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RESEARCH ON FUZZY PROGRAMMING METHOD BASED ON UTILITY FREQUENCY

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Abstract. *Fuzzy programming is a widespread problem in many fields such as resource allocation, and optimization decision. It is also the widespread attention in today's academic circles and application fields. In this paper, firstly, we analyze the essential features of fuzzy programming and the features and shortcomings of existing methods. Secondly, combining the influence on decision-making in different membership statuses, we propose a fuzzy programming method based on utility frequency. Then we give the expression of fuzzy satisfaction solution. Finally, we further analyze its natures and the validity of this method in combination with a concrete case. Theoretical analysis and calculation results show that this method not only has good structure characteristic, but also has strong interpretability and operability. Therefore, it can enrich the existing fuzzy programming theory to some extent.*

Keywords: Fuzzy programming, Fuzzy decision, Utility frequency, Fuzzy satisfaction solution

1. **Introduction.** As we all know, fuzziness is a widespread phenomenon. With the development of computer science and information, fuzziness cannot but be faced in many practical domains. In 1965, Zadeh [1] proposed the concept of fuzzy sets and established fuzzy set theory, which formed the foundation for describing and processing uncertain information. Thereafter, combined with the background of different theories and applications, many scholars have developed the fuzzy set theory, and formed many useful theory and application results in [2]. And the theory is also used to solve practical problems. For example, [3] illustrated a fuzzy goal-programming model using exponential membership function for health-care organization, and in combination with the hot topic of energy resources allocation, [4] discussed a hybrid fuzzy satisfying optimization model. With the deepening of the fuzzy set theory, fuzzy programming developed rapidly. For example, for multi-objective decision, Bellman and Zadeh [5] proposed the basic model of fuzzy decision, and [6] discussed the Newton method to find a non-dominated solution. At present, in fuzzy programming problems, the relatively mature research mainly concentrates on the determination of fuzzy optimal solution in linear ones. The basic feature of the research is to turn the problem into crisp linear or nonlinear one by a certain strategy. For example, for nonlinear fuzzy fractional integral and integrodifferential equations, [7] employed the method of upper and lower solutions to solve them. Using the deviation degree measures and weighted max-min method, [8] proposed a method for solving fuzzy multi-objective linear programming problems where all the coefficients are triangular fuzzy numbers and all the constraints are fuzzy equality or inequality. According to the structure properties of fuzzy numbers, [9-11] transformed a fuzzy linear programming problem to a classical one. In [12], it computed the extended weighted L-R approximation of a given fuzzy number by a method based on general results in Hilbert spaces. For linear programming

problems with coefficient, constraints and objectives being all fuzzy, [13] established an interactive fuzzy satisfaction method. [14] gave a method based on the numeric sequence structure of fuzzy number.

Although the existing methods of fuzzy programming and decision-making have been successfully applied in many fields, there are still the following limitations. 1) Because we often allow a certain error in practical decision, the existing abstract model cannot fully fit the practical problems. And it lacks integrity description system of the decision in fuzzy environment. 2) Due to the incomplete decision information and decision-makers' preference varying in practical problems, there is often no optimal single precise solution, but a set of satisfactory ones.

According to above analysis, aiming at the shortcomings of fuzzy programming, we mainly do the following work: 1) We analyze the features and shortcomings of the existing decision (programming) methods; 2) With the comprehensive optimal decisions of different threshold values, we propose the method of fuzzy satisfaction solution based on utility frequency, and then discuss its properties; 3) Finally, we analyze the effectiveness of the proposed method combining with a concrete case.

The rest of this paper is organized as follows. Section 2 introduces some basic definitions and the formal representation of fuzzy programming, and then discusses two methods which are commonly used in fuzzy programming. In Section 3, we propose a new algorithm to solve the fuzzy satisfaction solutions based on effect frequency. Here, we prove the theorems having been used, and give the principles and notations that we should pay attention to. In Section 4, a simple example is used to illustrate the proposed algorithm. After that, the former method and the proposed algorithm have been compared to each other. Finally, conclusion is derived in Section 5.

