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STATE ESTIMATION FOR NETWORKED CONTROL SYSTEMS UNDER INFORMATION LIMITATION

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Abstract. *This paper investigates state estimation problem for linear continuous time-invariant systems over a stationary memoryless uncertain digital channel without data dropout and time delay. In particular, the case with information limitation is examined. A sufficient condition on zero-error channel capacity for asymptotic observability is derived. It is shown that, there exists an encoder, decoder, and estimator such that the system is asymptotically observable if zero-error channel capacity is larger than a low bound given in our results. An illustrative example is given to demonstrate the effectiveness of the lower bound given.*

Keywords: State estimation, Information limitation, Asymptotic observability, Networked control systems

1. **Introduction.** The problem of state estimation for networked control systems has received an increasing interest in recent years [1,2]. This problem arises when the information of the plant states is transmitted over a band-limited communication channel. It becomes an active research area motivated by many engineering applications, such as industrial automation, sensor networks, vehicle systems, and aerospace industry.

A high-water mark in the study of quantized feedback using data rate limited feedback channels is known as the data rate theorem. The intuitively appealing result was proved in [3-5], indicating that it quantifies a fundamental relationship between unstable physical systems and the rate at which information must be processed in order to stably control them. When the feedback channel capacity is near the data rate limit, control designs typically exhibit chaotic instabilities. This result was generalized to different notions of stabilization and system models, and was also extended to multi-dimensional systems [6-8]. Control under communication constraints inevitably suffers signal transmission delay, data packet dropout and measurement quantization which might be potential sources of instability and poor performance of control systems [9-11].

In [12], a quantized-observer based encoding-decoding scheme was designed, which integrated the state observation with encoding-decoding. [13] addressed some of the challenging issues on moving horizon state estimation for networked control systems in the presence of multiple packet dropouts. It was shown in [14] that maxmin information was used to derive tight conditions for uniformly estimating the state of a linear time-invariant system over a stationary memoryless uncertain digital channel without channel feedback. [15] investigated the quantized feedback control problem for stochastic time-invariant linear control systems. A predictive control policy under data-rate constraints was proposed to stabilize the unstable plant in the mean square sense. [16] addressed LQ (linear quadratic) control of MIMO (multi-input multi-output), discrete-time linear systems, and gave the inherent tradeoffs between LQ cost and data rates.

Although there are many results on control under information limitation in the literature, the difference is that we restrict attention to the stationary memoryless uncertain digital channel, and give a lower bound on zero-error capacity for asymptotic observability. This paper is concerned with state estimation problem for networked control systems over a stationary memoryless uncertain digital channel without data dropout and time delay. The zero-error capacity of such a channel should be large enough to send the transmission of the state information without error. Clearly, zero-error capacity has important effect on asymptotic observability of the system. Our purpose here is to derive the condition on zero-error channel capacity for asymptotic observability. Our work here differs in that we present a lower bound on zero-error capacity for asymptotic observability of networked control systems.

The remainder of this paper is organized as follows: Section 2 introduces problem formulation; Section 3 deals with state estimation problem under information limitation; The results of numerical simulation are presented in Section 4; Conclusions are stated in Section 5.

2. **Problem Formulation.**

Consider the following linear continuous time-invariant system

$$\begin{aligned} \dot{X}(t) &= AX(t), \\ Y(t) &= CX(t) \end{aligned} \tag{1}$$

where $X(t) \in \mathbb{R}^n$ is the state process, and $Y(t) \in \mathbb{R}^m$ is the measured output, assumed to be Lebesgue-measurable. A and C are known constant matrices with appropriate dimensions. The initial state $X(0)$ is an uncertain variable with range $\Omega \subseteq B_l(0)$. Here, let $B_l(z)$ denote the l-ball$\{x : \|x - z\| \leq l\}$ centered at z, where $\|\cdot\|$ denotes either the maximum norm on a finite-dimensional real vector space or the matrix norm it induces.

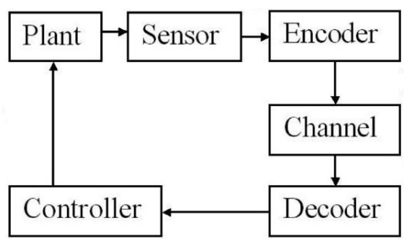

Figure 1. Networked control systems

Assume that there exists a real orthogonal matrix $H \in \mathbb{R}^{n\times n}$ that diagonalizes A. Namely, $A = H'\Lambda H$ with $\Lambda := \text{diag}[\lambda_1, \cdots, \lambda_n]$. Clearly, λ_i is the ith eigenvalue of system matrix A. Here, we choose to examine system (1) in this form because it makes our results most transparent.

Then, after a coordinate transformation, system (1) may be rewritten as

$$\begin{aligned} \dot{X}_c(t) &= \Lambda X_c(t), \\ Y(t) &= CH'X_c(t) \end{aligned}$$

where we define $X_c(t) := HX(t)$.

