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POSITION TRACKING CONTROL FOR PERMANENT MAGNET SYNCHRONOUS MOTORS VIA ADAPTIVE NEURAL NETWORK BACKSTEPPING

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Abstract. *This paper focuses on the problem of neural networks (NNs)-based adaptive backstepping control for permanent magnet synchronous motors (PMSMs) with parameter uncertainties and load torque disturbance. Based on backstepping technique, an adaptive neural network control method is proposed by using neural network systems to approximate unknown nonlinearities of permanent magnet synchronous motor drive system. The proposed adaptive neural network controller guarantees the tracking error converges to a small neighborhood of the origin. Then, the simulation results demonstrate the effectiveness of the proposed approach.*
Keywords: Permanent magnet synchronous motor, Adaptive control, Neural networks, Backstepping

1. **Introduction.** Permanent magnet synchronous motors (PMSMs) have been widely used in many industrial control fields due to its high power density and high efficiency over other kinds of motors such as induction motors and DC motors. However, on the other hand it is still a challenging problem to control permanent magnet synchronous motors to get the perfect dynamic performance because its dynamic model is usually multivariable, coupled and highly nonlinear. And the main disadvantage of the motor drive is that PMSM needs a more complex controller for high performance industrial applications. The control strategies based on recent modern control theories such as backstepping control [1], sliding mode control [2] and other control methods [3] are put forward to meet high performance application requirements of industrial applications. The backstepping-based adaptive control technique has become one of the most popular nonlinear control approaches because of its ability to clear up the influence of the uncertain parameter, particularly those systems that do not satisfy the matching conditions. The most appealing point of it is to use the virtual control variable to make the original high order system simple; thus, the final control outputs can be derived systematically through the suitable Lyapunov functions. Recently, neural network (NN) approximation [4,5] method has attracted great attention in PMSM drive systems because of its inherent capability for modeling and controlling highly uncertain, nonlinear and complex systems. The controller based on neural network has been applied to a broad range of engineering problems. Hence, neural networks can be used to deal with uncertain factors in nonlinear systems, and furthermore, be applied to controlling these systems which are ill-defined or too complex to have a mathematical model. It has been found one of the popular and conventional tools in functional approximations.

This paper is based on the dynamic mathematical model of PMSMs and designs an adaptive NN controller to realize the position tracking control. During the controller design process, NN systems are employed to approximate the nonlinearities. And the adaptive technique and backstepping are used to construct NN controller. The simulation results show that the adaptive NN control guarantees that PMSMs servo drives have a good tracking performance [6] even with the unknown parameters and load disturbances.

The remainder of this paper is organized as follows. The model of PMSM drive system is described in Section 2. Then the controller design of PMSM system is developed in Section 3. And its stability is analyzed in Section 4. The simulation results of the PMSM position control system are given in Section 5. Finally, some conclusions are presented.

2. **Modeling of PMSM Drive System.** The mathematical model of PMSM drive system can be described in the well-known d-q frame as follows [7]:

$$\begin{cases} \dfrac{d\Theta}{dt} = \omega, \\ \dfrac{d\omega}{dt} = \dfrac{3n_p}{2J}[(L_d - L_q)i_d i_q + \Phi i_q] - \dfrac{B\omega}{J} - \dfrac{T_L}{J}, \\ \dfrac{di_d}{dt} = \dfrac{(-R_s i_d + n_p \omega L_q i_q + u_d)}{L_d}, \\ \dfrac{di_q}{dt} = \dfrac{(-R_s i_q - n_p \omega L_d i_d - n_p \omega \Phi + u_q)}{L_q}. \end{cases} \tag{1}$$

where Θ, ω, n_p, J, B, T_L, Φ denote the rotor position, rotor angular velocity, pole pair, rotor moment of inertia, viscous friction coefficient, load torque and magnet flux linkage. i_d and i_q stand for the d-q axis currents. u_d and u_q are the d-q axis voltages. L_d and L_q are the stator inductors.

