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SEQUENTIAL SELECTION AMONG AUCTIONS UNDER INCOMPLETE INFORMATION IN COGNITIVE RADIO NETWORKS

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Abstract. *In this paper, the problem of how to sequentially select an auction for satisfying SUs' different objectives under incomplete information in cognitive radio networks has been studied. An intelligent secondary user will decide whether to attend an auction or go to the next auction based on stopping rules. Dirichlet process is used to estimate the winning probability (WP) of each auction by utilizing the historical data of winning bids in each auction. The whole selection process does not need any prior information of other SUs' bidding strategies and WP of each auction. Three auction selection policies with different purposes have been proposed and simulation results show that the proposed auction selection policies are effective.*
Keywords: Cognitive radio networks, Auction theory, Dirichlet process, Optimal stopping theory

1. **Introduction.** In cognitive radio networks (CRNs) where wireless devices are allowed to access idle licensed spectrum bands (also called spectrum opportunities in both time domain and space domain) to improve the spectrum usage efficiency [1], the users who have the licenses of spectrum bands are called primary users (PUs) and the users who can utilize spectrum opportunities are described as secondary users (SUs). In order to improve the utilization of spectrum opportunities further, an appropriate economic mechanism is needed [2]. Auction is a very important market mechanism and suitable for allocating the idle spectrum bands among SUs efficiently [3, 4]. There are many works which have used auction models to research opportunistic spectrum allocation problems in CRNs. In [5], authors have proposed an auction mechanism based on Dirichlet process (DP), which enables fair and efficient allocating of spectral resources among SUs. From the view of an individual SU, there may be many PUs who act as auctioneers to sell spectrum opportunities at the same time. Under this situation, the problem of how to allocate multiple PUs' channels among SUs has been studied in [6] based on the concept of Nash equilibrium. However, the situation where each SU will select an auction one by one without any prior information of other SUs' bidding strategies and WP of each auction has not been considered in [6], which is a sequential selection problem. Luckily, some work [7, 8, 9] has analyzed some sequential selection problems in CRNs based on optimal stopping theory (OST). In [8], authors have proposed a simple channel sensing order based on OST without a priori knowledge of primary user activities. The process of SUs sensing the state of each primary channel will consume SUs' time and energy. Therefore, an optimal energy-efficient channel exploration strategy is very important for SUs in practice, which has been investigated in [9] based on OST. Although these work has used OST to research sequential selection problems in different CRNs models, previous studies

have not tackled the problem of how to select an auction one by one under incomplete information.

In this paper, we assume there is an intelligent SU (ISU) who is different from other SUs in CRNs. The ISU can optimize its auction selection dynamically by utilizing the available information produced by the previous auction rounds. However, other SUs who are ordinary SUs (OSUs) do not change their own auction selection policies during the whole auction process, which means OSUs will always attend the auction which has been chosen at the first selection. This assumption is reasonable, because if there are many SUs who attend auctions in each slot, each SU just cares about the previous winning bids and WP of each auction in the next slot. Therefore, each SU can consider other SUs do not change their selection strategies during the whole auction process. In other words, the performance of the ISU is the key in this paper and OSUs just provide a dynamic external environment for the ISU. In our proposed model, the auction selection process for the ISU is modeled as a finite horizon optimal stopping problem. The remainder of this paper is organized as follows. The system model is described in Section 2. In Section 3, the expression of WP has been given and three auction selection policies with different purposes have been proposed. The simulation results are shown in Section 4. Finally, Section 5 concludes this paper.