Furthermore, we examine the case where the sensors and the controller are geographically separated and connected by a stationary memoryless uncertain digital channel without data dropout and time delay. The case involves digital control with a uniform sampling interval h. Then, the corresponding discrete-time system is given by

$$\begin{aligned} X(k+1) &= GX(k), \\ Y(k) &= FX(k) \end{aligned} \tag{2}$$

where we have $X(k) := X_c(kh)$, $Y(k) := Y(kh)$, $G = e^{\Lambda h}$, and $F = CH'$.

The information of the measured output is transmitted over the communication channel. Then, the measured output $Y(k)$ is encoded via an operator Φ. The encoder at time k is defined as

$$\Phi : E(k) = \Phi[k, Y(0), Y(1), \cdots, Y(k)]. \tag{3}$$

Each symbol $E(k)$ is then sent to the decoder over the channel without feedback. Let $D(k)$ denote the received symbol. The decoder at time k is defined as

$$\Psi : D(k) = \Psi[k, E(0), E(1), \cdots, E(k)]. \tag{4}$$

Let $\hat{X}(k)$ denote the state estimate. The estimator at time k is defined as

$$\Theta : \hat{X}(k) = \Theta[k, D(0), D(1), \cdots, D(k)]. \tag{5}$$

Here, the prediction error at time k is defined as

$$Z(k) := X(k) - \hat{X}(k).$$

System (1) is surely asymptotically observable if there exists an encoder (3), decoder (4) and estimator (5) such that

$$\limsup_{k\to\infty} \|Z(k)\| \to 0.$$

We know that, zero-error capacity has important effect on observability of system (1). Namely, zero-error capacity must be larger than a lower bound such that there exists an encoder (3), decoder (4) and estimator (5) to achieve asymptotic observability of system (1). Thus, the main task here is to derive the condition on zero-error capacity C_0 for asymptotic observability of system (1).

3. **State Estimation under Information Limitation.** In this section, we examine the state estimation problem for linear time-invariant systems over a stationary memoryless uncertain digital channel without data dropout and time delay. Here, we address asymptotic observability of system (1) under information limitation. Clearly, zero-error capacity has important effect on observability. Our main task here is to present a lower bound on zero-error capacity for observability, and design an encoder, decoder, and estimator on the basis of such a lower bound.

The main result of this section is given below.

Theorem 3.1. *Consider system (1) with uncertain initial state $X(0)$ and plant outputs that are encoded by the encoder (3), decoded by the decoder (4), and estimated by the estimator (5) via a stationary memoryless uncertain digital channel without data dropout and time delay. Let $C_0 \geq 0$ denote zero-error capacity of such a channel. Then, there exists an encoder, decoder, and estimator such that system (1) is asymptotically observable if zero-error capacity C_0 satisfies the following condition:*

$$C_0 > \textstyle\sum_{i\in\Xi} \lambda_i \log e \quad (bits/s) \tag{6}$$

with $\Xi := \{i \in [1, n] : \lambda_i \geq 1\}$.

Proof: Notice that, system (1) is asymptotically observable if $\forall \varepsilon > 0$, $\forall X(0) \in B_l(0)$, $\exists T(\varepsilon, l) > 0$ such that

$$\|Z(k)\| \in B_\varepsilon(0)$$

holds as $k > T(\varepsilon, l)$. This means that

$$\limsup_{k\to\infty} \|Z(k)\| \to 0$$

holds. Then, zero-error channel capacity must be large enough to ensure that the codewords can be transmitted over such a channel without error.

In order to obtain the lower bound on zero-error channel capacity for asymptotic observability of system (1), we may compute the number of regions of diameter less than 2ε, which takes to cover the nonempty l-ball $B_l(0)$ of the plant states.

Then, we define

$$\alpha \in \left(0, 1 - \max_i \frac{1}{e^{\lambda_i h}}\right)$$

with $i \in \Xi$. The interval $[-l, l]$ on the ith axis is divided into d_i equal subintervals, for each $i \in \Xi$. Then, we have

$$d_i = \left\lceil \left[(1+\alpha)e^{\lambda_i h}\right]^k \right\rceil \tag{7}$$

with $i \in \Xi$. Here, we define

$$\lceil x \rceil := \min\{z \in \mathbb{Z} : z > x\}.$$

Let $I_*\left[X(0), D(0), D(1), \cdots, D(k)\right]$ denote the maxmin information between $D(0)$, $D(1), \cdots, D(k)$ and $X(0)$. The definition of the maxmin information is given by [14]. Then, it follows from [14] that,

$$2^{I_*[X(0),D(0),D(1),\cdots,D(k)]} \geq \prod_{i\in\Xi} d_i. \tag{8}$$

Substitute (7) into (8), and we get

$$2^{I_*[X(0),D(0),D(1),\cdots,D(k)]} \geq \prod_{i\in\Xi} \left\lceil \left[(1+\alpha)e^{\lambda_i h}\right]^k \right\rceil.$$

