $b(t)$: $b(t) = e^{-\beta t}$, $0 < b(t) < 1$, $t_0 \leq t < T$ where β is the scale factor, and t_0 is the time length that is not allowed shortage during a cycle.

(5) A constant fraction θ ($0 < \theta < 1$) of on-hand inventory deteriorates per unit of time and the time of the deterioration point is t_h.

(6) Instantaneous replenishment is considered, i.e., the replenishment lead time is 0.

(7) Some cost parameters are involved in the inventory model. The cost per ordering is C_b. The inventory holding cost per unit time and per unit perishable items is d, the shortage loss cost per unit time and per unit perishable items is f, and the deterioration cost per unit time and per unit perishable items is g. The total inventory holding cost over a cycle is C_d, the total shortage loss cost is C_f, the total deterioration cost is C_g, and the total inventory cost over the finite time horizon H is C_H.

3. **Inventory Control Model Construction.** The last cycle over the time horizon is not allowed to be shortage, and thus there are two periods in the nth cycle, namely non-deterioration period $[T_n, t_{n,h})$ and deterioration period $(t_{n,h}, T_{n+1}]$ where $t_{n,h}$ is the deterioration point of the nth cycle. However, for the previous $n-1$ cycles, $[T_i, T_{i+1}]$ ($i = 1, \ldots, n-1$), two scenarios should be considered. If the shortage point is earlier than the deterioration point, the cycle $[T_i, T_{i+1}]$ includes the inventory holding period $[T_i, t_{i,0})$ and the shortage period $[t_{i,0}, T_{i+1}]$; otherwise, three periods should be modeled, i.e., inventory

holding and non-deterioration period $[T_i, t_{i,h})$, inventory holding and deterioration period $(t_{i,h}, t_{i,0})$ and shortage period $[t_{i,0}, T_{i+1}]$.

3.1. **Inventory control model with shortage point before the previous $n-1$ cycles.** Here, we discuss the first $n-1$ cycles including two periods mentioned above.

(1) In the inventory holding period $[T_i, t_{i,0})$, the on-hand inventory items do not deteriorate and the inventory level is positive. Thus, the change rate of inventory level is equal to the demand rate of retailers for perishable items when $I(t) \geq 0$. So the instantaneous state of the inventory level should satisfy the following condition:

$$dI_{i,1}(t)/dt = -D - \delta I_{i,1}(t), \quad t \in [T_i, t_{i,0}) \tag{2}$$

The boundary condition is $I_{i,1}(t_{i,0}) = 0$, and then the inventory level of retailers at time t is:

$$I_{i,1}(t) = D\left[e^{\delta(t_{i,0}-t)} - 1\right]/\delta, \quad t \in [T_i, t_{i,0}) \tag{3}$$

(2) In the shortage period $(t_{i,0}, T_{i+1}]$, the inventory level is negative and there is no deterioration. The change rate of inventory level is equal to the demand rate of retailers for perishable items when $I(t) < 0$. The instantaneous state of inventory level should satisfy the following condition:

$$dI_{i,2}(t)/dt = Db(t), \quad t \in (t_{i,0}, T_{i+1}] \tag{4}$$

The boundary condition is $I_{i,2}(t_{i,0}) = 0$, and then the inventory level of retailer at t is:

$$I_{i,2}(t) = D\left(e^{-\beta t_0} - e^{-\beta t}\right)/\beta, \quad t \in (t_{i,0}, T_{i+1}] \tag{5}$$

The ordering cost of the retailer in the ith cycle is C_b. The total holding cost of the retailer is:

$$C_d = d\int_{T_i}^{t_{i,0}} I_{i,1}(t)dt = dD\left(e^{\delta t_{i,0}} - \delta t_{i,0} - 1\right)/\delta^2 \tag{6}$$

The shortage loss cost of the retailer in the ith cycle is given by:

$$C_f = f\int_{t_{i,0}}^{T_{i+1}} I_2(t)dt = fD\left[(T_{i+1} - t_{i,0})e^{-\beta t_{i,0}} + \left(e^{-\beta T_{i+1}} - e^{-\beta t_{i,0}}\right)/\beta\right]/\beta \tag{7}$$

The inventory cost of the retailer over a finite time horizon H (in addition to the nth cycle) is obtained by summing over the cost before the first $n-1$ cycles:

$$C_H = (n-1)C_b + \sum_{i=1}^{n-1}(C_d + C_f) \tag{8}$$

(3) Optimal solution analysis

The total inventory cost C_H is the continuous function of the variable $t_{i,0}$. By determining $t_{i,0}$, the objective function C_H can be obtained to the minimum, which corresponds to minimize $(C_d + C_f)$. The necessary condition for $(C_d + C_f)$ to the minimum is given as:

$$d(C_d + C_f)/dt_{i,0} = 0 \tag{9}$$

$$e^{-\beta t_{i,0}} f\left(t_{i,0} - T_{i+1}\right) + dDe^{\delta t_{i,0}} - dD/\delta = 0 \tag{10}$$