For simplicity, the following notations are introduced:

$$\begin{cases} x_1 = \Theta,\ x_2 = \omega,\ x_3 = i_q,\ x_4 = i_d,\ a_1 = \dfrac{3n_p\Phi}{2}, \\ a_2 = \dfrac{3n_p(L_d - L_q)}{2},\ b_1 = -\dfrac{R_s}{L_q},\ b_2 = -\dfrac{n_p L_d}{L_q},\ b_3 = -\dfrac{n_p\Phi}{L_q}, \\ b_4 = -\dfrac{1}{L_q},\ c_1 = -\dfrac{R_s}{L_q},\ c_2 = \dfrac{n_p L_q}{L_d},\ c_3 = \dfrac{1}{L_d}. \end{cases} \tag{2}$$

By using these notations, the dynamic model of PMSM driver system can be described by the following differential equations:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = (a_1 x_3 + a_2 x_3 x_4 - B x_2 - T_L)/J, \\ \dot{x}_3 = b_1 x_3 + b_2 x_2 x_4 + b_3 x_2 + b_4 u_q, \\ \dot{x}_4 = c_1 x_4 + c_2 x_2 x_3 + c_3 u_d. \end{cases} \tag{3}$$

In this paper, the radial basis function (RBF) NN [8] will be used to approximate the unknown continuous function $\phi(z) : R^q \to R$ as $\hat{\varphi}(z) = \phi^{*T} P(z)$, where $z \in \Omega_z \subset R^q$ is the input vector with q being NN input dimension, $\phi^* = [\varphi_1^*, \ldots, \varphi_n^*]^T \in R^n$ is the weight vector with $n > 1$ being the NN node number, and $P(z) = [p_1(z), \ldots, p_n(z)]^T \in R^n$ is the basis function vector with $p_i(z)$ chosen as the commonly used Gaussian function in the following form: $p_i(z) = \exp\left[\frac{-(z-\nu_i)^T(z-\nu_i)}{q_i^2}\right]$, $i = 1, 2, \ldots, n$, where $\nu_i = [\nu_{i1}, \ldots, \nu_{iq}]^T$ is the center of the receptive field and q_i is the width of the Gaussian function. It has been shown in that, for a given scalar $\varepsilon > 0$, by choosing sufficiently large l, the RBF NN can approximate any continuous function over a compact set $\Omega_z \in R^q$ to an arbitrary accuracy as $\varphi(z) = \phi^T P(z) + \delta(z)$, $\forall z \in \Omega_z \subset R^q$, where $\delta(z)$ is the approximation error satisfying $|\delta(z) \leq \varepsilon|$ and ϕ is an unknown ideal constant weight vector, which is an artificial quantity required for analytical purpose. Typically, ϕ is chosen as the value of ϕ^* that minimizes $|\delta(z)|$ for all $z \in \Omega_z$, $\varphi := \arg\min_{\varphi* \in R^n} \left\{ \sup_{z\in\Omega_z} \left|\phi(z) - \varphi^{*T} P(z)\right| \right\}$.

3. Adaptive Neural Network Controller Design with Backstepping Technique. In this section, we will design a controller for the PMSMs based on backstepping.

Step 1: For the reference signal x_d, we define the tracking error variable as $z_1 = x_1 - x_d$.

From the first differential equation of (1), the error dynamic system is computed by $\dot{z}_1 = x_2 - \dot{x}_d$. Choose a Lyapunov function candidate as $V_1 = \frac{z_1^2}{2}$, and then the time derivative of V_1 is computed by

$$\dot{V}_1 = z_1\dot{z}_1 = z_1(x_2 - \dot{x}_d) \tag{4}$$

Construct the virtual control law α_1 as $\alpha_1 = -k_1z_1 + \dot{x}_d$, with $k_i > 0$ $(i = 1, 2, 3, 4)$ being a design parameter and $z_2 = x_2 - \alpha_1$. Then, $\dot{V}_1$ can be written as $\dot{V}_1 = -k_1z_1^2 + z_1z_2$.

Step 2: Differentiating z_2 gives $\dot{z}_2 = (a_1x_3 + a_2x_3x_4 - Bx_2 - T_L)/J - \dot{\alpha}_1$.

Choose the Lyapunov function candidate as $V_2 = V_1 + \frac{Jz_2^2}{2}$, and then the time derivative of V_2 is computed by

$$\dot{V}_2 = -k_1z_1^2 + z_2(a_1x_3 + f_2) \tag{5}$$

where $f_2(Z) = z_1 + a_2x_3x_4 - Bx_2 - T_L - J\dot{\alpha}_1$ and $Z = [x_1, x_2, x_3, x_4, x_d, \dot{x}_d]$. According to the RBF neural network approximation property, for given $\varepsilon_2 > 0$, there exists an RBF neural network $\phi_2^TP_2(Z)$ such that $f_2(Z) = \phi_2^TP_2(Z) + \delta_2(Z)$, where $\delta_2(Z)$ is the approximation error satisfying $|\delta_2| \leq \varepsilon_2$. Consequently, we can get

$$z_2f_2 \leq \frac{1}{2l_2^2}z_2^2\|\phi_2\|^2P_2^TP_2 + \frac{1}{2}\left(l_2^2 + z_2^2 + \varepsilon_2^2\right) \tag{6}$$