2. **Formal Representation of Fuzzy Programming.** Under some constraint conditions to seek optimal decision scheme is the essence of programming problem, and its general form [15] is as follows:

$$\begin{cases} \max f(x), \\ \text{s. t. } x \in A. \end{cases} \tag{1}$$

Here, A denotes a collection of universe U (called as feasible region), and $f(x)$ denotes the function of a certain number of features on U (called as objective function), which is to measure the decision scheme x good or bad.

According to the different characteristics of feasible region and the objective function, we can divide (1) into determine programming problem (i.e., when $f(x)$ is a real function, A is a crisp subset on U) and multi-parametric problem (i.e., U or $f(x)$ has some uncertainty). Particularly, when $f(x)$ has ambiguity on U or A is a fuzzy set on U, we call (1) fuzzy programming problem.

We know that fuzziness is caused by inconsistent understanding; therefore, the model of fuzzy programming problem such as (1) is just a formal model, being lack of recognition algorithm. Below we use a programming problem with crisp objectives (that is, the objective function is a real function) and fuzzy constraints (here, the feasible domain is a fuzzy set) to analyze the characteristics and limitations of the existing methods. Finally, we give a formalized description system of fuzzy optimal value. For convenience, $A(x)$ and $A_\lambda = \{x|A(x) \geq \lambda\}$ respectively denote the membership function of fuzzy set A (that is, the degree of x belongs to the fuzzy set A) and λ-level set (a partial description way or a relative clarity description of fuzzy set) of fuzzy set A.

Method 1: The method based on comprehensive effect. This method regards some comprehensive utility value of $f(x)$ and $A(x)$ as the standard of describing the performance of x, and makes model (1) be converted to a common programming problem.

Its general description [16] is as follows:

$$\begin{cases} \max S(f(x), A(x)), \\ \text{s. t. } x \in U. \end{cases} \tag{2}$$

Here, $S(u, v)$ is called synthesizing effect function if it satisfies the following conditions: 1) It is monotone non-decreasing on u and v; 2) $S(u, v)$ is monotone increasing on u.

Model (2) considers the satisfaction of targets and constraints at the same time. Compared with the following model (3), it can better reflect the basic feature of fuzzy decision. However, because the constraints of practical problems vary, the importance of each constraint is different and they are often mutual coupling in real problems. The restrictive relationship of targets and constraints is complex, and simple function relation fails to describe this phenomenon. Therefore, this method is lack of enough generality and operability.

Method 2: The method based on the requirement of a certain satisfaction. This method uses some level cut set to approximatively describe the fuzzy set A, and then the model (1) can be turned into a programming problem with crisp constraints. Its general form [17] is as follows:

$$\begin{cases} \max f(x), \\ \text{s. t. } x \in A_\lambda. \end{cases} \tag{3}$$

Obviously, (3) has good interpretability, and as a relatively crisp description on the threshold level λ of (1), its decision results vary with the variety of $\lambda \in [0, 1]$. The fuzzy set A can be understood as a family of crisp sets $\{A_\lambda | \lambda \in [0, 1]\}$; therefore, if we regard the fuzzy programming (1) as a crisp one with the objective function $f(x)$ and the feasible region A_λ, then

$$\left\{(\lambda, x^*_\lambda) \,\middle|\, \lambda \in (0, h(A)],\ x^*_\lambda,\ \text{and}\ f(x^*_\lambda) = \max f(x) | x \in A_\lambda \right\} \tag{4}$$

can be thought as a basic description of the optimal solution of (1). Here, $h(A) = \max\{A(x) | x \in U\}$.

It is straightforward to show that: 1) when U is a limited field, it is easy to get the concrete form of (4); 2) when U is an infinite universe, we can combine the appropriate numerical or intelligent computation method, and get the mode (5) to curve (4) approximately through discretization method:

$$\left\{ \left(\lambda_i,\ x^*_{\lambda_i}\right) \,\middle|\, \begin{array}{l} \lambda_i \in [0, h(A)],\ x^*_{\lambda_i} \in A_{\lambda_i},\ f(x^*_{\lambda_i}) = \max\{f(x) | x \in A_{\lambda_i}\}, \\ i = 0, 1, 2, \cdots, n,\ \text{and}\ 0 = \lambda_0 < \lambda_1 < \cdots < \lambda_n = h(A) \end{array} \right\}. \tag{5}$$

