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AN IMPROVED PARTHENO-GENETIC ALGORITHM FOR THE VEHICLE ROUTING PROBLEM WITH DYNAMIC DEMANDS AND TIME WINDOWS

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Abstract. *Considering the demand from customers may make changes to the original order contract upon arrival of the vehicle at the location of each customer, this paper focuses on the vehicle routing problem with dynamic demands and time windows (VRP_DDTW). The mixed-integer mathematical optimization model is established to minimize the total cost. Since the VRP is classified as an NP-hard problem, an improved Partheno-genetic algorithm incorporating heuristic algorithm (IPGA_HA) is presented to determine the optimal routing policy. Numerical example shows that the proposed algorithm achieves promising results by comparing with genetic algorithm.*
Keywords: Vehicle routing problem, Demand, Time windows, Genetic algorithm, Heuristic algorithm

1. **Introduction.** The classical VRP is first introduced by Dantzig and Ramser [1] in 1959, which aims to seek the optimal routing policy by minimizing the cost and deliver goods from a depot to geographically dispersed customers having deterministic demands [2]. All vehicles with limited capacity begin and end the route at the depot, and each customer is serviced only once, by only one vehicle. Numerous studies have been developed; see a recent overview [3] in which the available exact and heuristic algorithms for the VRP are provided. However, there are some ideal assumptions in the VRP which restrict the practical application in the real-life cases. Many efforts have been made for the variations of the VRP by relaxing assumptions or incorporating more constraints [4,5]. Among the most common are multi-depot VRP [6,7], capacitated VRP [6,8], stochastic demands [8], time windows [9,10], split deliveries [10] and pickup and delivery [7,9], which are closer to the practical situations.

In the studies for the VRP with stochastic demands (VRPSD), it is assumed that customers' demand follows a known or unknown probability distribution and the actual demand is known only when the vehicle arrives at the customer location [8]. It is the main difference from the capacitated VRP where the demand is known in advance. It is, however, difficult to estimate the specific distribution parameters for VRPSD since the field data is unavailable. Each location is allowed to be visited by vehicles within a specific time window referring to the VRP with (hard or soft) time windows (VRPTW), which will be considered in our following work. If the vehicle reaches the location before the lower limit of the time window, it has to wait; otherwise the service should be immediate. Nevertheless, a penalty cost is usually imposed for the vehicle once the arrival time is beyond the deadline of the time window.

This paper is motivated by a practical problem, where the initial demand and acceptance time for service (time window) are known according to the contract signed by the central depot and the customer. Based on the contract items for demands and time windows, the loaded and capacitated vehicles leave the central depot and begin to visit customers. However, customers may increase or decrease the order when the vehicle arrives to them; a new order even occurs. The time window restriction is shared by the original demand for all customers due to the contract requirements, but it is free of constraint for the demands caused by the increased and new order. The assumption that one customer is only visited once is unrealizable when the demand exceeds heavily the vehicle capacity, so the demand has to be split into several small-scale orders and delivered more than once.

It is difficult to find the exact solution for the VRP variations due to that it is a NP-hard problem and is computationally very demanding. Many algorithms have been wildly proposed to approximately optimize the VRP and have achieved excellent performance [11]; for example, Tabu Search (TS), Simulated Annealing (SA), Genetic Algorithm (GA), Evolution Strategies (ES), Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO). The hybrid algorithm is also presented to optimize the best route for vehicles, please refer to [11]. The most common meta-heuristics used to NP-hard is GA because of its global search strategy, especially Partheno-Genetic Algorithm (PGA) [12]. Since there is no crossover operator in PGA, it avoids the invalid chromosome after the crossover operator in GA. However, the determination of an initial solution for PGA has an essential role on searching for the optimization solution and the search effectiveness. In our following proposed algorithm, heuristic algorithm is developed to determine the initial population for GA and then apply genetic algorithm to them while performing an optimization search.

The main contributions of this paper different from the previous work are: (1) the demand of the original order for customers may be changed once the vehicle arrivals to them, referring to the dynamic demand; (2) a mathematic optimization model for VRP_DDTW is constructed to minimize the total cost; (3) an improved PGA incorporating heuristic algorithm is presented to find the optimal routes for vehicles. The remaining outline of this work is organized as follows: Section 2 introduces the modeling assumptions and the mathematic model; in Section 3, the algorithm IPGA_HA is developed for solving the proposed model of Section 2; this is followed by computational results and comparison in Section 4; Section 5 concludes the paper.