Then, we choose $\alpha_2 = \frac{1}{a_1}\left(-k_2z_2 - \frac{1}{2}z_2 - \frac{1}{2l_2^2}z_2\hat{\theta}P_2^TP_2\right)$, where $\hat{\theta}$ is the estimation of the unknown constant θ which will be specified later and define $z_3 = x_3 - \alpha_2$, with l_2 being positive constant.

$$\dot{V}_2 \leq -\sum_{i=1}^{2}k_iz_i^2 + a_1z_2z_3 + \frac{1}{2l_2^2}z_2^2\left(\|\phi_2\|^2 - \hat{\theta}\right)P_2^TP_2 + \frac{1}{2}\left(l_2^2 + \varepsilon_2^2\right) \tag{7}$$

Step 3: Differentiating z_3 obtains $\dot{z}_3 = b_1x_3 + b_2x_2x_4 + b_3x_2 + b_4u_q - \dot{\alpha}_2$.

Choose the Lyapunov function candidate as $V_3 = V_2 + \frac{z_3^2}{2}$, and the time derivative of V_3 is given by

$$\dot{V}_3 \leq -\sum_{i=1}^{2}k_iz_i^2 + \frac{1}{2l_2^2}z_2^2\left(\|\phi_2\|^2 - \hat{\theta}\right)P_2^TP_2 + \frac{1}{2}\left(l_2^2 + \varepsilon_2^2\right) + z_3(f_3 + b_4u_q) \tag{8}$$

where $f_3(Z) = b_1x_3 + b_2x_2x_4 + b_3x_2 + a_1z_2 - \dot{\alpha}_2 = \phi_3^TP_3(Z) + \delta_3(Z)$. Similarly, for given $|\delta_3| \leq \varepsilon_3$, $\varepsilon_3 > 0$, we can obtain

$$z_3f_3 \leq \frac{1}{2l_3^2}z_3^2\|\phi_3\|^2P_3^TP_3 + \frac{1}{2}\left(l_3^2 + z_3^2 + \varepsilon_3^2\right) \tag{9}$$

Then, construct the control law u_q

$$u_q = \frac{1}{b_4}\left(-k_3z_3 - \frac{1}{2}z_3 - \frac{1}{2l_3^2}z_3\hat{\theta}P_3^TP_3\right) \tag{10}$$

Furthermore, using Equations (8), (9) and (10), it can be verified easily that

$$\dot{V}_3 \leq -\sum_{i=1}^{3}k_iz_i^2 + a_1z_2z_3 + \sum_{i=2}^{3}\frac{1}{2l_i^2}z_i^2\left(\|\phi_i\|^2 - \hat{\theta}\right)P_i^TP_i + \sum_{i=2}^{3}\frac{1}{2}\left(l_i^2 + \varepsilon_i^2\right) \tag{11}$$

Step 4: At this step, we will construct the control law u_d. Define $z_4 = x_4$ and choosing $V_4 = V_3 + \frac{z_4^2}{2}$, then V_4 is computed by

$$\dot{V}_4 = \dot{V}_3 + z_4\dot{z}_4 = \dot{V}_3 + z_4(f_4 + c_3u_d) \tag{12}$$

where $f_4(Z) = c_1x_4 + c_2x_2x_3 = \phi_4^T P_4(Z) + \delta_4(Z)$. Similarly, for given $|\delta_4| \le \varepsilon_4$, $\varepsilon_4 > 0$, we can get

$$z_4 f_4 \le \frac{1}{2l_4^2} z_4^2 \|\phi_4\|^2 P_4^T P_4 + \frac{1}{2}\left(l_4^2 + z_4^2 + \varepsilon_4^2\right) \tag{13}$$

Construct the virtual control law u_d as

$$u_d = \frac{1}{c_3}\left(-k_4 z_4 - \frac{1}{2} z_4 - \frac{1}{2l_4^2} z_4 \hat{\theta} P_4^T P_4\right) \tag{14}$$