Notice that

$$\left\lceil \left[(1+\alpha)e^{\lambda_i h}\right]^k \right\rceil \geq \left[(1+\alpha)e^{\lambda_i h}\right]^k.$$

Then, we have

$$2^{I_*[X(0),D(0),D(1),\cdots,D(k)]} \geq \prod_{i\in\Xi} \left[(1+\alpha)e^{\lambda_i h}\right]^k.$$

Take logarithms, and we may obtain

$$I_*[X(0), D(0), D(1), \cdots, D(k)] \geq \log \prod_{i\in\Xi} \left[(1+\alpha)e^{\lambda_i h}\right]^k. \tag{9}$$

Furthermore, it follows from [14] that

$$k \cdot C_0 > I_*[X(0), D(0), D(1), \cdots, D(k)]. \tag{10}$$

It follows from (9) and (10) that

$$C_0 > \frac{1}{k} \log \prod_{i\in\Xi} \left[(1+\alpha)e^{\lambda_i h}\right]^k = \sum_{i\in\Xi} \log \left[(1+\alpha)e^{\lambda_i h}\right].$$

Notice that α may be arbitrarily small. Then, letting $\alpha \to 0$ yields

$$C_0 > \textstyle\sum_{i\in\Xi} \log e^{\lambda_i h} = \sum_{i\in\Xi} \lambda_i h \log e = \sum_{i\in\Xi} \lambda_i \log e \text{ (bits/s)}.$$

Thus, the proof is complete. □

4. **Numerical Example.** In this section, we give a numerical example to illustrate the effectiveness of the lower bound on zero-error capacity given in our results. We consider a class of networked control problems which arises in the coordinated motion control of unmanned air vehicles (UAVs), where the plant states evolve in discrete-time according to

$$X(k+1) = \begin{bmatrix} 3.246 & 2.512 & -1.462 \\ 0.265 & -2.512 & 4.561 \\ 0.231 & 6.321 & 8.431 \end{bmatrix} X(k).$$

Let $X(0) = [10\ 20\ -10]'$. Here, we employ the encoder (3), decoder (4), and estimator (5) subject to the condition on zero-error capacity presented in Theorem 3.1. The corresponding simulation is given in Figure 2. It is shown that the system is asymptotically observable. However, if zero-error capacity is too low, it leads to instability of the system. The corresponding simulation is given in Figure 3. It is shown in the simulation results that zero-error capacity has important effects on observability of networked control systems.

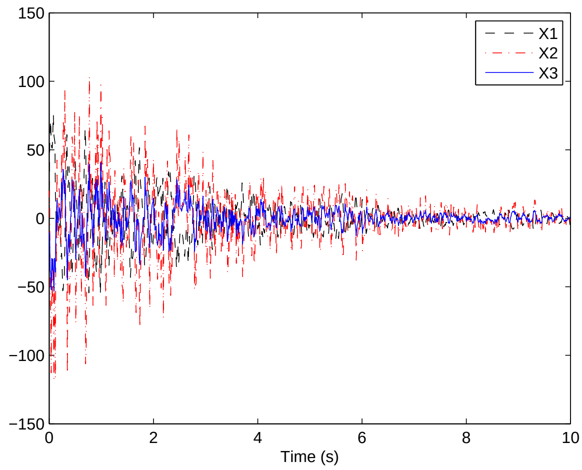

FIGURE 2. The plant state responses with zero-error capacity $C = 18$ bits/s

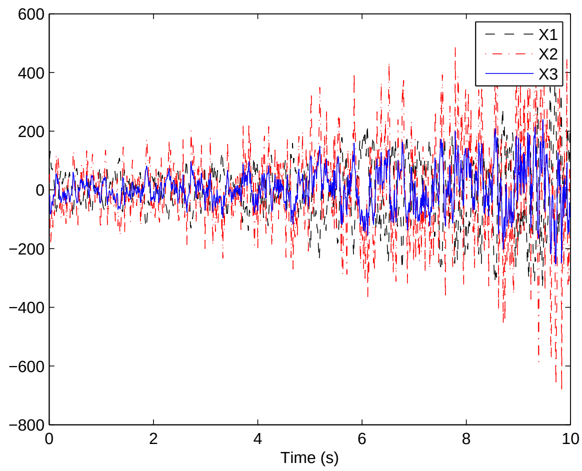

FIGURE 3. The plant state responses with zero-error capacity $C = 11$ bits/s

5. **Conclusions.** In this paper, we examined state estimation for networked control systems, and considered the case with information limitation. Our results state that, zero-error channel capacity has important effect on asymptotic observability of the system. A lower bound on zero-error channel capacity was derived to ensure that there exists an encoder, decoder, and estimator such that the system is asymptotically observable. The simulation results have illustrated the effectiveness of the lower bound given. The study of nonlinear system with limited information will be our future work.

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