2. **Problem Description and Mathematical Model of VRP_DDTW.**

2.1. **Problem assumptions.** The VRP_DDTW is defined as an undirected graph $G = (V, E)$ with vertex set $V = \{0, 1, \dots, N\}$, where 0 denotes the depot and the other vertices denote customers, and E is the edge set. The VRP_DDTW aims to create efficient vehicle routes, which will satisfy the demand of all customers. Initially, a set of identical vehicles with limited capacity leave the central depot and begin to service customers with time windows in terms of the contract. However, the demand may be randomly changed upon arrival of the vehicle at the location of each customer. In the following, we first present some modeling assumptions and notations.

(1) There is only one central depot "0".
(2) A single type of products at the depot is delivered by vehicles and ordered by customers.

(3) The vehicles are identical with the maximum capacity Q ($Q > 0$) and fixed cost c.
(4) The duration of a vehicle from departing from depot to ending at depot after going through some customers is defined as a cycle.
(5) Split deliveries are allowed in which each customer may be visited by several vehicles, and this means that a route may be subject to multiple cycles in the subsequent modeling.
(6) The initial demand of each customer i, $i \in V - \{0\}$ is d_i ($d_i \geq 0$) with time windows $[E_i, L_i]$, where E_i is an earliest arrival time and L_i is a latest arrival time.
(7) Upon the first arrival of the vehicle at the location of each customer i, the random demand d'_i occurs in which $d'_i > 0$ (< 0) corresponds to the increased (decreased) order, respectively. Additionally, the demand of customer i keeps unchanged if $d'_i = 0$.
(8) There is no limit of time windows for the demand caused by the changed order.
(9) A vehicle has to wait until the lower limit of the time window E_i if the arrival time at the customer location is earlier than E_i, and the penalty cost per unit time resulting from waiting for service is c_p. However, an immediate service is provided for the customer if the arrival time of a vehicle at the customer location falls into the time windows $[E_i, L_i]$ or is larger than the upper limit L_i. Exceptionally, vehicles arriving later than the latest arrival time L_i are also penalized with the cost per unit time c_q ($c_q > c_p$).
(10) There are enough products in the depot providing customers, i.e., no shortage.

The following notations will be used in the proposed model and algorithm.

K ($k = 1, 2, \ldots, K$), the total number of vehicles; L_k, the cycle number of a route for vehicle k; f_i, service time at customer i; t_{ij} ($i, j \in E$), travel time between i and j with the travel cost per unit time c_t; ta^k_{il}, arrival time of the lth cycle for vehicle k at customer i; td^k_{il}, departure time of the lth cycle for vehicle k from customer i; x^k_{ijl}, referring to a binary variable which takes the value 1 if and only if (i, j) belong to one of the routes of the lth cycle for vehicle k; and the associated discharging amount is b^k_{jl} at customer j; Q^k_{il} is the remaining amount after the discharging of the vehicle k in the lth cycle at customer i.

2.2. **Mathematical formulation.** The VRP_DDTW can be described as follows:

$$\begin{aligned}\text{Minimize: } C_{total} = {} & cK + \sum_{k=1}^{K}\sum_{l=1}^{L_k}\sum_{i=0}^{N}\sum_{j=0}^{N} x^k_{ijl} \cdot t_{ij} \cdot c_t \\ & + \sum_{k=1}^{K}\sum_{l=1}^{L_k}\sum_{j=1}^{N} \left[c_p\left(E_j - ta^k_{jl}\right) + c_q\left(ta^k_{jl} - L_j\right)\right]\end{aligned} \tag{1}$$

Subject to

$$\sum_{j=1}^{N} x^k_{0jl} = 1, \quad \sum_{j=1}^{N} x^k_{j0l} = 1 \tag{2}$$

$$\sum_{i=0(i \neq j)}^{N} x^k_{ijl} = 1 \tag{3}$$

$$\sum_{i=0}^{N} x^k_{ihl} - \sum_{j=0}^{N} x^k_{hjl} = 0, \quad h = \{1, 2, \ldots, N\} \tag{4}$$

$$\sum_{i=0}^{N}\sum_{j=1}^{N} x^k_{ijl} \cdot b^k_{jl} \leq Q \tag{5}$$

$$\sum_{k=1}^{K}\sum_{l=1}^{L_k}\sum_{i=0}^{N} x_{ijl}^{k}\cdot b_{jl}^{k} = d_j + d_j' \tag{6}$$

$$ta_{jl}^{k} + f_j \leq \sum_{j\in V(i\neq j)}^{N} x_{ijl}^{k}\cdot td_{ijl}^{k} \tag{7}$$