Defining $\theta = \max\{\|\phi_2\|^2, \|\phi_3\|^2, \|\phi_4\|^2\}$, $\tilde{\theta} = \hat{\theta} - \theta$ and using Equations (13) and (14), one has

$$\dot{V}_4 \le -\sum_{i=1}^{4} k_i z_i^2 + \sum_{i=2}^{4} \frac{1}{2l_i^2} z_i^2 \left(\theta - \hat{\theta}\right) P_i^T P_i + \sum_{i=2}^{4} \frac{1}{2}\left(l_i^2 + \varepsilon_i^2\right) \tag{15}$$

Choose the Lyapunov function candidate as $V = V_4 + \frac{\tilde{\theta}^2}{2r_1}$, where r_1 is a positive constant. By differentiating V, one has

$$\dot{V} \le -\sum_{i=1}^{4} k_i z_i^2 + \sum_{i=2}^{4} \frac{1}{2}\left(l_i^2 + \varepsilon_i^2\right) + \frac{1}{r_1}\tilde{\theta}\left(-\sum_{i=2}^{4} \frac{r_1}{2l_i^2} z_i^2 P_i^T P_i + \dot{\hat{\theta}}\right) \tag{16}$$

According to Equation (16), the corresponding adaptive law is chosen as follows:

$$\dot{\hat{\theta}} = \sum_{i=2}^{4} \frac{r_1}{2l_i^2} z_i^2 P_i^T P_i - m_1 \hat{\theta} \tag{17}$$

where m_1 and l_i $(i = 2, 3, 4)$ are positive constants.

Remark 3.1. *Consider the system (1) and the reference signals x_d. Then under the action of the adaptive neural controllers (10), (14) and adaptive laws (17), the tracking error of the closed-loop controlled system will converge to a small neighborhood of the origin and all the closed-loop signals are bounded.*

4. **Stability Analysis of PMSM Position Control.** Lyapunov stability theorem is used to analyze stability of PMSM position system in this paper; substituting (17) into (16), one has

$$\dot{V} \le -\sum_{i=1}^{4} k_i z_i^2 + \sum_{i=2}^{4} \frac{1}{2}\left(l_i^2 + \varepsilon_i^2\right) - \frac{m_1}{r_1}\tilde{\theta}\hat{\theta} \tag{18}$$

For the term $-\tilde{\theta}\hat{\theta}$, one has $-\tilde{\theta}\hat{\theta} \le -0.5\tilde{\theta}^2 + 0.5\theta$. Consequently, by using these inequalities, (18) can be rewritten in the following form

$$\dot{V} \le -\sum_{i=1}^{4} k_i z_i^2 + \sum_{i=2}^{4} \frac{1}{2}\left(l_i^2 + \varepsilon_i^2\right) + \frac{m_1}{2r_1}\tilde{\theta}^2 - \frac{m_1}{2r_1}\theta^2 \le -aV + b \tag{19}$$

where $a = \min\left\{2k_1, \frac{2k_2}{J}, 2k_3, 2k_4, m_1\right\}$ and $b = \sum_{i=2}^{4} \frac{1}{2}\left(l_i^2 + \varepsilon_i^2\right) + \frac{m_1}{2r_1}\theta^2$.

Then, (19) implies that

$$V(t) \le \left(V(t_0) - \frac{b}{a}\right) e^{-a(t-t_0)} + \frac{b}{a} \le V(t_0) + \frac{b}{a},\ \forall t \ge t_0 \tag{20}$$

All z_i $(i = 1, 2, 3, 4)$, θ belong to the compact set $\Omega = \left\{\left(z_i, \tilde{\theta}\right) \middle| V \le V(t_0) + \frac{b}{a},\ \forall t \ge t_0\right\}$. Namely, all the signals in the closed-loop system are bounded. From (20), we have

$$\lim_{t \to \infty} z_1^2 \le \frac{2b}{a} \tag{21}$$

By the definitions of a and b, we can set r_1 large enough to get a small tracking error, with l_i and ε_i small enough after giving the parameters k_i and m_1.