3.2. **Inventory control model with deterioration point.**

3.2.1. *The inventory level of retailer before previous $n-1$ cycles.*

(1) Within the interval $[T_i, t_{i,h}]$, the on-hand inventory items do not deteriorate and the inventory level is positive. The change rate of inventory level is equal to the demand rate of retailers for perishable items when $I(t) \geq 0$. So the instantaneous state of inventory level within $[T_i, t_{i,h}]$ should satisfy the following condition:

$$dI_{i,3}(t)/dt = -D - \delta I_{i,3}(t), \quad t \in [T_i, t_{i,h}] \tag{11}$$

Then, the inventory of retailer at t is given as:

$$I_{i,3}(t) = e^{\delta(t_{i,h}-t)} \left[D/\delta + D\left(e^{(\theta+\delta)(t_{i,0}-t_{i,h})} - 1\right)/(\theta+\delta)\right] - D/\delta, \quad t \in [T_i, t_{i,h}] \tag{12}$$

(2) Within the interval $(t_{i,h}, t_{i,0})$, the on-hand inventory items deteriorate and the inventory level is positive. The change rate of inventory level is equal to the summation of the demand rate of retailers and the deterioration rate of perishable items when $I(t) \geq 0$. The instantaneous state of inventory level should satisfy the following condition:

$$dI_{i,4}(t)/dt = -D - \delta I_{i,4}(t) - \theta I_{i,4}(t), \quad t \in (t_{i,h}, t_{i,0}) \tag{13}$$

The boundary condition is $I_{i,4}(t_{i,0}) = 0$, so the inventory level of retailers at t is given as:

$$I_{i,4}(t) = D\left(e^{(\theta+\delta)(t_{i,0}-t)} - 1\right)/(\theta+\delta), \quad t \in (t_{i,h}, t_{i,0}) \tag{14}$$

(3) Within the interval $[t_{i,0}, T_{i+1}]$, the inventory level is negative. The change rate of inventory level is equal to the demand rate of retailers when $I(t) < 0$. The instantaneous state of inventory level within $[t_{i,0}, T_{i+1}]$ should satisfy the condition shown in Equation (4), so the inventory level of retailers at t, $I_{i,5}(t)$, can be expressed as Equation (5).

The total inventory holding cost of the retailer in the ith cycle is given as:

$$\begin{aligned} C_d &= d\left[\int_{T_i}^{t_{i,h}} I_{i,3}(t)dt + \int_{t_{i,h}}^{t_{i,0}} I_{i,4}(t)dt\right] \\ &= d\left[\frac{D}{\delta} + \frac{D}{\theta+\delta}\left(e^{(\theta+\delta)(t_{i,0}-t_{i,h})} - 1\right)\right]\frac{e^{\delta t_{i,h}} - 1}{\delta} - d\frac{Dt_{i,h}}{\delta} \\ &\quad + \frac{dD}{(\theta+\delta)}\left[\frac{e^{(\theta+\delta)\left(t_{i,0}-t_{i,h}\right)} - 1}{\theta+\delta} - t_{i,0} + t_{i,h}\right] \end{aligned} \tag{15}$$

The shortage loss cost of the retailer in the ith cycle, C_f, can be found in Equation (7).

The deterioration cost of the retailer in the ith cycle is given as:

$$C_g = g\int_{t_{i,h}}^{t_{i,0}} I_{i,4}(t)dt = gD\left[\left(e^{(\theta+\delta)(t_{i,0}-t_{i,h})} - 1\right)/(\theta+\delta) - t_{i,0} + t_{i,h}\right]/(\theta+\delta) \tag{16}$$

The inventory cost of the retailer is obtained by summing up the cost for the previous $n-1$ cycles:

$$C_h = (n-1)C_b + \sum_{i=1}^{n-1}(C_d + C_f + C_g) \tag{17}$$

3.2.2. *Inventory level of retailers in the nth cycle.*

(1) Within the interval $[T_n, t_{n,h})$, the on-hand inventory items do not deteriorate. The change rate of inventory level equals to the demand rate of retailers for perishable items when $I(t) \geq 0$. Hence the instantaneous state of inventory level within $[T_n, t_{n,h})$ should satisfy:

$$dI_{n,1}(t)/dt = -D - \delta I_{n,1}(t), \quad t \in [T_n, t_{n,h}) \tag{18}$$

$I_{n,1}(t)$ is a continuous function, and then the inventory level of retailers at t is given as:

$$I_{n,1}(t) = \left[D\left[e^{(\theta+\delta)(T_{n+1}-t_{n,h})} - 1\right]/(\theta+\delta) + D/\delta\right]e^{\delta(t_{n,h}-t)} - D/\delta, \quad t \in [T_n, t_{n,h}) \tag{19}$$

(2) During the interval $(t_{n,h}, T_{n+1}]$, the on-hand inventory items turn to deterioration. The change rate of inventory level equals the summation of the demand rate of retailers and the deterioration rate of perishable items when $I(t) \geq 0$. Thus, we have:

$$dI_{n,2}(t)/dt = -D - \delta I_{n,2}(t) - \theta I_{n,2}(t), \quad t \in (t_{n,h}, T_{n+1}] \tag{20}$$