Synthesizing the above discussion we can see that the results of the same fuzzy programming may be different under different decision consciousness. The cause for this difference is that the emphasis of objectives and constraints is often inconsistent to different decision makers. Therefore, the optimal solution of fuzzy programming should be understood as a fuzzy set on the decision domain. And this fuzzy set has no generally accepted concrete form. Formula (4) can be regarded as a formal description which wholly reflects the optimal solution of fuzzy programming. Because there is often some difference between the theory and the actual optimal value (i.e., the optimal value allows some volatility), how to construct a determination method of fuzzy satisfaction solution on the basis of (4) not only can make up for the inadequacy of the existing decision method, but also has importantly practical value. Below, combining the basic feature of fuzzy decision and in view of (4), we will discuss the determination mechanism of fuzzy satisfaction solution based on the utility.

3. **Fuzzy Satisfaction Solution Based on Utility Frequency.** Because the analytic form of programming problem (1) cannot completely characterize the actual decision problem, it is just a theoretic model. We may have no optimal solution through that model, and also we often allow a certain deviation. At this point, if we want to solve fuzzy problems, we must consider the existing deviation. Thus, in this paper, we call

$$M_\lambda(\varepsilon) = \left\{x \middle| x \in A_\lambda \text{ and } f(\lambda, x_\lambda^*) - f(x) \le \varepsilon\right\} \tag{6}$$

as ε-satisfied solution set on the level λ of (1) to describe the deviation. Here, $f(\lambda, x_\lambda^*) = \max\{f(x)|x \in A_\lambda\}$ denotes the optimal value of (3), and $\varepsilon \ge 0$ denotes the optimal satisfaction threshold.

Obviously, the satisfaction threshold ε is a satisfactory decision criterion based on $f(\lambda, x_\lambda^*)$. To a certain degree, it reflects the vagueness of the decision scheme of (1). This deviation in actual process of decision is used frequently. However, we should limit the range of ε; otherwise it will lead to the persuasion of the satisfied solution set reducing. The bigger one will make it lower to distinguish the satisfaction (Especially, when ε gets bigger to some degree, it will cause all alternatives being satisfied and make the decision lose real meaning). The smaller one may lose some satisfaction solution which meets the actual requirements.

No matter what is the form of λ and ε, $M_\lambda(\varepsilon)$ cannot be recognized as the optimal solution set of (1). That is because the optimal solution of fuzzy programming is different as fuzzy processing consciousness varies. Therefore, the solution of the model (1) should be a fuzzy set on U and is expressed as B. Then $M_\lambda(\varepsilon)$ can be considered as the basic factor reflecting the features of B. Due to the fact that the different levels λ characterizes a compatible (or recognized) degree of decision-making on A_λ, and the greater (smaller) λ is, the higher (lower) the recognized degree of decision is. So the utility (function) of decision on different levels λ is different to the overall decision. If we understand the level utility as a mapping $L(\lambda)$ which is from $[0, 1]$ to $[0, \infty)$ (called ***level effect function***), $L(\lambda)$ should satisfy the following basic principles.

Principle 1: The effect monotonicity of the threshold, namely, $L(\lambda)$ is non-decreasing on $[0, 1]$.

Principle 2: The effect continuity of the threshold, namely, $L(\lambda)$ is continuous on $[0, 1]$.

Principle 3: The normalization of total effect, namely, $\int_0^1 L(\lambda)d\lambda = 1$.

Here, Principle 1 and Principle 2 must be satisfied, but Principle 3 is to maintain the consistency with regular information processing mode. According to the above discussion as well as the basic idea of fuzzy statistics, for $x \in U$, if

$$\eta(x, \lambda, \varepsilon) = \begin{cases} 1, & x \in M_\lambda(\varepsilon) \\ 0, & x \notin M_\lambda(\varepsilon) \end{cases} \tag{7}$$

denotes the coverage frequency of $M_\lambda(\varepsilon)$ to x, then

$$B(x, L(\lambda), \varepsilon) = \int_0^1 L(\lambda)\eta(x, \lambda, \varepsilon)d\lambda \tag{8}$$

is a kind of cover frequency of x based on the level effect function (called as utility coverage). Here, $L(\lambda)$ is a parameter to reflect the effect of the level λ in comprehensive decision process, and it is a kind of quantification strategy to describe decision consciousness.