5. **Simulation Results.** In order to illustrate the effectiveness of the proposed results, the simulations are performed to evaluate the performance of closed-loop system by using Matlab/Simulink. The motor parameters of the PMSM are:

$$R_s = 0.68\Omega,\ J = 0.003798\text{kg}\cdot\text{m}^2,\ n_p = 3,\ L_d = 0.00285\text{H},$$
$$L_q = 0.00315\text{H},\ \Phi = 0.1245\text{Wb}$$

The RBF NNs are chosen in the following way. Then, an adaptive neural network controller is used to control this permanent magnet synchronous motors. The control parameters are chosen as follows:

$$k_1 = 150,\ k_2 = 60,\ k_3 = 80,\ k_4 = 100,\ r_1 = 1.25,\ m_1 = 0.005,\ l_2 = l_3 = l_4 = 0.05$$

The simulation is carried out under the zero initial condition for the permanent magnet synchronous motors. Give the reference signals:

$$x_d = \sin 2t, \quad T_L = \begin{cases} 1.5N\cdot m, & 0 \le t \le 1 \\ 3N\cdot m, & t \ge 1 \end{cases}$$

Figure 1 shows the reference signal x_1 and x_d. Figure 2 shows the error curve. It can be observed from Figure 1 and Figure 2 that the system can track the given reference signal well. Figure 3 and Figure 4 show the trajectories of u_q and u_d. It can be seen that the controllers are bounded. From the above simulation results, it is clearly seen that the proposed controllers can track the reference signal quite well even under parameter uncertainties and load torque disturbance.

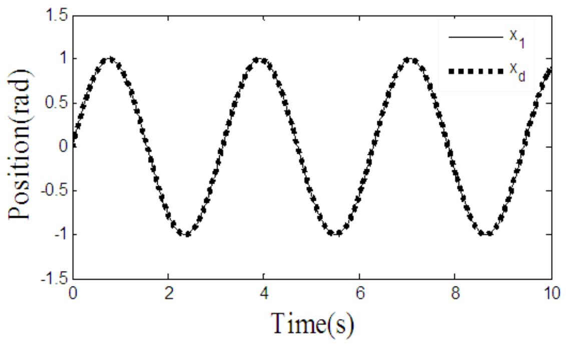

Figure 1. Position curve

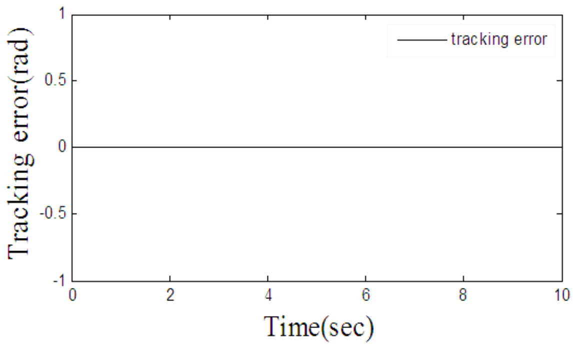

Figure 2. Error curve

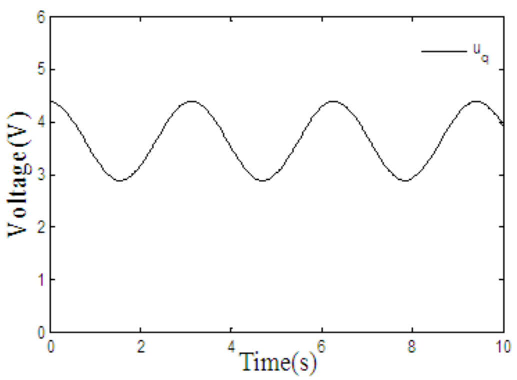

Figure 3. Voltage u_q curve

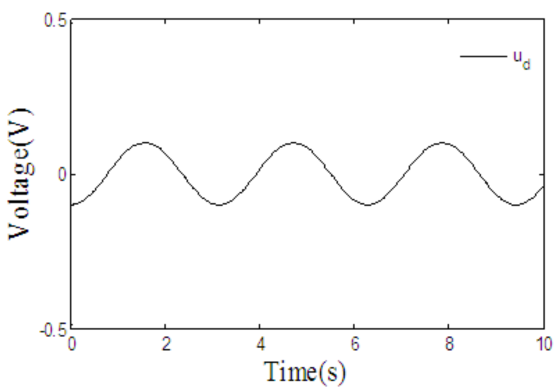

Figure 4. Voltage u_d curve

6. **Conclusions.** Based on backstepping technique, an adaptive neural network controller method is designed to control permanent magnet synchronous motors. The proposed controller is able to overcome the problem of "explosion of complexity" inherent in the traditional backstepping design. And the designed controller guarantees that the position tracking error can converge to a small neighborhood of the origin. Simulation results testify its effectiveness in the PMSM drive system. In the future work, we will focus on the practical application of the proposed control algorithm.

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