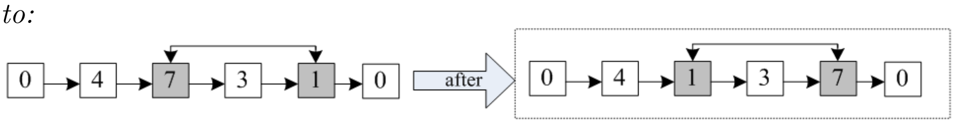
Function (1) is designed to minimize the total cost by summing up the fixed cost of all vehicles, the travelling cost due to delivery, and the penalty cost. Constraint (2) assures vehicles depart and return to the depot "0" in a cycle; Constraint (3) guarantees that each customer is visited only once by one vehicle in a cycle; Constraint (4) ensures the sequence of a route $i \to h \to j$; Constraint (5) restricts the load on a vehicle; Constraint (6) states that the demand of each customer is satisfied; Constraint (7) defines the timing constraint, i.e., for a cycle of each vehicle, the departure time from customer j cannot be smaller than the time sum of the arrival time at j and the service time.

3. **Improved Partheno-Genetic Algorithm Incorporating Heuristic Algorithm.** In this section, we design a hybrid algorithm IPGA_HA in which an initial population for VRP_DDTW is created by heuristic algorithm, based on which the Partheno-genetic algorithm is used to search the optimal solution. The procedure of the proposed algorithm IPGA_HA is shown in Algorithm 1.

Algorithm 1:
Input: Population size M, probability of recombination operator P_r, stopping criteria
Output: Solution to VRP_DDTW, i.e., the optimal routing policy

Step 1: Encoding of chromosomes
Using natural number coding rule.
Step 2: Generation of the initial population with HA (See the following **Algorithm 2**)
Step 3: Construct the fitness function
The fitness function can be expressed as $f = 1/(1 + C_{total} + G)$, *where* G *is a very large integer.*
Step 4: **While** (stopping criteria not met) do

- Generate a new population with size M' by recombination operator with probability P_r;
  Single point conversion operator is performed on gene over a cycle; please refer to:



- Evaluate the fitness of each individual;
- Select individuals through some selection scheme.

The tournament selection strategy is used for the selection of individuals for development of next generation.

End;

In the above Algorithm 1, an initial population is generated using HA. The pseudo code is presented as Algorithm 2.

4. **Numerical Example.** The parameters in model (1) are assumed as $Q = 50$, $c = 1000$, $c_t = 20$, $c_p = 25$ and $c_q = 40$. The customer information is shown in Table 1, where the travel time t_{ij} between i and j is also given. The proposed algorithm is performed using Matlab, where the algorithm parameters are set in Table 2. Depending on these parameters, we obtain the optimal route policy using the proposed algorithm IPGA_HA, as shown in Table 3. The changed demand for customers is determined during the search, $d_1' = 25$, $d_2' = -20$, $d_3' = 45$, $d_4' = 0$, $d_5' = -5$, $d_6' = 0$, $d_7' = 80$, $d_8' = -8$.

Algorithm 2:

Step 1: Initialize parameters c, c_t, c_p, c_q, d_i, $[E_i, L_i]$, $K = 4$, $l = 1$, $s_i = 0$, $d_i^{'} = 0$;

Step 2: Each vehicle k departs from the central depot, and record td_{il}^k;

Step 3: Vehicle k $(k = 1, \ldots, K)$ randomly visits a customer j with $d_j > 0$ or $d_j^{'} > 0$, and record $ta_{jl}^k = tl_{il}^k + t_{ij}$;

If $s_j = 0$ *% Customer j is not yet visited by vehicles such that the demand may be changed.*

Generate a random number r in $(0, 0.9]$ and there are three scenarios that should be considered as follows.