By fuzzy set theory, we can determine a fuzzy set B in the universe U for a given level of effect function $L(\lambda)$, according to Formula (8). The membership function systemically reflects the satisfaction solutions of fuzzy programming (1). For convenience, we call B the fuzzy satisfaction solution of (1) based on level utility function $L(\lambda)$ (called as ***fuzzy satisfaction solution based on utility*** for short).

Theorem 3.1. *For fuzzy programming problem (1) and $x_0 \in U$, if there exists $[\alpha_1, \alpha_2] \subset [0, 1]$, such that $x_0 \in M_{\alpha_1}(\varepsilon)$ and $x_0 \in M_{\alpha_2}(\varepsilon)$, then $x_0 \in M_\lambda(\varepsilon)$ holds for any $\lambda \in [\alpha_1, \alpha_2]$.*

Proof: Let $f(\lambda, x^*_\lambda) = \max\{f(x)|x \in A_\lambda\}$, then using $A_{\alpha_1} \supset A_\lambda \supset A_{\alpha_2}$ for any given $\lambda \in [\alpha_1, \alpha_2]$, we can know that $f\left(\alpha_1, x^*_{\alpha_1}\right) \geq f(\lambda, x^*_\lambda) - f(x_0) \geq f\left(\alpha_2, x^*_{\alpha_2}\right)$. By this and $x_0 \in M_{\alpha_1}(\varepsilon)$, $x_0 \in M_{\alpha_2}(\varepsilon)$, we have $x_0 \in A_\lambda$ and $\varepsilon \geq f\left(\alpha_1, x^*_{\alpha_1}\right) - f(x_0) \geq f\left(\lambda, x^*_\lambda\right) - f(x_0) \geq f\left(\alpha_2, x^*_{\alpha_2}\right) - f(x_0)$ for any given $\lambda \in [\alpha_1, \alpha_2]$, that is, $x_0 \in M_\alpha(\varepsilon)$ holds for any $\lambda \in [\alpha_1, \alpha_2]$.

Theorem 3.1 and the properties of level set show that for any given $x \in U$ and $\varepsilon \geq 0$, there exists $[\underline{\alpha}(x, \varepsilon), \overline{\alpha}(x, \varepsilon)] \subset [0, 1]$ and it satisfies: 1) $\eta(x, \lambda, \varepsilon) = 1$ when $\lambda \in (\underline{\alpha}(x, \varepsilon), \overline{\alpha}(x, \varepsilon))$; 2) $\eta(x, \lambda, \varepsilon) = 0$ when $\lambda \in 1 - [\underline{\alpha}(x, \varepsilon), \overline{\alpha}(x, \varepsilon)]$. Model (8) is significant based on this and the definition of utility function, and we have $B(x, L(\lambda), \varepsilon) = \int_{\underline{\alpha}(x,\varepsilon)}^{\overline{\alpha}(x,\varepsilon)} L(\lambda)d\lambda$. $\underline{\alpha}(x, \varepsilon)$ and $\overline{\alpha}(x, \varepsilon)$ rely on a variety of $\lambda \in [0, 1]$ corresponding to the optimal solution of programming problem (3). There exists no operational formalization method to it. So the calculation model is just a theoretical expression. Below, we will discuss the determination method of $\underline{\alpha}(x, \varepsilon)$ and $\overline{\alpha}(x, \varepsilon)$ for the discrete universe U.

Theorem 3.2. *For fuzzy programming problem (1), if $U = \{x_1, x_2, \cdots, x_n\}$, $\lambda_1, \lambda_2, \cdots, \lambda_m$ are the different values of $A(x_1), A(x_2), \cdots, A(x_n)$ and these values satisfy $0 = \lambda_0 \leq \lambda_1 < \lambda_2 < \cdots < \lambda_m$, then for any $x \in M(\varepsilon) = \cup_{k=0}^m M_{\lambda_k}(\varepsilon)$, we have $B(x, L(\lambda), \varepsilon) = \int_{\underline{\alpha}(x,\varepsilon)}^{\overline{\alpha}(x,\varepsilon)} L(\lambda)d\lambda$. Here,*

$$\underline{\alpha}(x, \varepsilon) = \begin{cases} \lambda_0, & x \in M_{\lambda_0}(\varepsilon), \\ \min\{\lambda_{k-1}|x \in M_{\lambda_k}(\varepsilon)\}, & x \notin M_{\lambda_0}(\varepsilon). \end{cases} \tag{9}$$

$$\overline{\alpha}(x, \varepsilon) = \max\{\lambda_k|x \in M_{\lambda_k}(\varepsilon)\}. \tag{10}$$