The boundary condition is $I_{n,2}(T_{n+1}) = 0$, so the inventory level of retailers at t is given as:

$$I_{n,2}(t) = D\left[e^{(\theta+\delta)(T_{n+1}-t)} - 1\right]/(\theta+\delta), \quad t \in (t_{n,h}, T_{n+1}] \tag{21}$$

The total inventory holding cost of the retailer in the nth cycle is given as:

$$\begin{aligned} C'_d =& d\left[\int_{T_n}^{t_{n,h}} I_{n,1}(t)dt + \int_{t_{n,h}}^{T_{n+1}} I_{n,2}(t)dt\right] \\ =& d\left[\left\{\frac{D}{(\theta+\delta)}\left[e^{(\theta+\delta)(T_{n+1}-t_{n,h})} - 1\right] + \frac{D}{\delta}\right\}\left(e^{\delta t_{n,h}} - 1\right) - Dt_{n,h}/\delta \right. \\ &\left. + \frac{D}{\theta+\delta}\left[\frac{e^{(\theta+\delta)\left(T_{n+1}-t_{n,h}\right)} - 1}{\theta+\delta} - T_{n+1} + t_{n,h}\right]\right] \end{aligned} \tag{22}$$

The deterioration cost of the retailer in the nth cycle is:

$$C'_g = g\int_{t_{n,h}}^{T_{n+1}} I_{n,2}(t)dt = gD\left[\left[e^{(\theta+\delta)(T_{n+1}-t_{n,h})} - 1\right]/(\theta+\delta) - T_{n+1} + t_{n,h}\right]/(\theta+\delta) \tag{23}$$

Then the total inventory cost of the retailer in the nth cycle is:

$$C_n = C_b + C'_d + C'_g \tag{24}$$

Then the total inventory cost over the time horizon H is obtained as $C_H = C'_h + C_n$.

4. **Numerical Example.**

Table 1 shows the parameters mentioned in Section 3, which are collected and obtained from the managers' experience in a supermarket of Beijing.

Table 1. The modeling parameters

H	T	n	D	δ	β	θ	C_b	d	f	g	$t_{i,h}$
90 days	15 days	6 times	600/day	0.15	0.2	0.35	200 RMB	0.2 RMB	0.8 RMB	0.6 RMB	13 days

The optimal shortage point is obtained using Matlab as 13.1 days the parameters in Table 1, with the minimal inventory cost 168100 RMB. Table 2 shows the sensitivity analysis of modeling parameters to decide which parameters have more impact on the inventory policy.

From the results in Table 2, it is obvious that the impact of the parameters δ, θ and g on the shortage point is greater compared with other parameters. It is consistent with the real case since these three parameters are related to the deteriorating process. For example, the larger the deterioration rate θ is, the smaller the optimal shortage point is; and the higher the deterioration cost g is, the smaller the optimal shortage point is. With the increase of the coefficient of inventory impacting sale rate δ, the inventory level of retailers decreases such that the shortage is easier to happen. Inventory managers should concern these indexes to avoid the cost loss caused by shortage.

Table 2. Results of sensitivity analysis for modeling parameters

D	580	590	600	610	620
t_0^* (days)	13.1	13.1	13.1	13.1	13.1
C^* (RMB)	162530	165320	168100	170880	173660
δ	0.13	0.14	0.15	0.16	0.17
t_0^* (days)	13.122	13.111	13.1	13.091	13.083
C^* (RMB)	149650	158500	168100	178610	190070
β	0.18	0.19	0.20	0.21	0.22
t_0^* (days)	13.1	13.1	13.1	13.1	13.1
C^* (RMB)	153090	153090	168100	153090	153090
θ	0.33	0.34	0.35	0.36	0.37
t_0^* (days)	13.106	13.103	13.1	13.098	13.095
C^* (RMB)	167950	168020	168100	168270	168310
d	0.18	0.19	0.20	0.21	0.22
t_0^* (days)	13.108	13.104	13.1	13.098	13.095
C^* (RMB)	138350	145730	168100	160520	167910
f	0.78	0.79	0.80	0.81	0.82
t_0^* (days)	13.1	13.1	13.1	13.1	13.1
C^* (RMB)	153090	153090	168100	153090	153090
g	0.58	0.59	0.60	0.61	0.62
t_0^* (days)	13.103	13.103	13.1	13.097	13.065
C^* (RMB)	153000	153030	168100	153200	153260

5. **Conclusions and Further Studies.** This paper deals with an inventory model over a finite time horizon for perishable items with constant deterioration rate and the time-varying demand, where according to the sequence of deterioration point and shortage point two scenarios are considered. Total inventory cost is chosen as the objective function including the ordering cost, the inventory holding cost, the shortage loss cost and the deterioration cost to optimize the shortage point. In the numerical example, based on the collected data, the sensitivity analysis is carried out. The inventory control policy for different perishable items should be focused in the further research.

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