2. **System Model.** Consider a CRN with M PUs and N SUs as well as one SU base station S in the same transmission area. All SUs are composed of two parts, namely, one ISU and multiple OSUs. All PUs and SUs operate in the slot transmission structure and the ISU knows the total number M of PUs. Each slot t is composed of the auction selection time (ST) $M\tau$ and the data transmission time (DT) $(t - M\tau)$. Furthermore, we assume $M\tau \ll t$. An illustration of the ISU auction selection process is given in Figure 1. Each PU who acts as an auctioneer owns one primary channel and each SU who acts as a bidder involves only one auction at a time. If any SU wants to use idle slot t of primary channel m $(m \in M)$ to send its data to the base station S, each SU must attend the channel auction holden by PU m at the beginning of slot t. The winner who will be decided at the moment $M\tau$ in each auction will obtain DT $(t - M\tau)$ of the channel m. All auctioneers decide who are the winners at the moment $M\tau$ for the synchronization of CRNs. We also assume the total slots are T and all primary channels are always idle. In each slot t, each PU will hold an auction to sell DT of its channel, where ISU can select an auction with higher WP one by one only based on historical data and each OSU can just attend a fixed auction through all slots. The transmission rates between SUs and SU base station through each primary channel are assumed unchanged in each slot t. At

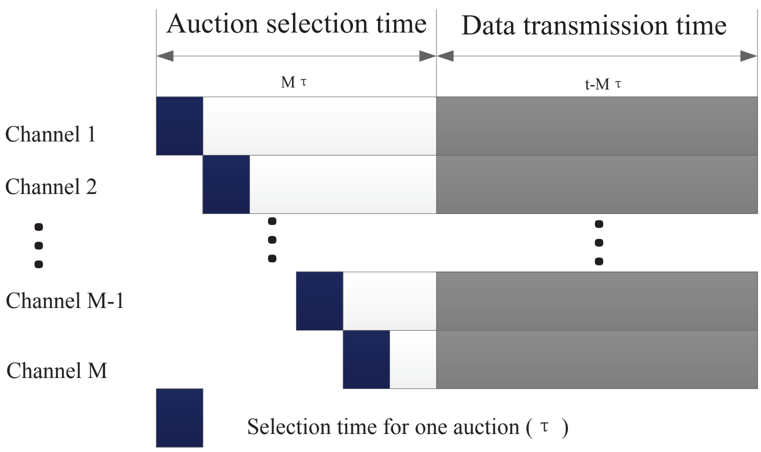

FIGURE 1. An illustration of auction selection process

the beginning of slot t, each OSU estimates the transmission rate $R^t_{n,S,m}$ of the idle slot by channel estimation technologies [5], and then sends its own true estimation value as a bid [10] to the PU which has been fixed previously. The transmission rate $R^t_{n,S,m}$ of channel m estimated by SU n in slot t is $R^t_{n,S,m} = W \log_2 \left(1 + P g^t_{n,S,m}/\sigma^2\right)$, where W is the channel bandwidth, P is the transmit power and σ^2 is the thermal noise power, which are the same to all SUs for the simplification of research. Note that $g^t_{n,S,m}$ which denotes the channel gain between SUs and SU base station in slot t and changes over the time but keeps unchanged during each slot (block-fading). The ISU not only estimates the transmission rate $R^t_{n,S,m}$ but also evaluates the WP using the historical data of winning bids of auction m. Here, we choose the second price sealed auction to allocate idle slots among SUs because this auction mechanism is incentive compatibility which means that reporting the true private value is the weak dominant strategy for all bidders [10]. In the auction held by PU m, there is $K_m(\sum_m K_m = N)$ participators and any SU n with the highest bid wins the DT of the channel in slot t, i.e.,

$$\Omega^t_n = \begin{cases} 1, & R^t_{n,S,m} > R^t_{n',S,m},\ \forall n' \neq n, n', n \in K_m \\ 0, & \text{else.} \end{cases} \tag{1}$$

Hence, the highest bid $b^{(1)}_{m,t} = R^t_{n,S,m}$ and the secondary highest bid $b^{(2)}_{m,t} = \max(R^t_{n',S,m}, n' \in K_m \setminus n)$ in slot t.