Theorem 3.2 provides a concrete method for fuzzy satisfaction solution of fuzzy programming, and the steps are as follows:

Step 1 To determine $M_{\lambda_k}(\varepsilon)$, $k = 0, 1, 2, \cdots, m$, $M(\varepsilon) = \cup_{k=0}^m M_{\lambda_k}(\varepsilon)$;

Step 2 To determine $\underline{\alpha}(x, \varepsilon)$ and $\overline{\alpha}(x, \varepsilon)$ according to (9) and (10) for any $x \in M(\varepsilon)$;

Step 3 To calculate $B(x, L(\lambda), \varepsilon) = \int_{\underline{\alpha}(x,\varepsilon)}^{\overline{\alpha}(x,\varepsilon)} L(\lambda)d\lambda$.

Remark 3.1. *The infinite domain always can approximately turn into the discrete one combined with some kind of strategy (for example, when $U = [a, b]$, we can combine some accuracy to make U discrete into $a = x_0 < x_1 < x_2 < \cdots < x_n = b$). So the calculation steps can be used as a numerical calculation basis of $B(x, L(\lambda), \varepsilon)$.*

Remark 3.2. *When A is a family of crisp set on U, $B(x, L(\lambda), 0)$ has nothing to do with the level effect function $L(\lambda)$, and it satisfies $B(x, L(\lambda), 0) = \begin{cases} 1, & f(x) = \max\{f(u)|u \in A\} \\ 0, & f(x) \neq \max\{f(u)|u \in A\}. \end{cases}$ That is, $B(x, L(\lambda), 0)$ is the optimal solution set of the programming problem. It suggests the discussion in this paper is an expending of crisp programming.*

All of above analysis and discussion show that (8) is an abstract fuzzy programming model. It can sum up the basic features of fuzzy decision. It not only reflects the function of the fuzzy membership state for decision (i.e., the level effect $L(\lambda)$), but contains qualitative phenomenon in actual decision (i.e., satisfaction threshold ε). We can find it has good interpretation and quantification of structure system. Therefore, (8) has a certain guiding significance to propose the method in complex environment.

4. The Case Analysis.

In this section we further expound the application of the fuzzy satisfaction solution in decision problem combined with a concrete case.

Case description: In order to improve the production system, a group company plans to invest 10 million yuan to build a processing factory of raw materials. Due to different production environment having a huge impact on the production effect, the company makes an extensive research and argumentation for the location choice, production targets, equipment configuration, operating mode and so on. In the research, it has 10 alternatives $\{x_1, x_2, \cdots, x_{10}\} \triangleq U$. And the expected return rate $r(x_i)$ and the support rate $\mu(x_i)$ of each alternative x_i are given in Table 1. Try to determine the building scheme which conforms to the actual requirements.

TABLE 1. The expected return rate and the support rate of each alternative project

Alternative project x_i	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
expected return rate $r(x_i)$	0.14	0.27	0.19	0.28	0.16	0.24	0.26	0.17	0.30	0.25
support rate $\mu(x_i)$	0.64	0.82	0.2	0.24	0.72	0.37	1	0.76	0.5	0.58

Because there are various alternatives in the project options and every support is different, all the alternatives have possibility to be selected. If we consider the expected return rate $r(x_i)$ as a measure of production effect, the support rate $\mu(x_i)$ as a degree of x_i satisfying the decision requirement according to fuzzy statistics method, and remember A as a fuzzy set on U, whose membership function is $\mu(x_i)$, then the choice of building scheme can be expressed as the following fuzzy programming problem:

$$\begin{cases} \max r(x), \\ \text{s. t. } x \in A. \end{cases} \tag{11}$$

Obviously, (11) is a fuzzy programming problem, and all of the alternatives cannot be as the recognized optimal solution. So what we have to do is to determine the fuzzy satisfaction solutions of (11). Through analysis, we can see the difference between the highest and the lowest return rate is $0.30 - 0.14 = 0.16$; as a result, the measure of similar return rates should not be too big. Below, combined with the discussion of part 3, we will determine the fuzzy satisfaction solutions under some different situations of ε.