(1) If $r \in (0, 0.3]$, the demand increases with $d_j^{'} > 0$ and update $s_j = 1$; (2) if $r \in (0.3, 0.6]$, the demand decreases with $d_j^{'} < 0$, $abs(d_j^{'}) \le d_j$ and update $s_j = 1$; (3) otherwise, keep the demand unchanged.

Go to Step 4;

Else *go to Step 4*

End;

Step 4: Calculate the discharging amount b_{jl}^k.

Scenario 1: $d_j' > 0$ ***If*** $Q_{il}^k > d_j$ *and* $d_j' > 0$ *then* $b_{jl}^k = d_j + \min\left\{d_j', Q_{il}^k - d_j\right\}$ *Update* $d_j = 0$, $d_j' = \max\left\{0, d_j' - \left(Q_{il}^k - d_j\right)\right\}$, $Q_{jl}^k = Q_{il}^k - b_{jl}^k$, $x_{ijl}^k = 1$ ***End;*** ***If*** $Q_{il}^k \le d_j$ *and* $d_j' > 0$ *then* $b_{jl}^k = Q_{il}^k$ *Update* $d_j = d_j - Q_{il}^k$, $Q_{jl}^k = 0$, $x_{ijl}^k = 1$ ***End;*** ***Scenario 2:*** $d_j^{'} <= 0$ ***If*** $Q_{il}^k > d_j$ *then* $b_{jl}^k = d_j$, *update* $d_j = 0$, $Q_{jl}^k = Q_{il}^k - b_{jl}^k$, $x_{ijl}^k = 1$ ***End;*** ***If*** $Q_{il}^k \le d_j$ *then* $b_{jl}^k = Q_{il}^k$, *update* $d_j = d_j - Q_{il}^k$, $Q_{jl}^k = 0$, $x_{ijl}^k = 1$ ***End;***	***Considering time windows:*** *1)* $ta_{jl}^k < E_j$ *Uploading until* E_j $td_{jl}^k = E_i + f_j$ *2)* $ta_{jl}^k \in [E_j, L_j]$ *or* $> L_j$ *Uploading immediately* $td_{jl}^k = ta_{jl}^k + f_j$

Step 5: Continue to deliver? Return to the central depot?

If $\sum_{i=1}^N (d_i + d_i') = 0$ *% The demand of all customers have been delivered.*

Vehicle k returns to the depot and ends delivery; record $L_k = l$ and $x_{j0L_k}^k = 1$;

Else *% Goods for several customers are still being in demand.*

If $Q_{jl}^k = 0$ % Empty vehicle; vehicle k returns to the depot, and ends the lth cycle; loading.

$x_{j0l}^k = 1$, *update* $l = l + 1$, $Q_{0l}^k = \min\left\{Q, \sum_{i=1}^N (d_i + d_i')\right\}$; *Go to Step 2;*

Else Go to Step 2;

End;

End;

Step 6: Output the route of all vehicles according to L_k, x_{ijl}^k, i.e., an initial solution.

Step 7: End this procedure until M individuals are generated.

Taking the route of vehicle 2 as example, the "0 $\rightarrow$ 2 $\rightarrow$ 1 $\rightarrow$ 0'$\rightarrow$ 8 $\rightarrow$ 6 $\rightarrow$ 7 $\rightarrow$ 0" implies that there are two cycles, 0 $\rightarrow$ 2 $\rightarrow$ 1 $\rightarrow$ 0' and 0'$\rightarrow$ 8 $\rightarrow$ 6 $\rightarrow$ 7 $\rightarrow$ 0. Also, the discharging amount at customers 2, 1, 8, 6, 7 are 40, 10, 7, 15, 28, where the value in bold denotes the changed random demand upon arrival of a vehicle at the customer location. The comparison of the proposed algorithm with the classical GA for VRP_DDTW in Section 2 is done and Figure 1 gives the convergence effect of the total cost. From Figure 1, it can be seen that the total cost decreases with the increase of the number of iterations, and the minimal total cost caused by the vehicle routing problem with dynamic demands