3. **Online Selection Policy.** In this section, we firstly introduce the expression of WP and then we will propose three online selection policies which satisfy different objectives of ISU. Let $b^{(1)}_{m,1}, b^{(1)}_{m,2}, \ldots, b^{(1)}_{m,t-1}$ denote the historical winning bids data of auction m in previous $(t-1)$ rounds. At the beginning of slot t, ISU estimates the channel transmission rate $R^t_{n,S,m}$ at first, and then it uses $R^t_{n,S,m}$ to evaluate the WP based on the historical winning bids data of the auction m. The expression of WP which is denoted by P^t_m can be written as follows [11],

$$\begin{aligned} &P^t_m\left(b^{(1)}_{m,t} \leq R^t_{n,S,m} \,\middle|\, b^{(1)}_{m,1}, b^{(1)}_{m,2}, \ldots, b^{(1)}_{m,t-1}\right) \\ &= \frac{1}{\beta + t - 1}\left(\beta H\left(R^t_{n,S,m}\right) + \sum_{i=1}^{t-1} \delta_{b^{(1)}_{m,i}}\left(R^t_{n,S,m}\right)\right), \end{aligned} \tag{2}$$

where $\sum_{i=1}^{t-1} \delta_{b^{(1)}_{m,i}}\left(R^t_{n,S,m}\right)$ denotes the number of winning bids which are less than $R^t_{n,S,m}$. The base distribution H is assumed to be the uniform distribution over $\left(0, \max b^{(1)}_{m,t}\right)$, where $b^{(1)}_{m,t}$ denotes the any possible winning bid in the auction m [5]. Furthermore, in fact, $\max b^{(1)}_{m,t}$ denotes the maximal capacity between SUs and SU base station S, which is restricted by SUs' hardware. In simulations of Section 4, we assume the maximal capacity among the SUs is produced through all random channel conditions. Formula (2) is deduced by the predictive distribution of DP [11]. If readers want to know more details about DP, please refer to [11].

3.1. **Sequential selection with finding the optimal auction.** First, we propose a simple selection policy with recall in which ISU can evaluate the WPs of all auctions and always selects the optimal auction (the auction with $\max_m(P^t_m)$) to be present which is also called Policy-1 in simulation results. Second, we propose a selection policy based on the classical secretary problem [12] for the maximal probability of finding the optimal auction, which is called sequential auction selection without recall (also called Policy-2 in the simulation part). No recall means ISU is not allowed to choose the optimal auction again if ISU loses the optimal auction in the past. When ISU evaluates P^t_m of auction m after having observed $P^t_1, P^t_2, \ldots, P^t_{m-1}$, it will make a decision to stop at auction m or go to evaluate auction $m+1$. If ISU has evaluated the first m auctions, the WPs of the first m auctions can be ranked from the maximal to the minimal. Here, we use $X_1, X_2, \ldots, X_M$ to denote the absolute ranks of WPs of M auctions, for example, $X_1 = 1$ denotes the maximal WP among M auctions. The relative rank of observed WPs is denoted as $y_m = \{\text{the number of } (P_1^t, P_2^t, \ldots, P_{m-1}^t > P_m^t) + 1\}$, where $y_1 = 1$ represents the maximal WP among the first m auctions. We assume y_m has a uniform distribution over the integers from 1 to m. However, this assumption is not entirely reasonable, which will be showed in simulation results. Note that an auction should be attended only if it is relatively maximum among those already evaluated. The stopping rule in classical secretary problem is called a threshold rule with threshold r, which means the decision-maker will reject the first $r-1$ auctions and then accept the next relatively best auction. The following analyzed technology is adopted from [12]. The probability p_r of finding optimal auction using stopping rule is $p_r = \sum_{k=r}^{M} p$ (kth auction with maximal WP and selected) $= \sum_{k=r}^{M} \frac{1}{M} p$ (the maximal of the first $k-1$ appears before stage r) $= \sum_{k=r}^{M} \frac{1}{M}\frac{r-1}{k-1} = \frac{r-1}{M}\sum_{k=r}^{M}\frac{1}{k-1}$, where $\frac{r-1}{r-1}$ represents 1 if $r=1$. The optimal rule r_o is the value of r which maximizes p_r. Therefore, $p_{r+1} \leq p_r$ if and only if $\frac{r}{M}\sum_{r+1}^{M}\frac{1}{k-1} \leq \frac{r-1}{M}\sum_{r}^{M}\frac{1}{k-1}$ if and only if $\sum_{r+1}^{M}\frac{1}{k-1} \leq 1$. Therefore, the optimal rule $r_o = \min\left\{r \geq 1 : \sum_{r+1}^{M}\frac{1}{k-1} \leq 1\right\}$. For example, if $M=4$, the optimal rule $r_o = 2$, which means ISU will not select the first auction and will select the auction with a higher relative rank than the first one. The Policy-2 is that if ISU can find an auction using the optimal rule r_o, it will stop at the auction and sends its bid to the auction; otherwise, it will stop at the last auction and sends its bid to the last auction. The main objective of Policy-2 is to maximize the probability of finding the optimal auction without recall.