From Theorem 3.2 and Table 1, the different threshold levels λ_i $(i = 1, 2, \cdots, 10)$ of $A(x_1), A(x_2), \cdots, A(x_{10})$ are 0.20, 0.24, 0.37, 0.5, 0.58, 0.64, 0.72, 0.76, 0.82, 1. Its local optimal solution M_{λ_i} and local satisfied solutions $M_{\lambda_i}(\varepsilon)$ corresponding to model (3) are given in Table 2.

TABLE 2. The local fuzzy satisfaction solutions under some different accuracies

λ_i	$\max\{r(x)\|x \in A_{\lambda_i}\}$	M_{λ_i}	$M_{\lambda_i}(\varepsilon)$			
			$\varepsilon = 0.01$	$\varepsilon = 0.03$	$\varepsilon = 0.04$	$\varepsilon = 0.06$
0.2	0.3	$\{x_9\}$	$\{x_9\}$	$\{x_2, x_4, x_9\}$	$\{x_2, x_4, x_7, x_9\}$	$\{x_2, x_4, x_6, x_7, x_9, x_{10}\}$
0.24	0.3	$\{x_9\}$	$\{x_9\}$	$\{x_2, x_4, x_9\}$	$\{x_2, x_4, x_7, x_9\}$	$\{x_2, x_4, x_6, x_7, x_9, x_{10}\}$
0.37	0.3	$\{x_9\}$	$\{x_9\}$	$\{x_2, x_9\}$	$\{x_2, x_7, x_9\}$	$\{x_2, x_6, x_7, x_9, x_{10}\}$
0.5	0.3	$\{x_9\}$	$\{x_9\}$	$\{x_2, x_9\}$	$\{x_2, x_7, x_9\}$	$\{x_2, x_7, x_9, x_{10}\}$
0.58	0.27	$\{x_2\}$	$\{x_2, x_7\}$	$\{x_2, x_7, x_{10}\}$	$\{x_2, x_7, x_{10}\}$	$\{x_2, x_7, x_{10}\}$
0.64	0.27	$\{x_2\}$	$\{x_2, x_7\}$	$\{x_2, x_7\}$	$\{x_2, x_7\}$	$\{x_2, x_7\}$
0.72	0.27	$\{x_2\}$	$\{x_2, x_7\}$	$\{x_2, x_7\}$	$\{x_2, x_7\}$	$\{x_2, x_7\}$
0.76	0.27	$\{x_2\}$	$\{x_2, x_7\}$	$\{x_2, x_7\}$	$\{x_2, x_7\}$	$\{x_2, x_7\}$
0.82	0.27	$\{x_2\}$	$\{x_2, x_7\}$	$\{x_2, x_7\}$	$\{x_2, x_7\}$	$\{x_2, x_7\}$
1	0.26	$\{x_7\}$	$\{x_7\}$	$\{x_7\}$	$\{x_7\}$	$\{x_7\}$
$M(\varepsilon)$			$\{x_2, x_7, x_9\}$	$\{x_2, x_4, x_7, x_9, x_{10}\}$	$\{x_2, x_4, x_7, x_9, x_{10}\}$	$\{x_2, x_4, x_6, x_7, x_9, x_{10}\}$

From Table 2, as what we have discussed above, if the value of ε is bigger in a certain degree, it will lead the range of the solutions increasing (sometimes the same). Below, in Table 3, we will analyze the effect coverage $B(x, L(\lambda), \varepsilon)$ of different satisfaction solutions.

Combined with the analysis process and calculation results in Table 3, we can know: 1) The satisfactory solutions are closely related to the choice of satisfaction threshold ε.