Table 1. Customer information

Customer i	d_i	$[E_i, L_i]$	f_i	t_{ij} **0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0	–	–	–	0	29	17	15	27	23	11	16	19
1	35	$[35, 80]$	10	–	0	14	10	9	19	8	10	21
2	60	$[24, 40]$	8	–	–	0	11	15	7	12	8	7
3	0	–	13	–	–	–	0	10	9	16	9	15
4	45	$[20, 85]$	7	–	–	–	–	0	12	4	13	9
5	30	$[18, 60]$	6	–	–	–	–	–	0	13	5	10
6	70	$[15, 90]$	4	–	–	–	–	–	–	0	6	8
7	0	–	9	–	–	–	–	–	–	–	0	17
8	40	$[20, 80]$	11	–	–	–	–	–	–	–	–	0

Table 2. Algorithm parameters

Population size M	Probability of recombination operator P_r	Maximum number of iterations	G
10	0.4	300	20000

Table 3. Optimal route policy

Vehicle	Route	Discharging amount
1	0 → 1 → 0'→ 6 → 0	0 → 35/**15** → 0'→ 50/− → 0
2	0 → 2 → 1 → 0'→ 8 → 6 → 7 → 0	0 → 40/− → −/**10** → 0'→ 7/− → 15/− → −/**28** → 0
3	0 → 4 → 6 → 0'→ 3 → 7 → 0	0 → 45/− → 5/− → 0'→ 45/− → −/**2** → 0
4	0 → 5 → 8 → 0'→ 7 → 0	0 → 25/− → 25/− → 0'→ −/**50** → 0

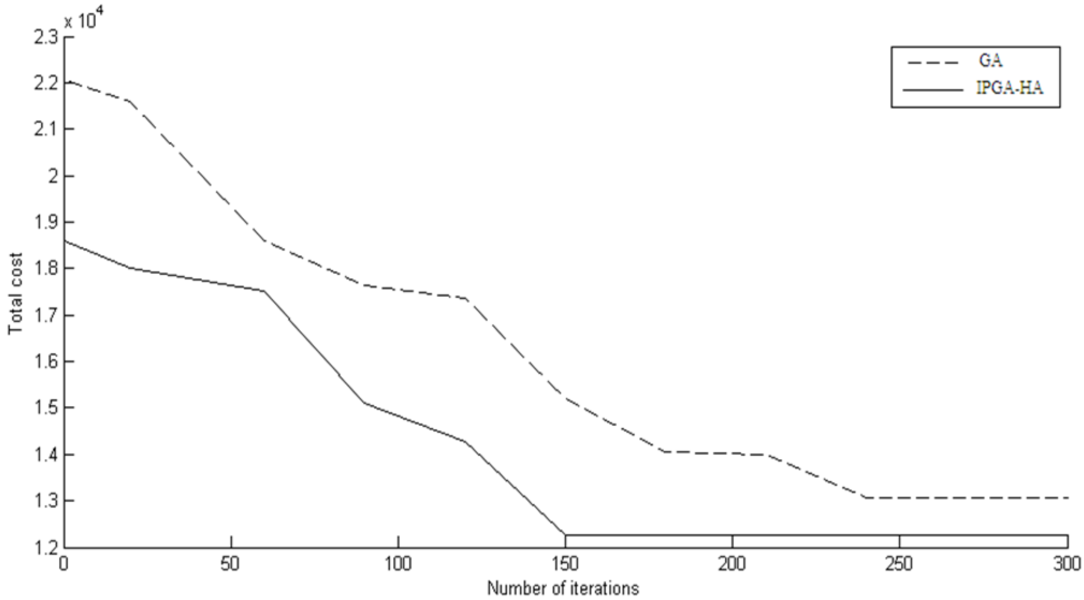


Figure 1. Convergence effect of the total cost

and time windows (VRP_DDTW) is obviously smaller than that obtained by GA, which is what we expected. This is because the initial solution in IPGA_HA is determined through HA, which fastens the search for the optimal solution. Moreover, the individuals after recombinant in IPGA_HA are still a feasible solution, which avoids that the IPGA_HA falls into the local optimum.

5. **Conclusions.** A mathematical model for the vehicle routing problem with dynamic demands and time windows (VRP_DDTW) is established to find the optimal route by minimizing the total cost. Since the demand may be changed once the vehicle arrives at a customer, dynamic demand is modeled. The limit of time windows is more common in practice such that it is considered, but it has no effect on the changed demand. IPGA_HA which employs HA to find the initial solution is then proposed to search the optimal route and delivery goods. The simulation results show the proposed algorithm can obtain better solution than GA with much less number of iterations. Since products may be deteriorated in the transport process, VRP for perishable products will be studied in the future.

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