3.2. **Sequential selection with minimal expected absolute rank.** As above mentioned, the ISU evaluates the WP of each auction one by one, and how to set an optimal stopping rule for minimizing the expected absolute rank of WPs before stopping to attend the auction is an interesting problem, which is meaningful for ISU who cares about the cost of searching an auction. We assume there is a stopping policy which is the vector $\boldsymbol{r} = \{r_1 \leq r_2 \leq r_3 \ldots \leq r_M\}$ (also called Policy-3 in the simulation part). In Policy-3, the recall is also not allowed. After evaluating the first m auctions, if the relative rank y_m is less than or equal to r_m, ISU will stop at the mth auction. Otherwise, ISU will continue to evaluate the WP of the next auction. The following technical analyses for determining $\boldsymbol{r}$ are adopted from [13]. If each permutation of the WPs $M!$ is equally likely, the relative rank y_m of the mth auction takes the value from 1 to m with equal probability $1/m$. This assumption in our model is also not entirely reasonable but is effective. Define an event w_m: the event whose stopping does not occur at the mth auction or earlier, and its probability is $Q(m) = \hat{P}(w_m)$. Hence, the probability of ISU stopping at the mth auction is $\hat{p} = \hat{P}\{w_{m-1}, y_m\} = Q(m-1)\frac{r_m}{m}$. We can also get a recurrence formula $Q(m) = Q(m-1)\left(1 - \frac{r_m}{m}\right)$.

Consider $\hat{P}\left(y_m = j | X_m = \hat{m}\right) = \binom{\hat{m}-1}{j-1}\binom{M-\hat{m}}{m-j}\Big/\binom{M-1}{m-1}$. $\hat{P}\left(X_m = \hat{m} | y_m = j\right)$ can be written as $\hat{P}\left(X_m = \hat{m} | y_m = j\right) = \hat{P}\left(X_m = \hat{m}, y_m = j\right)/\hat{P}(y_m = j) = \binom{\hat{m}-1}{j-1}\binom{M-\hat{m}}{m-j}\Big/\binom{M}{m}$, where $\hat{P}(X_m = \hat{m}, y_m = j) = \hat{P}\left(X_m = \hat{m}\right)\hat{P}(y_m = j | X_m = \hat{m})$. The expected absolute rank under condition $y_m = j$ is given as $\boldsymbol{E}(X_m | y_m = j) = \sum_{\hat{m}}^{M} \hat{m}\hat{P}(X_m = \hat{m} | y_m = j) = (M + 1/m + 1)\, j$. Let C_m denote the expected absolute rank under the condition w_m, $C_m = \boldsymbol{E}(\hat{m} | w_m)$ and the recurrence formula is as follows,

$$C_{m-1} = \frac{1}{Q(m-1)}\sum_{k=m}^{M} Q(k-1)\frac{1}{k}\frac{M+1}{k+1}\sum_{j=1}^{r_m} j$$

$$= \frac{(M+1)r_m(r_m+1)}{2m(m+1)} + \left(1 - \frac{r_m}{m}\right) C_m. \tag{3}$$

Our objective is to minimize the expected absolute rank C_0. For achieving this goal, (3) can be rewritten as follows,

$$C_{m-1} = C_m + \frac{1}{m}\sum_{j=1}^{r_m}\left(\frac{M+1}{m+1}j - C_m\right). \tag{4}$$

For given C_m, we can determine r_m to minimize C_{m-1}, which is based on the principle of optimality in dynamic programming [13]. Therefore, r_m can be chosen as the largest integer j with satisfied $\frac{M+1}{m+1}j \le C_m$ and then,

$$r_m = \left\lfloor \frac{m+1}{M+1} C_m \right\rfloor. \tag{5}$$

Hence, starting with $r_M = M$, $C_{M-1} = \frac{M+1}{2}$ and using (4) and (5), the optimal rule $\boldsymbol{r}$ can be deduced.