Table 3. The satisfaction solutions under some different decision consciousness

ε	$x_i \in M(\varepsilon)$	$\underline{\alpha}(x_i,\varepsilon)$	$\overline{\alpha}(x_i,\varepsilon)$	$B(x,L(\lambda),\varepsilon)$		
				$L(\lambda)=1$	$L(\lambda)=1.5\lambda^{0.5}$	$L(\lambda)=0.5\lambda^{-0.5}$
$\varepsilon=0.01$	x_2	0.5	0.82	0.32	0.3889	0.1984
	x_7	0.5	1	0.5	0.6464	0.2929
	x_9	0.00	0.5	0.5	0.3536	0.7071
$\varepsilon=0.03$	x_2	0.00	0.82	0.82	0.7425	0.9055
	x_4	0.00	0.24	0.24	0.1176	0.4899
	x_7	0.5	1	0.5	0.6464	0.2929
	x_9	0.00	0.5	0.5	0.3536	0.7071
	x_{10}	0.5	0.58	0.08	0.0882	0.0545
$\varepsilon=0.04$	x_2	0.00	0.82	0.82	0.7425	0.9055
	x_4	0.00	0.24	0.24	0.1176	0.4899
	x_7	0.00	1	1	1	1
	x_9	0.00	0.5	0.5	0.3536	0.7071
	x_{10}	0.5	0.58	0.08	0.0882	0.0545
$\varepsilon=0.06$	x_2	0.00	0.82	0.82	0.7425	0.9055
	x_4	0.00	0.24	0.24	0.1176	0.4899
	x_6	0.00	0.37	0.37	0.2251	0.6083
	x_7	0.00	1	1	1	1
	x_9	0.00	0.5	0.5	0.3536	0.7071
	x_{10}	0.00	0.58	0.58	0.4417	0.7616

To some extent, ε can reflect fuzziness of the decision. The local satisfaction solutions are often different under different ε. However, the overall may be same. And the number of overall satisfaction solutions will be non-decreasing along with the increase of ε. 2) The satisfactory solutions are closely related to the choice of utility frequency λ. And also the utility coverage $B(x, L(\lambda), \varepsilon)$ can directly reflect the degree of satisfied solutions conforming to the optimal. With fuzzy processing consciousness being different, the decision result is different. For example, when $\varepsilon = 0.01$, if $L(\lambda) = 1$, the degree of $\{x_7\}$ and $\{x_9\}$ conforming to the optimal is the biggest, and their coincidence degree is 0.5; if $L(\lambda) = 1.5\lambda^{0.5}$, $\{x_7\}$ is the biggest, and its coincidence degree is 0.6464; if $L(\lambda) = 0.5\lambda^{-0.5}$, $\{x_9\}$ is the biggest, and its coincidence degree is 0.7071. So how to select the final decision, the decision-maker should be based on the specific situation, and choose an appropriate function. In this way, it could better reflect the actual situation and decision consciousness.

Compared with the fuzzy optimal value of original problem, proposing this method makes the choice of the decision results diversity, rather than a single one in theory. The solutions not only characterize the fuzziness of programming problem, but also have more practical meaning. In practice, because of the different consciousness of decision-makers, the final selection will be different. However, the fuzzy satisfaction solutions proposed here can be used as a reasonable reference of the decision result.

5. **Conclusion.** It is a basic way to realize scientific management by establishing an evaluation method for fuzzy decision according to different membership states and decision preferences. In this paper, for solving fuzzy programming problems, we analyzed the features and shortcomings of the existing method systematically. Then we discussed the judgment method of fuzzy satisfaction solution based on utility frequency from the essence of fuzzy decision. Finally we analyzed the effectiveness of this method in combination with

a concrete case. Although this article is just in view of the form of the fuzzy programming solution, this method has good structure and can effectively make fuzzy consciousness into the decision-making process through quantification method. The universality of this method is also the basis of some fuzzy decision problems to determine the optimal value. Thus the work of this paper enriches the existing theory and method of fuzzy decision. It also has a certain reference significance for further establishing decision methods under complex environments.

Although the calculation model (8) proposed in the paper can describe the fuzzy satisfaction solutions, it is just a theoretical expression. At present we discuss the solving model in the discrete domain, but continuous programming problem is more common in practice, so we will discuss the solving method combined with model (8) in the continuous domain next.

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