4. **Simulation Results.** In this section, we evaluate the performances of the proposed selection policies in the CRN. We consider all SUs randomly distribute in a 500m × 500m coverage area, and SU base station S locates in the center of the area. PUs are also in this area and all SUs can send the bids to PUs. We assume each auction always has 10 OSUs at a time. Hence, the total number of all SUs $N = 10M + 1$. The range of M is from 2 to 20. The total times $T = 10000$ and all SUs adopt the channel model of Flat/Light tree density proposed in [14]. $P = 10^{-3}$ of all SUs and $\sigma^2 = 10^{-12}$W as well as $W = 1$Hz and $\beta = 0.5$. For each M, the simulation will repeat 100 times.

In Figure 2, we compare the winning ratio of all auction selection policies. The winning ratio is defined as the ratio between the total winning times of ISU and the total slots T. In Policy-1, ISU always selects the optimal auction and sends its bid to the auctioneer. Therefore, the winning ratio of Policy-1 should be the best policy among all policies. The main goal of the Policy-2 is helping the ISU to find the optimal auction as much as possible. However, this effort is not helpful to increase the winning ratio during the whole selection process. Another fact in Figure 2 that the winning ratios of Policy-1 and Policy-3 increase with M increasing is discovered. This is because the probability of finding an auction with a lower absolute rank under the condition of the same relative rank will increase when M increases. However, this trend is not obvious in the curve of

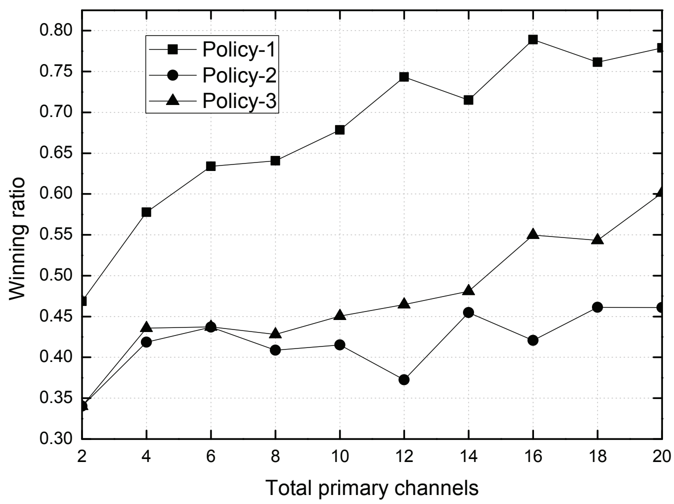

Figure 2. Winning ratio of three auction selection policies

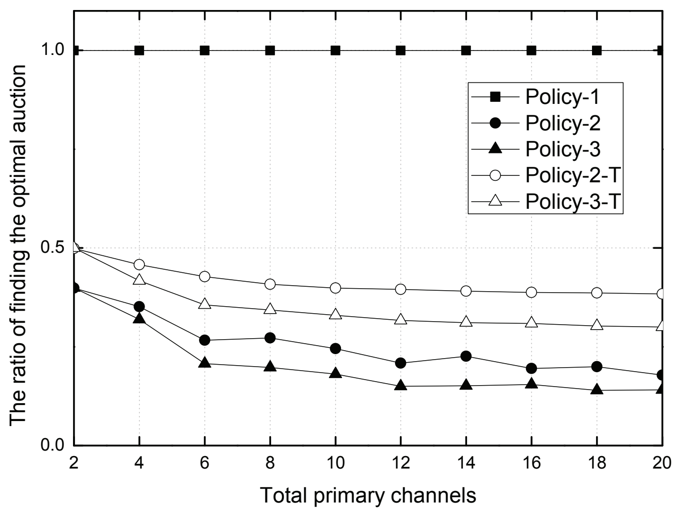

FIGURE 3. The ratio of finding the optimal auction

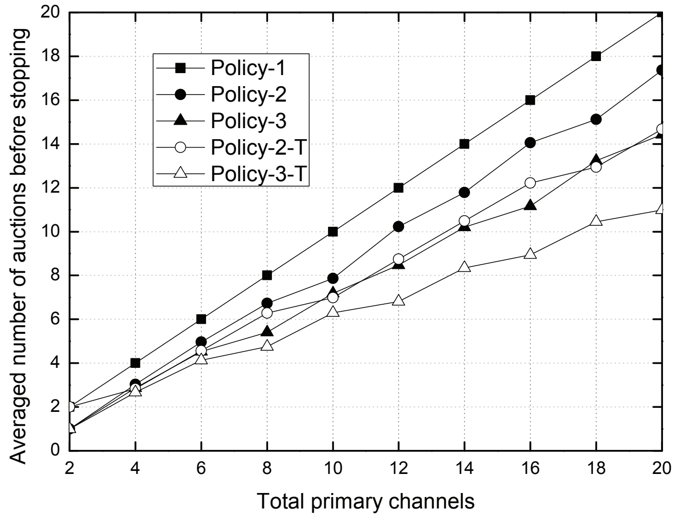

FIGURE 4. Averaged number of auctions before stopping

Policy-2 in Figure 2. This is because, in Policy-2, if ISU cannot find the optimal auction, it will have to stop at the last auction which will reduce the winning ratio of ISU.

In Figure 3, we define the ratio of finding the optimal auction as the ratio between the number of finding the optimal auction and the total auction rounds T. The ISU always selects the optimal auction using Policy-1, hence the ratio is 1 for all different M. The Policy-2 outperforms the Policy-3 in the respect of finding the optimal auction. We also show the ratio of Policy-2 and Policy-3 under the condition that any ordering which is one permutation of $M!$ appears equally, which are Policy-2-T and Policy-3-T in Figure 3. Policy-2-T and Policy-3-T outperform Policy-2 and Policy-3. This is because the assumption of any ordering of $M!$ WPs being equally likely is not rigorously satisfied but is also effective.

In Figure 4, the averaged numbers of auctions before ISU stopping at an auction using different policies are demonstrated. The main objective of Policy-3 are minimizing the expected absolute rank of WPs before ISU stopping. This effort is effective, which can be seen from Figure 4. Minimizing the absolute rank of WPs is meaningful in practical CRNs because the processes of estimating channel transmission rate and evaluating WP will cost some resources. The Policy-3 can realize the tradeoff between obtaining auction winning times and costing resources (e.g., time and energy). This is because the averaged numbers of auctions before ISU stopping using Policy-3 are less than using Policy-2 while the winning ratio using Policy-3 is better than Policy-2 in Figure 2. We also show the averaged numbers of auctions before ISU stopping before stopping for Policy-2 and Policy-3 under the condition that any ordering which is one permutation of $M!$ appears equally, which are Policy-2-T and Policy-3-T in Figure 4. The performances of Policy-2-T and Policy-3-T are better than Policy-2 and Policy-3 in our model, which can reflect the assumption of the orderings of WPs of M auctions being equal probability is not completely rational. However, this assumption is helpful for ISU selecting a proper auction under incomplete information. Policy-1 will evaluate WPs of all auctions each time, hence the averaged numbers of auctions before ISU stopping are the same with M.

5. **Conclusions.** In this paper, we propose three sequential selection policies based on OST. In the selection process, we have no need to assume any prior distribution information of the winning bids of each auction and WP of each auction. From the simulation results, although the assumption that each permutation of WPs is equally likely is not entirely satisfied in our model, the proposed policies are effective in finding a proper auction for ISU. Since no prior information is needed in the auction selection process, our proposed policies are suitable for practical CRNs. Furthermore, the multiple ISU model will be considered in our future work based on game theory, which is a multi-agent model.

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